PHYS 454
HANDOUT 1-*Basic Concepts*

1. In the theory of relativity the relation between energy-momentum is $E^2 = c^2p^2 + m^2c^4$. Writhe down the one-dimensional free wave equation for this case.

2. A statistical quantity A has only two possible values $a_1 = 4$ and $a_2 = 8$, which appear with probabilities $P_1 = 1/4$ and $P_2 = 3/4$ respectively. Calculate the uncertainty ΔA of this quantity.

3. Why we have chosen the quantity $\left|\psi\left(x,t\right)\right|^2$ as the probability density and not the quantity $\left|\psi\left(x,t\right)\right|$?

4. Show that if two wave-functions ψ and ψ' differ by a constant phase then they represent the same physical state. What happens if the phase depends on position?

5. Calculate the average value $\langle x\rangle$ and the position uncertainty Δx for a particle the state of which, is described, at a certain moment, from the function: $\psi\left(x\right) = Ne^{-\lambda x^2/2}$. It is given that

$$\int_{-\infty}^{+\infty} x^{2n}e^{-\lambda x^2}dx = \frac{1\cdot 3\cdots\left(2n-1\right)}{\left(2\lambda\right)^n}\sqrt{\frac{\pi}{\lambda}}$$

6. Is it possible the function $\psi\left(x\right) = Ne^{\lambda x}$, to describe the wavefunction of a particle? Explain.

7. The state of a particle is given by

$$\psi = N\left(\psi_1 + 2\psi_2 + \psi_3\right)$$

where ψ_1, ψ_2, ψ_3 are normalized eigenfunctions of some physical quantity (observable) A with eigenvalues $a_1 = -1,\ a_2 = 0,\ a_3 = 1$ respectively. Find the normalization constant N and calculate the average value $\langle A\rangle$ and the uncertainty ΔA of this quantity.

8. Write down the form of the quantum-mechanical operator of the angular momentum component along *z*.

9. What is the average position of a particle which is described from the wave function $\psi\left(x\right) = Nxe^{-\lambda x^2 + ikx}$?

10. At time *t*=0 a particle is represented by the wave function

$$\Psi(x,0)=\begin{cases} A\dfrac{x}{a} & 0\le x\le a \\ A\dfrac{(b-x)}{(b-a)} & a\le x\le b \\ 0 & otherwise \end{cases}$$

where A is an unknown constant and a and b are known constants. A) Normalize the wavefunction. B) What is the probability of finding the particle to the left of a?

11. Show with the help of the operators $A = x$ and $B=d/dx$ that $AB \neq BA$.

12. Somebody insists that the time-dependent wavefunction of the electron in the hydrogen atom is given by

$$\psi(\mathbf{r},t)=e^{-r/a_0}\cos\omega t$$

Is this possible? Explain.

13. Why the quantity $|\psi(x,t)|^2$ preserves the probability?

14. Derive Schrödinger's equation.

15. Compare the de Broglie wavelength of an electron, which moves at 5×10^3 m/s with the de Broglie wavelength of a car of mass 1000 kg moving at a speed of 36 km/h.

16. Determine where a particle is most likely to be found whose wave function is given by $\Psi(x)=(1+ix)/(1+ix^2)$