PRODUCTS OF TWO VECTORS

THE SCALAR PRODUCT OF TWO VECTORS

- The **scalar (**or **dot**) **product** of two vectors is defined by the following relation:

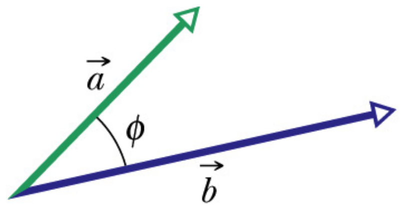
$$\vec{a} \cdot \vec{b} = ab\cos\varphi$$

- In unit-vector rotation it is defined as follows:

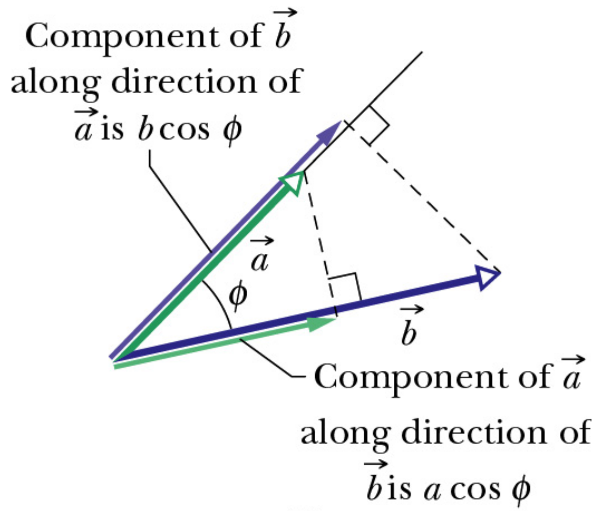
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

- vectors is commutative:

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$



(a)



(b)

THE VECTOR PRODUCT OF TWO VECTORS

- The **vector** (or **product**) of two vectors is written as:

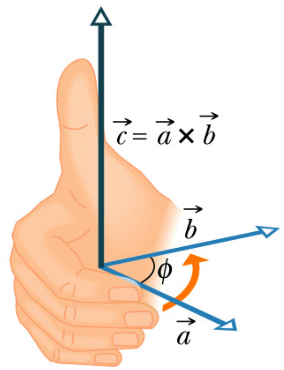
$$\vec{a} \times \vec{b}$$

- The magnitude of this vector is given by:

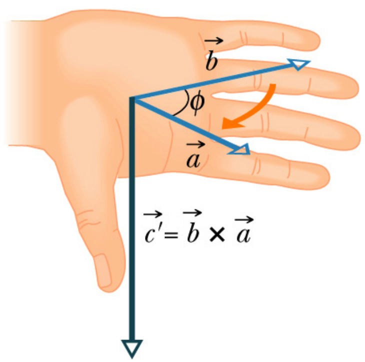
$$c = ab\sin\varphi$$

- In unit-vector notation the vector product is given by:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



(a)



(b)

PROPERTIES OF SCALAR PRODUCT OF TWO VECTORS

- In the special case where **a** = **b** we have:

$$\mathbf{a} \cdot \mathbf{a} = a_x a_x + a_y a_y + a_z a_z = \|\mathbf{a}\|^2$$

- From which we can write:

$$\|\mathbf{a}\| = \sqrt{\mathbf{a} \cdot \mathbf{a}}$$

PROPERTIES OF SCALAR PRODUCT OF TWO VECTORS

- For three vectors **u**, **v** and **w** we have the following properties:

a) $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$ (symmetry property)

b) $\mathbf{u} \cdot \left(\mathbf{v} + \mathbf{w}\right) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$ (distributive property)

c) $k\left(\mathbf{u} \cdot \mathbf{v}\right) = \left(k\mathbf{u}\right) \cdot \mathbf{v}$ (homogeneity property)

d) $\mathbf{v} \cdot \mathbf{v} \geq 0$ and $\mathbf{v} \cdot \mathbf{v} = 0$ if and only if $\mathbf{v} = 0$ (positivity property)

CAUCHY-SCHWARTZ INEQUALITY AND SCALAR PRODUCT-a

- From the definition of the scalar product it is easy to see that:

$$\theta = \cos^{-1}\left(\frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\cdot\|\mathbf{v}\|}\right)$$

- It is straightforward that

$$-1 \leq \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\cdot\|\mathbf{v}\|} \leq 1$$

CAUCHY-SCHWARTZ INEQUALITY AND SCALAR PRODUCT-b

- From the above relations it is easy to derive the famous Cauch-Schwartz inequality:

$$\left|\mathbf{u}\cdot\mathbf{v}\right| \leq \left\|\mathbf{u}\right\|\cdot\left\|\mathbf{v}\right\|$$