Q[1]: (321-Mid1)

Problem 3

The transfer function of a digital system is given by $H(z) = \dfrac{1-4z^{-2}}{(1+0.2z^{-1})(1-0.5z^{-1})}$.

a) Find the poles and zeros of this system, sketch the pole-zero plot,

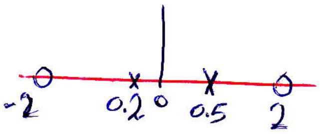
b) Based on the information given so far, list all the possibilities of the ROC. For each possibility, state: i) if DTFT exists or not, and ii) whether the system is left/right sided.

c) Suppose we tell you that the above system is causal, would this change your answers in b? Explain.

d) Assuming it exists, find the DTFT $H(e^{j\omega})$ of this system (note: no need to simplify). Then find its magnitude at $w = 0, w = \pi$

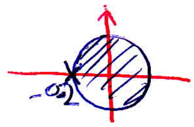
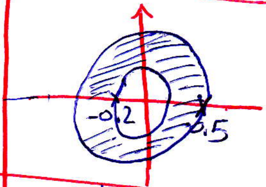
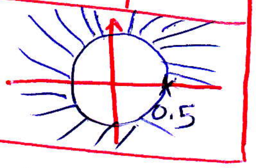
(a) poles at $z = -0.2$, $z = 0.5$

zeros at $z = \pm 2$



(Solve using Algebra or calculator!)

(b)

1	ROC → $\lvert z\rvert < 0.2$ * DTFT Does Not exist (unit circle not included) Not Stable, anti-causal, left-sided	
2	ROC → $\lvert 0.2\rvert < \lvert z\rvert < 0.5$ * DTFT Does Not exist. Not Stable, Not-causal, two-sided	
3	ROC → $\lvert z\rvert > 0.5$ * DTFT exists! * Stable, Causal, right sided	

(c) from table above, ROC → $|z| > 0.5$

(D) Substitute z by $(e^{j\omega}) \rightarrow H(e^{j\omega}) = \dfrac{1-4(e^{-j2\omega})}{(1+0.2e^{-j\omega})(1-0.5e^{-j\omega})}$

at $w=0$ → $\dfrac{1-4(e^{0})}{(1+0.2(e^{0}))(1-0.5(e^{0}))} = \dfrac{1}{(1+0.2)(0.5)} = -5$

at $w=\pi$ → $\dfrac{1-4(e^{-2j\pi})}{(1+0.2e^{-j\pi})(1-0.5e^{-j\pi})} = \dfrac{1-4}{(1-0.2)(1+0.5)} = -2.5$

Q[1]: (322-Mid1)

Problem 3

The transfer function of DT LTI system is given by

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-0.5z^{-1}}{(1-0.2z^{-1})(1+0.7z^{-1})}$$

a. Find the poles and zeros and sketch the pole-zero plot.

b. What are all the possibilities for the ROC? For each ROC state whether the system is stable and/or causal and explain why.

c. Assuming the system is causal, find the output $y[n]$ if the input is $x[n] = \left(\frac{1}{2}\right)^n u[n]$

c from the tabe of z-transform pairs

$$\boxed{\alpha^n \mu[n] \longleftrightarrow \frac{1}{1-\alpha z^{-1}}, |z| > |\alpha|}$$

hence, → $X[z] = \frac{1}{1-(0.5z^{-1})}$

→ $H(z) = \frac{Y(z)}{X(z)}$ → $Y(z) = H(z) \cdot X(z) = \frac{(1-\cancel{0.5z^{-1}})}{(1-0.2z^{-1})(1+0.7z^{-1})} \cdot \frac{1}{(1-\cancel{0.5z^{-1}})}$

$$\Rightarrow \boxed{Y(z) = \frac{1}{(1-0.2z^{-1})(1+0.7z^{-1})}}$$

→ $Y(z) = \frac{A}{1-0.2z^{-1}} + \frac{B}{1+0.7z^{-1}} = \frac{2/9}{(1-0.2z^{-1})} + \frac{7/9}{(1+0.7z^{-1})}$

for A → $A = (1-0.2z^{-1}) . Y(z)\Big|_{z=0.2} = \frac{(1-\cancel{0.2z^{-1}})}{(1-\cancel{0.2z^{-1}})(1+0.7z^{-1})} = \frac{1}{(1+0.7(0.2)^{-1})} = \frac{2}{9}$

for B → $B = (1+0.7z^{-1}) . Y(z)\Big|_{z=-0.7} = \frac{1}{(1-0.2(-0.7)^{-1})} = \frac{7}{9}$

Using the property above, → $y[n] = \frac{2}{9}(0.2)^n \mu[n] + \frac{7}{9}(-0.7)^n \mu[n]$