King Saud University

College of Science

Department of Mathematics

151 MATH EXERCISES

(6)

BOOLEAN ALGEBRAS

&

LOGIC GATES

&

MINIMIZATION OF CIRCUITS

Malek Zein AL-Abidin

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BOOLEAN ALGEBRAS

Introduction

Boolean algebra provides the operations and the rules for working with the set {0, 1}. Electronic and optical switches can be studied using this set and the rules of Boolean algebra. The three operations in Boolean algebra that we will use most are complementation, the Boolean sum, and the Boolean product. The **complement** of an element, denoted with a bar, is defined by $\bar{0} = 1$ and $\bar{1} = 0$. The Boolean sum, denoted by + or by *OR*, has the following values:

$$1 + 1 = 1, \quad 1 + 0 = 1, \quad 0 + 1 = 1, \quad 0 + 0 = 0.$$

The Boolean product, denoted by · or by *AND*, has the following values:

$$1 \cdot 1 = 1, \quad 1 \cdot 0 = 0, \quad 0 \cdot 1 = 0, \quad 0 \cdot 0 = 0 .$$

When there is no danger of confusion, the symbol · can be deleted, just as in writing algebraic products. Unless parentheses are used, the rules of precedence for Boolean operators are: first, all complements are computed, followed by all Boolean products, followed by all Boolean sums. This is illustrated in Example 1.

EXAMPLE 1 Find the value of $1 \cdot 0 + \overline{(0 + 1)}$.

Solution: Using the definitions of complementation, the Boolean sum, and the Boolean product, it follows that $1 \cdot 0 + \overline{(0 + 1)} = 0 + \bar{1} = 0 + 0 = 0$

Duality

The **dual** of a Boolean expression is obtained by interchanging Boolean sums and Boolean products and interchanging 0s and 1s. $E \underbrace{\longleftrightarrow}_{1\leftrightarrow 0 \ \wedge\ . \leftrightarrow +} E^d$

EXAMPLE 2 Find the duals of $x(y + 0)$ and $x \cdot 1 + (y + z)$.

Solution: Interchanging · signs and + signs and interchanging 0s and 1s in these expressions produces their duals.

The duals are $x + (y \cdot 1)$ and $(x + 0)(yz)$, respectively.

Boolean Identities.

Identity	*Name*
$\overline{\overline{x}} = x$	Law of the double complement
$x + x = x$ $x \cdot x = x$	Idempotent laws
$x + 0 = x$ $x \cdot 1 = x$	Identity laws
$x + 1 = 1$ $x \cdot 0 = 0$	Domination laws
$x + y = y + x$ $xy = yx$	Commutative laws
$x + (y + z) = (x + y) + z$ $x(yz) = (xy)z$	Associative laws
$x + yz = (x + y)(x + z)$ $x(y + z) = xy + xz$	Distributive laws
$\overline{(xy)} = \overline{x} + \overline{y}$ $\overline{(x + y)} = \overline{x}\,\overline{y}$	De Morgan's laws
$x + xy = x$ $x(x + y) = x$	Absorption laws
$x + \overline{x} = 1$	Unit property
$x\overline{x} = 0$	Zero property

EXAMPLE 2 Show that the distributive law $x(y + z) = xy + xz$ is valid.

x	y	z	$y + z$	xy	xz	$x(y + z)$	$xy + xz$
1	1	1	1	1	1	1	1
1	1	0	1	1	0	1	1
1	0	1	1	0	1	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

EXERCISES

Q1. Let $E = (xy)' + x' + y = 1$. Find E^d , (E^d or $\bar{E}$ is the **Duality** of E)

Q2. Let B is a Boolean Algebra and $x, y \in B$. Show that $x + y = xy + xy' + x'y$ is valid.

Q3. Let $f(x, y, z) = x(y + z') + y'$. Find $CSP(f)$ (sum-of-products expansion) and $CPS(f)$ (product-of-sums expansion) ?

Q4. Let $f(x,y,z) = x'z + yz'$.   Find $CSP(f)$ (sum-of-products expansion) and $CPS(f)$ (product-of-sums expansion) ?

Q5. Let $f(x,y,z) = (x' + z)(x + y)$. Find $CSP(f)$ (sum-of-products expansion) and $CPS(f)$ (product-of-sums expansion) ?

Q6. Let $f(x,y,z) = (x' + y)' \, (x + y + z)$

(i) Use NAND gates to construct circuits with this output.

(ii) Use NOR gates to construct circuits with this output.

Q7. Let $f(x, y, z) = (x + y)(x' + yz')$

(i) Find $CSP(f)$ and $CPS(f)$.

(ii) Find $MSP(f)$ and $MPS(f)$.

(iii) Construct a minimal circuit using (AND–OR) gates, with $f(x, y, z)$ output.

Q8. Let $f(x,y,z) = xy' + xz + y'z' + x'yz'$

(i) Find the Karnaugh -map for $f(x,y,z)$.

(ii) Find $MSP(f)$ and $MPS(f)$.

(iii) Construct a minimal circuit using (AND–OR) gates, with $f(x,y,z)$ output.

(iv) Use NAND gates to construct circuits with $f(x,y,z)$ output.

(v) Use NOR gates to construct circuits with $f(x,y,z)$ output.

Q9. Let $g(x,y,z) = xy'z + x'yz + x'y'z' + y'z$

(i) Find the Karnaugh -map for $g(x,y,z)$.

(ii) Find $MSP(g)$ and $MPS(g)$.

(iii) Construct a minimal circuit using (AND-OR) gates, with $g(x,y,z)$ output.

(iv) Use NAND gates to construct circuits with $g(x,y,z)$ output.

(v) Use NOR gates to construct circuits with $g(x,y,z)$ output.

Q10. Let

	yz	yz'	$y'z'$	$y'z$
x				1
x'	1	1		1

Be the Karnaugh -map for $g(x, y, z)$.

(i)Find $MSP(g)$ and $MPS(g)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $g(x, y, z)$ output.

(iii)Use NAND gates to construct circuits with $g(x, y, z)$ output.

(iv)Use NOR gates to construct circuits with $g(x, y, z)$ output.

Q11. Let

	yz	yz'	$y'z'$	$y'z$
x	1			1
x'	1	1		1

Be the Karnaugh -map for $g(x, y, z)$.

(i)Find $MSP(g)$ and $MPS(g)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $g(x, y, z)$ output.

(iii)Use NAND gates to construct circuits with $g(x, y, z)$ output.

(iv)Use NOR gates to construct circuits with $g(x, y, z)$ output.

Q12. Let

	zw	zw'	$z'w'$	$z'w$
xy		1		1
xy'	1	1	1	1
$x'y'$	1		1	
$x'y$				

Be the Karnaugh -map for $f(x, y, z)$.

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x, y, z)$ output.

(iii)Use NAND gates to construct circuits with $f(x, y, z)$ output.

(iv)Use NOR gates to construct circuits with $f(x, y, z)$ output.

Q13. Let

	zw	zw'	$z'w'$	$z'w$
xy		1		1
xy'	1	1	1	1
$x'y'$	1		1	
$x'y$				

Be the Karnaugh -map for $f(x,y,z)$.

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x,y,z)$ output.

(iii)Use NAND gates to construct circuits with $f(x,y,z)$ output.

(iv)Use NOR gates to construct circuits with $f(x,y,z)$

Q14. Let

	zw	zw'	$z'w'$	$z'w$
xy		1	1	
xy'	1	1	1	
$x'y'$		1	1	
$x'y$	1	1	1	1

Be the Karnaugh -map for $f(x,y,z)$.

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x,y,z)$ output.

(iii)Use NAND gates to construct circuits with $f(x,y,z)$ output.

(iv)Use NOR gates to construct circuits with $f(x,y,z)$

Q15. Let

	zw	zw'	$z'w'$	$z'w$
xy	1	1	1	1
xy'		1	1	
$x'y'$		1	1	
$x'y$			1	

Be the Karnaugh -map for $f(x, y, z)$.

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x, y, z)$ output.

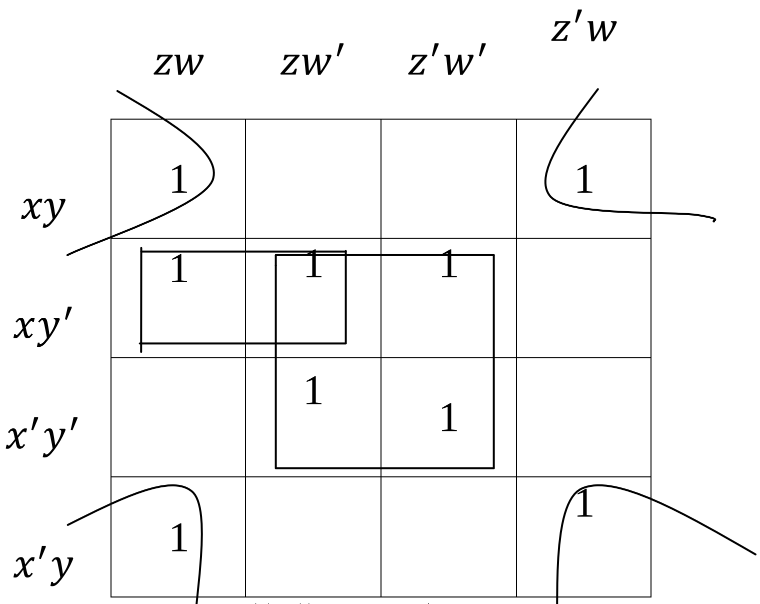
(iii)Use NAND gates to construct circuits with $f(x, y, z)$ output.

(iv)Use NOR gates to construct circuits with $f(x, y, z)$

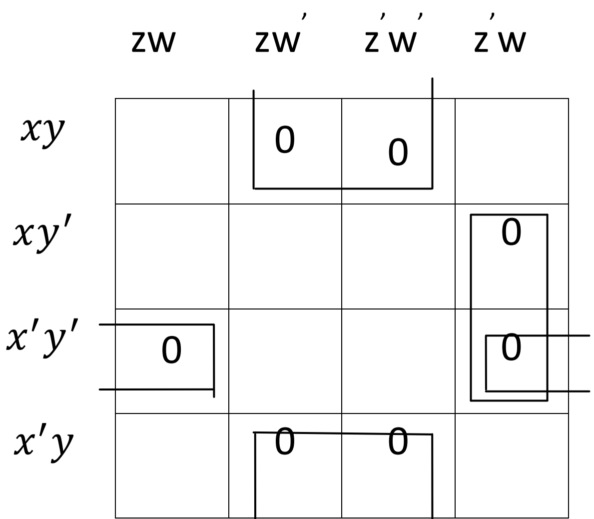
Q16 .

	zw	zw'	$z'w'$	$z'w$
xy	1	0	1	1
xy'	1	0	0	1
$x'y'$	0	0	0	0
$x'y$	1	1	0	0

Q17.



شكل كارنو للدالة f



متمم كارنو للدالة f'

ASSIGMENT

Q1. Find CSP(sum-of-products expansion) and CPS (product-of-sums expansion) for the following Boolean functions

1- $f(x,y,z) = (x+z)(x'+y)'$

2- $f(x,y,z) = z(y+x') + y'$.

3- $f(x,y,z) = (x+yz)(y+xz')$.

4- $g(x,y,z) = xy + z$.

5- $f(x,y,z) = xy' + z$.

6- $f(x,y,z) = (x' + y)'(y z')'$.

7- $g(x,y,z) = xz + y'z'$.

8- $f(x,y,z) = 1$ when $x = y$.

Q2. Let $h(x,y,z) = xy' + xyz + y'z' + x'yz'$

(i) Find the Karnaugh -map for $h(x,y,z)$.

(ii) Find $MSP(h)$ and $MPS(h)$.

(iii) Construct a minimal circuit using AND gates, OR gates, with $h(x,y,z)$ output.

(iv) Use NAND gates to construct circuits with $h(x,y,z)$ output.

(v) Use NOR gates to construct circuits with $h(x,y,z)$ output.

Q3. Let $f(x,y,z) = xy' + xz + yz + x'yz'$

(i) Find the Karnaugh -map for $f(x,y,z)$.

(ii) Find $MSP(f)$ and $MPS(f)$.

(iii) Construct a minimal circuit using AND gates, OR gates, with $f(x,y,z)$ output.

(iv) Use NAND gates to construct circuits with $f(x,y,z)$ output.

(v) Use NOR gates to construct circuits with $f(x,y,z)$ output.

Q4. Let the Karnaugh -maps for $f(x,y,z)$ given as bellow from exercise 1-15

(i)Find $MSP(f)$ and $MPS(f)$.

(ii)Construct a minimal circuit using (AND-OR) gates, with $f(x,y,z)$ output.

(iii)Use NAND gates to construct circuits with $f(x,y,z)$ output.

(iv)Use NOR gates to construct circuits with $f(x,y,z)$ output.

1-

	zw	zw'	$z'w'$	$z'w$
xy	1			1
xy'			1	1
$x'y'$			1	1
$x'y$	1		1	1

2-

	zw	zw'	$z'w'$	$z'w$
xy	1	1		1
xy'	1	1		1
$x'y'$				
$x'y$	1	1	1	

3-

	zw	zw'	$z'w'$	$z'w$
xy	1			1
xy'		1		
$x'y'$				
$x'y$	1		1	1

4-

	zw	zw'	$z'w'$	$z'w$
xy		1		
xy'				
$x'y'$	1			1
$x'y$	1	1	1	1

5-

	zw	zw'	$z'w'$	$z'w$
xy	1	1		1
xy'		1	1	
$x'y'$	1			1
$x'y$	1			1

6-

	zw	zw'	$z'w'$	$z'w$
xy	|	|	|	|
xy'	|	|		|
$x'y'$	|			
$x'y$	|			

7-

	zw	zw'	$z'w'$	$z'w$
xy	|	|	|	|
xy'	|			
$x'y'$				|
$x'y$	|	|	|	|

8-

	zw	zw'	$z'w'$	$z'w$
xy	|			|
xy'		|	|	|
$x'y'$		|	|	|
$x'y$	|			|

9-

	zw	zw'	z'w'	z'w
xy			1	
xy'		1	1	
$x'y'$		1	1	
$x'y$	1	1	1	1

10-

	zw	zw'	z'w'	z'w
xy		1	1	1
xy'		1	1	1
$x'y'$				1
$x'y$				

11-

	zw	zw'	z'w'	z'w
xy	1	1	1	1
xy'	1			
$x'y'$	1			
$x'y$	1		1	1

12-

	zw	zw'	$z'w'$	$z'w$
xy	1	1		1
xy'		1		
$x'y'$				
$x'y$	1		1	1

13-

	zw	zw'	$z'w'$	$z'w$
xy	1		1	1
xy'	1			1
$x'y'$				
$x'y$	1	1		

14-

	zw	zw'	$z'w'$	$z'w$
xy	1		1	1
xy'	1		1	1
$x'y'$			1	
$x'y$			1	1

15-

	zw	zw'	$z'w'$	$z'w$
xy	1	1	1	1
xy'	1	1	1	
$x'y'$		1	1	
$x'y$			1	