



CSC 220: Computer Organization

Unit 6 **Combinational Circuits-2**

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Overview

- Enabling
- Decoders
- Encoders
- Multiplexers
- DeMultiplexers

Chapter-3

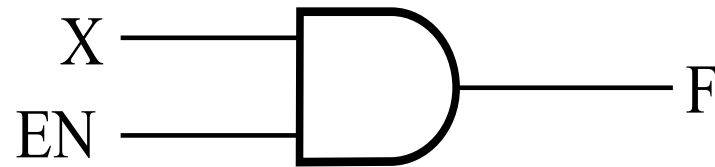
M. Morris Mano, Charles R. Kime and Tom Martin, **Logic and Computer Design Fundamentals**, Global (5th) Edition, Pearson Education Limited, 2016. ISBN: 9781292096124

Enabling

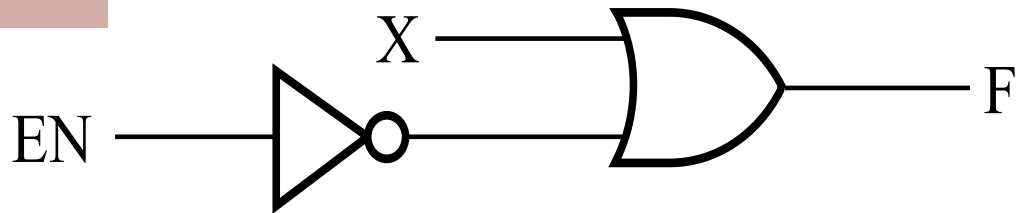
- **Enabling** permits an input signal to pass through to an output
- **Disabling** blocks an input signal from passing through to an output, replacing it with a *fixed value*
- The value on the output when it is disabled can be 0, 1 or Hi-Z

Note: it is possible to modify:

- EN = 0 enables X to reach the output
- EN = 1 blocks X.



(a) When disabled, 0 output



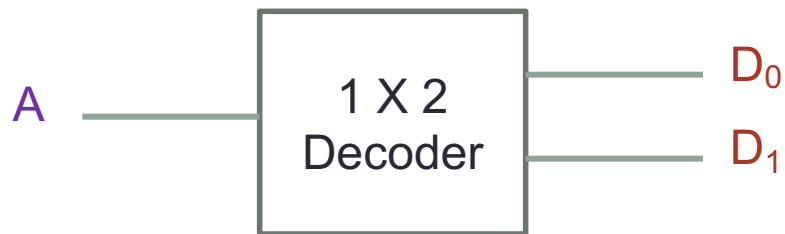
(b) When disabled, 1 output

Decoding

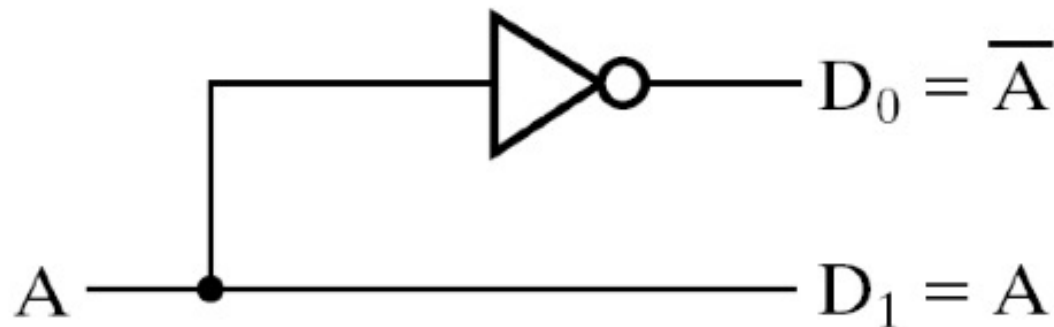
- A *decoder* is a combinational circuit that converts binary information from n input lines to (max. of) 2^n output lines.
- Generate 2^n (or fewer) minterms of n input variables.
 - it **activates** one and only one of its 2^n outputs based on the input
 - with all other outputs **deactivated**.

1-to-2-Line Decoder

- For $n=1$ input line and $m=2^n=2$ output lines

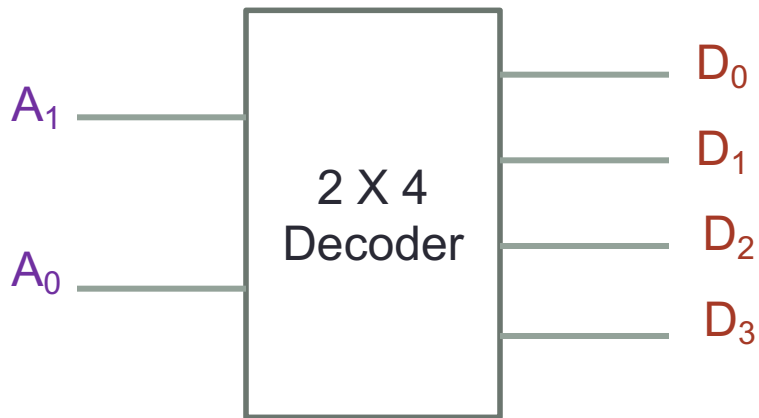


| A | D_0 | D_1 |
|---|-------|-------|
| 0 | 1 | 0 |
| 1 | 0 | 1 |

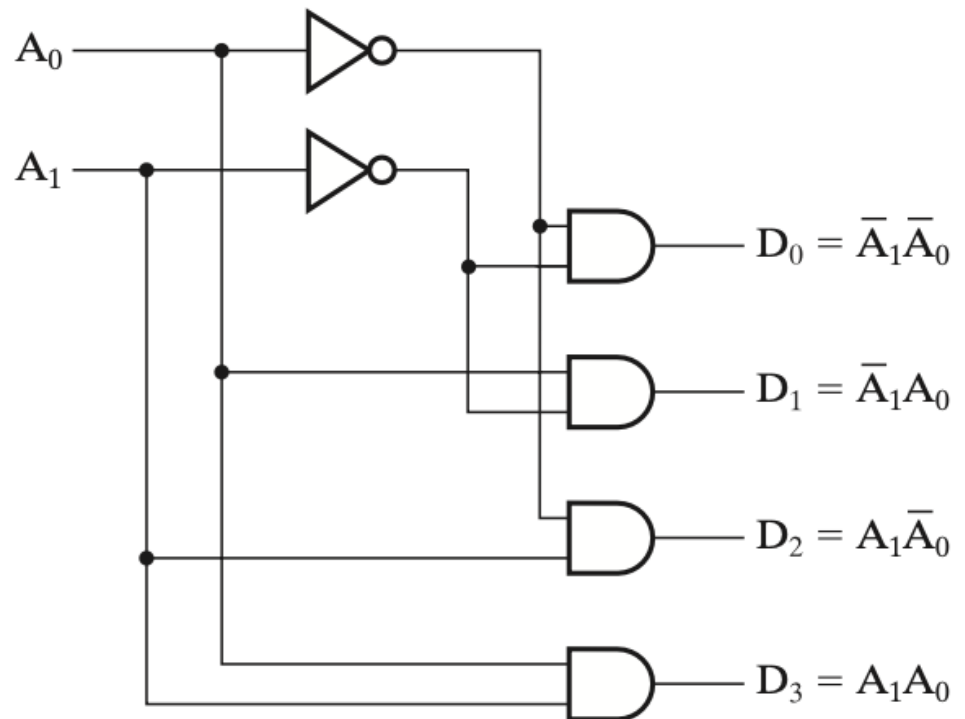


2-to-4-Line Decoder

- For n inputs = 2 and $m = 2^n = 4$ output lines



| A_1 | A_0 | D_0 | D_1 | D_2 | D_3 |
|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 |



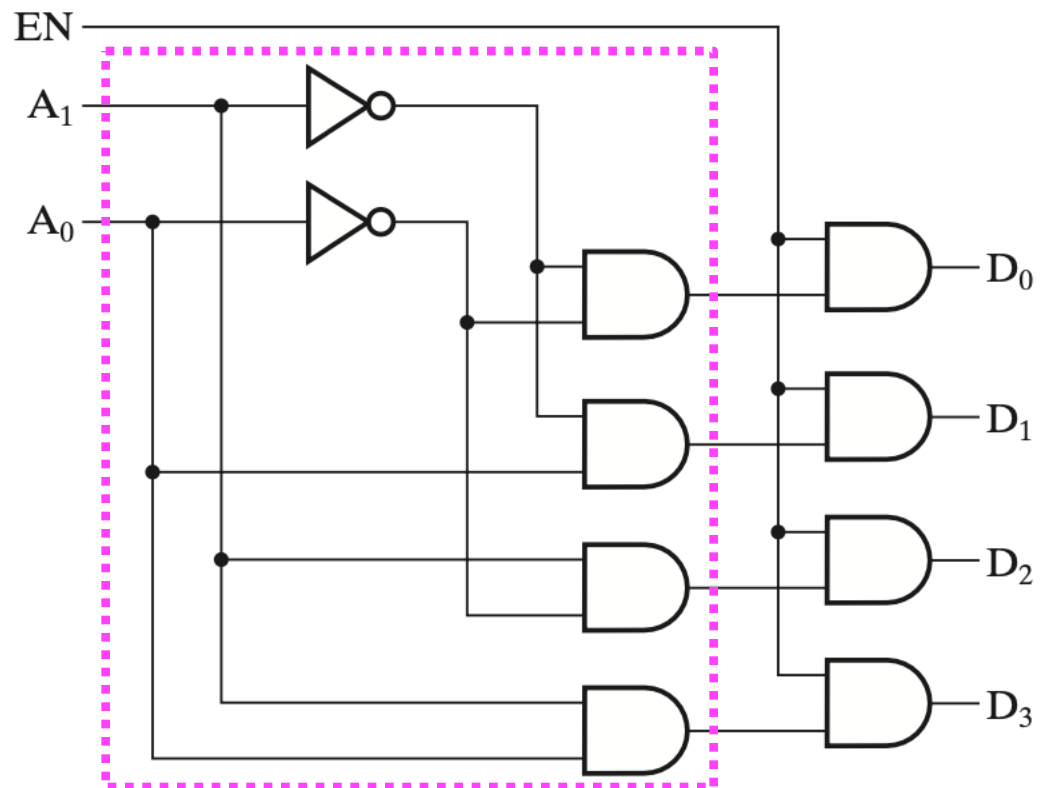
Decoder and Enabling Combinations

In general, attach m-enabling circuits to the outputs
See truth table below for function

- Note use of X's to denote both 0 and 1
- Combination containing two X's represent four binary combinations

| EN | A ₁ | A ₀ | D ₀ | D ₁ | D ₂ | D ₃ |
|----|----------------|----------------|----------------|----------------|----------------|----------------|
| 0 | X | X | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 1 |

(a)



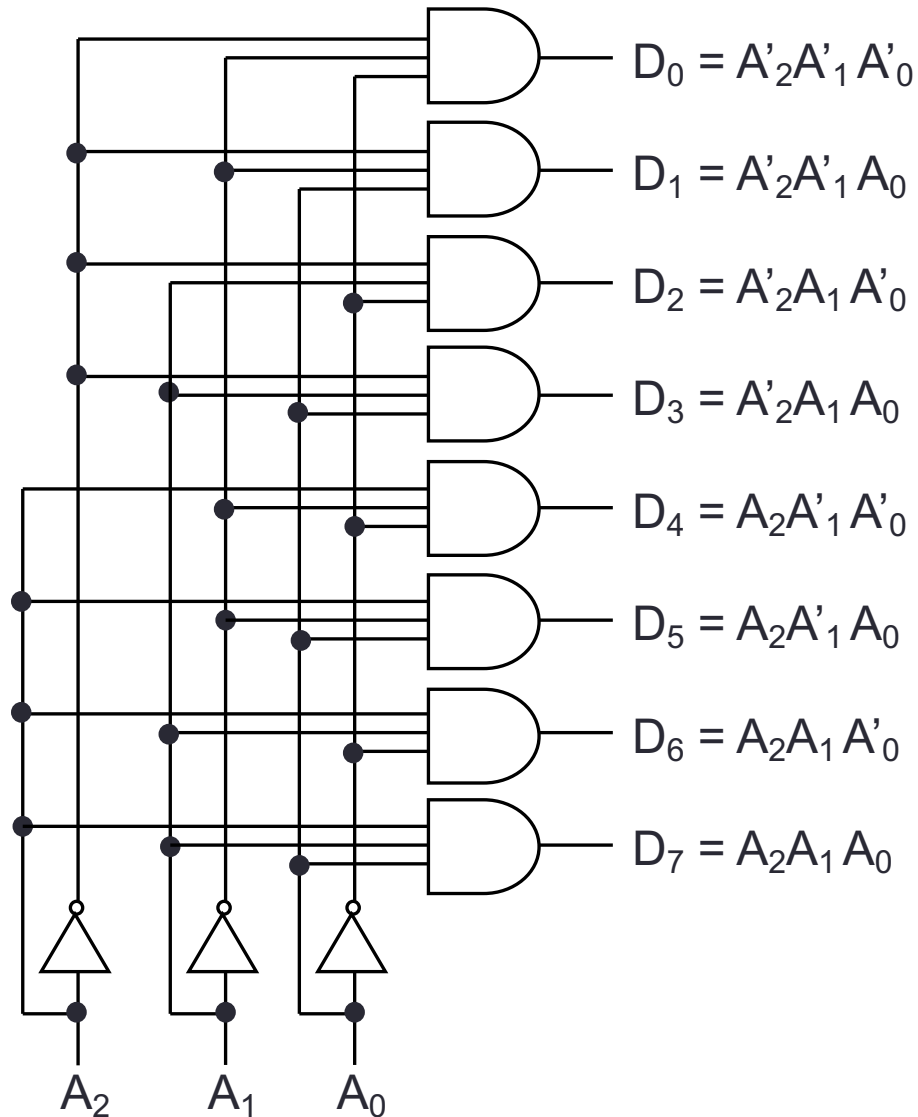
(b)

3-to-8-Line Decoder

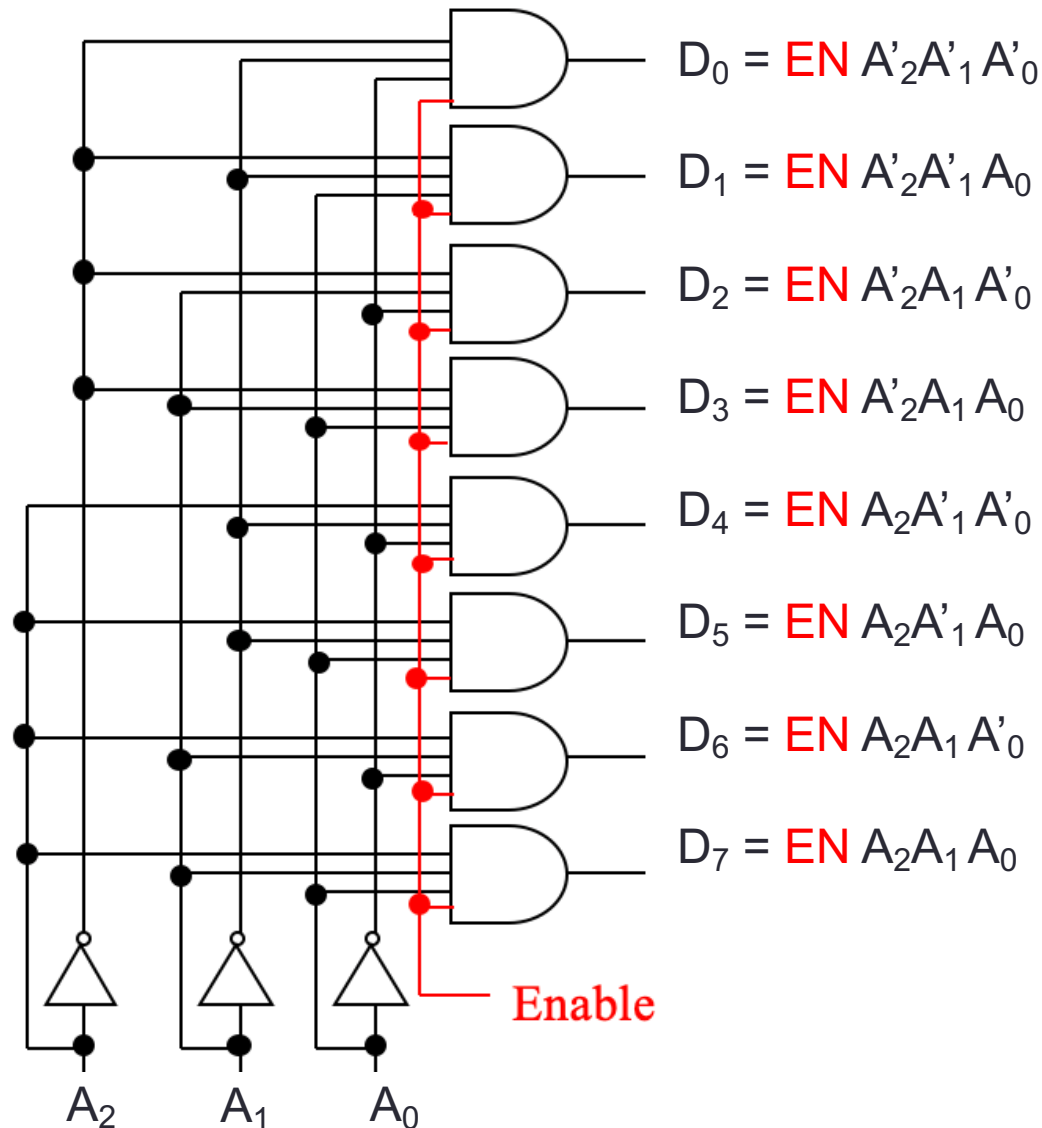
- For $n=3$ and $m=8$

| A_2 | A_1 | A_0 | D_0 | D_1 | D_2 | D_3 | D_4 | D_5 | D_6 | D_7 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

3-to-8-Line Decoder



3-to-8-Line Decoder with Enable

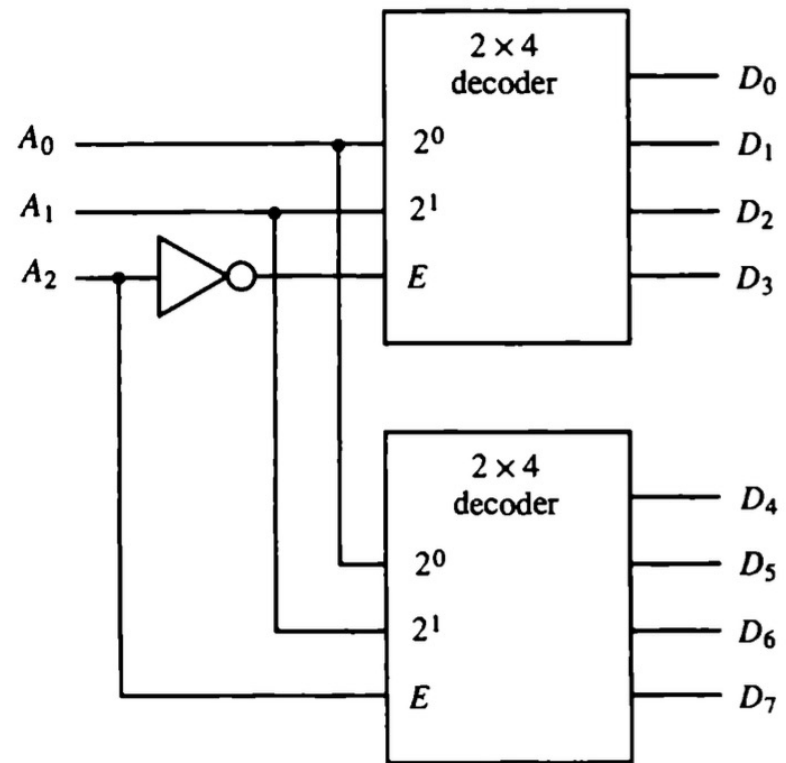


Decoder Expansion

- There are occasions when a certain-size decoder is needed but only smaller sizes are available.
- it is possible to combine two or more decoders with enable inputs to form a larger decoder

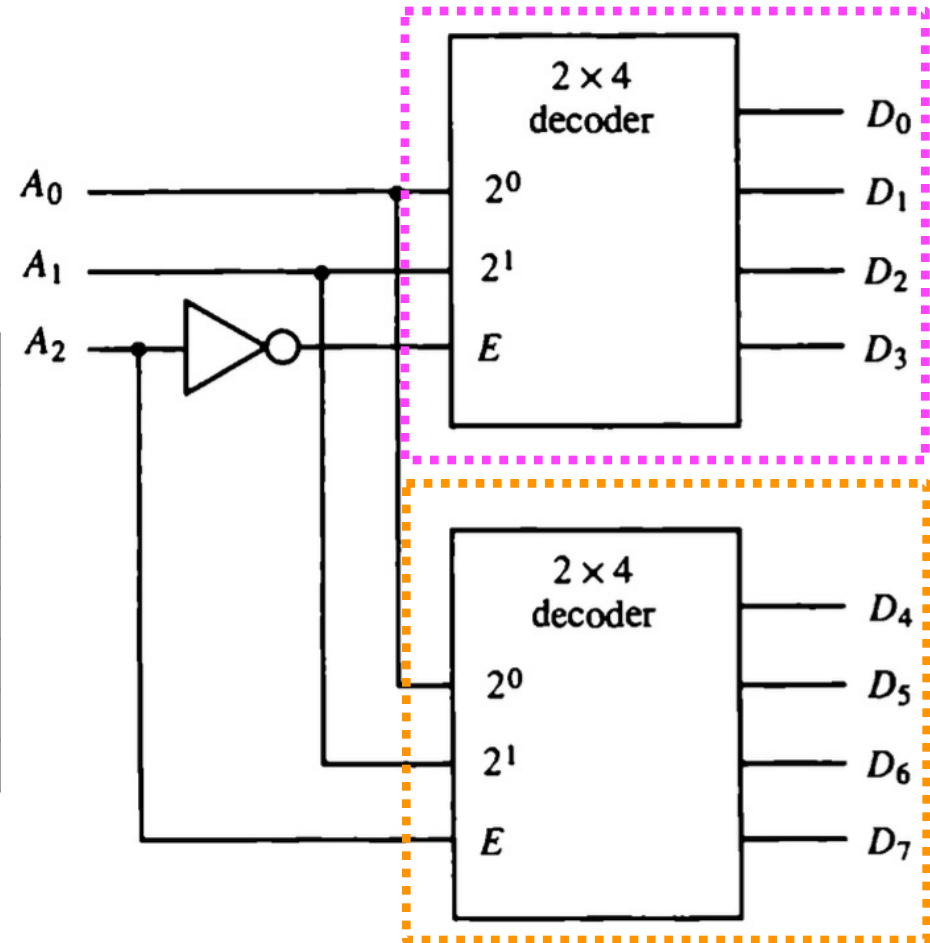
Two 2-to-4-line decoders are combined to achieve a 3-to-8-line decoder.

- When $A_2 = 0$, the upper decoder is enabled and the lower is disabled.
- When $A_2 = 1$, the lower decoder is enabled and the upper is disabled.



Decoder Expansion

| A_2 | A_1 | A_0 | D_0 | D_1 | D_2 | D_3 | D_4 | D_5 | D_6 | D_7 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |



Decoder-Based Combinational Circuits

- any Boolean function can be expressed as a sum of minterms:
- use a decoder to generate the minterms and combine them with an external OR gate to form a sum-of-minterms implementation
- any combinational circuit with n inputs and m outputs can be implemented with an n -to- 2^n -line decoder and m OR gates.

Example: Full adder

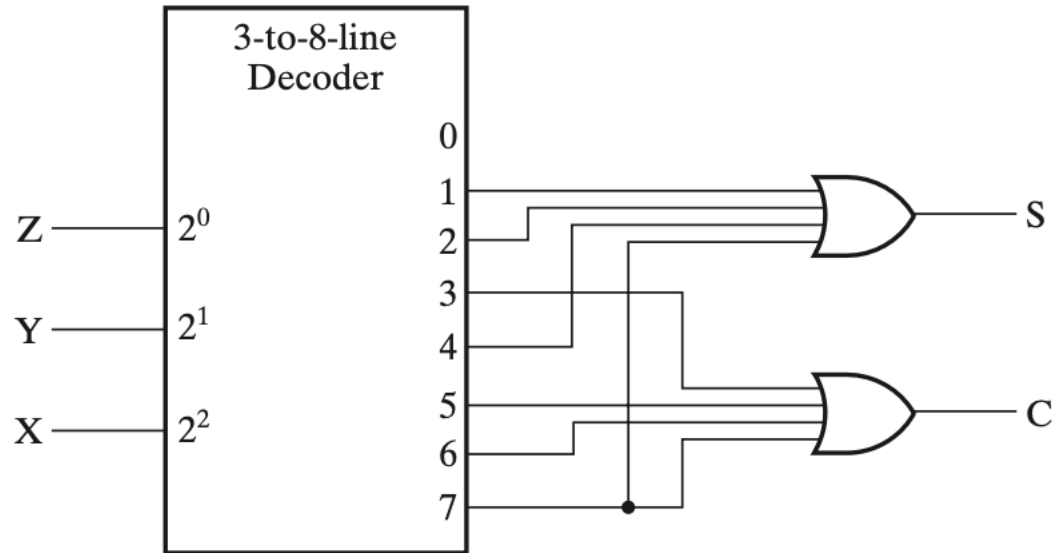
- Number of inputs: 3
- Number of outputs: 2 (S and C)
- Use 3-to- 2^3 line decoder and 2 OR gates

Example: Full Adder with a Decoder

| X | Y | Z | C | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$S = \sum(1, 2, 4, 7)$$

$$C = \sum(3, 5, 6, 7)$$



Encoders

- An encoder performs the **inverse** function of a decoder.
- It has **2^n (or fewer) input** lines and **n** output lines.
- The output lines generate the binary code corresponding to the input value.
- Example: an Octal to binary Encoder

Inputs: 8 ($D_7 \dots D_0$)

Outputs: 3 ($A_2 \dots A_0$)

$$A_0 = D_1 + D_3 + D_5 + D_7$$

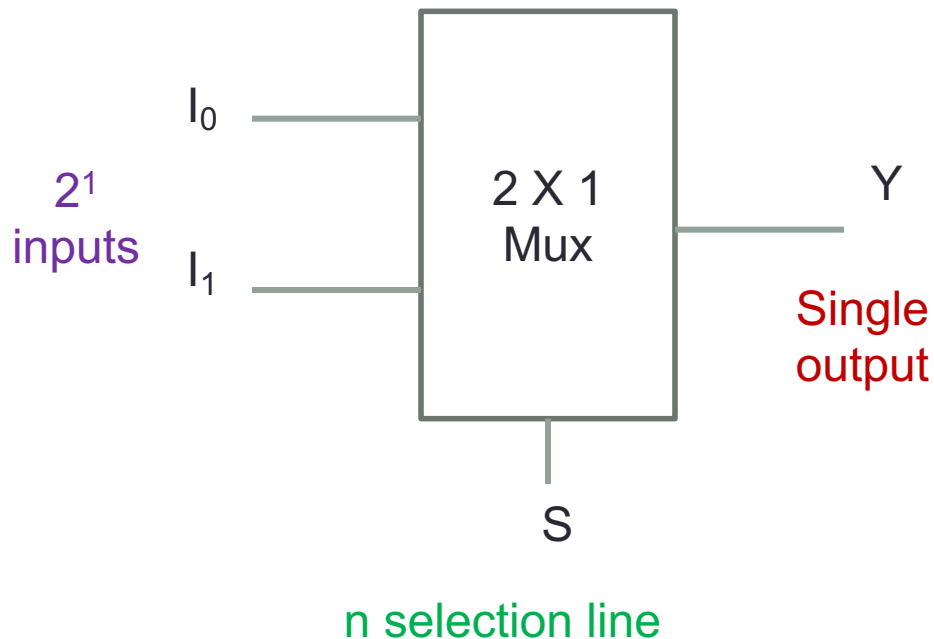
$$A_1 = D_2 + D_3 + D_6 + D_7$$

$$A_2 = D_4 + D_5 + D_6 + D_7$$

[illegible]

Multiplexer

- Logic circuits that perform selecting are called *multiplexers*
 - A set of **inputs** (2^n)
 - **A single output**
 - A set of **control lines** for making the selection (n)
 - **Example: $n=1$**



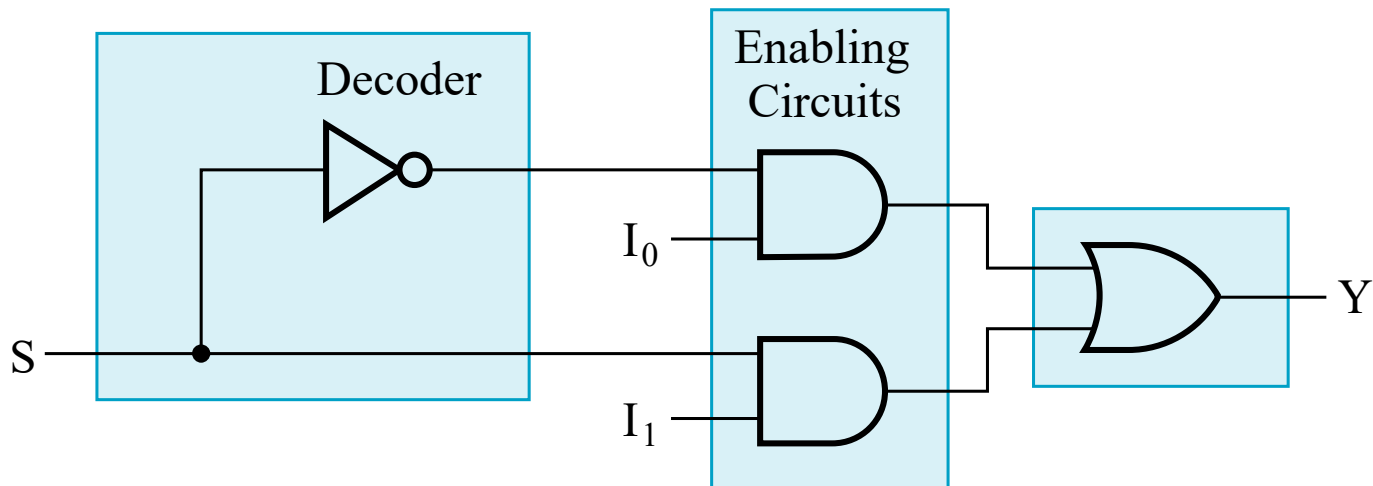
| S | I_0 | I_1 | Y |
|---|-------|-------|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

$$Y = \bar{S}I_0 + SI_1$$

2-to-1-Line Multiplexer

Note the regions of the multiplexer circuit shown:

- 1-to-2-line Decoder
- 2 Enabling circuits
- 2-input OR gate



$$Y = \bar{S}I_0 + SI_1$$

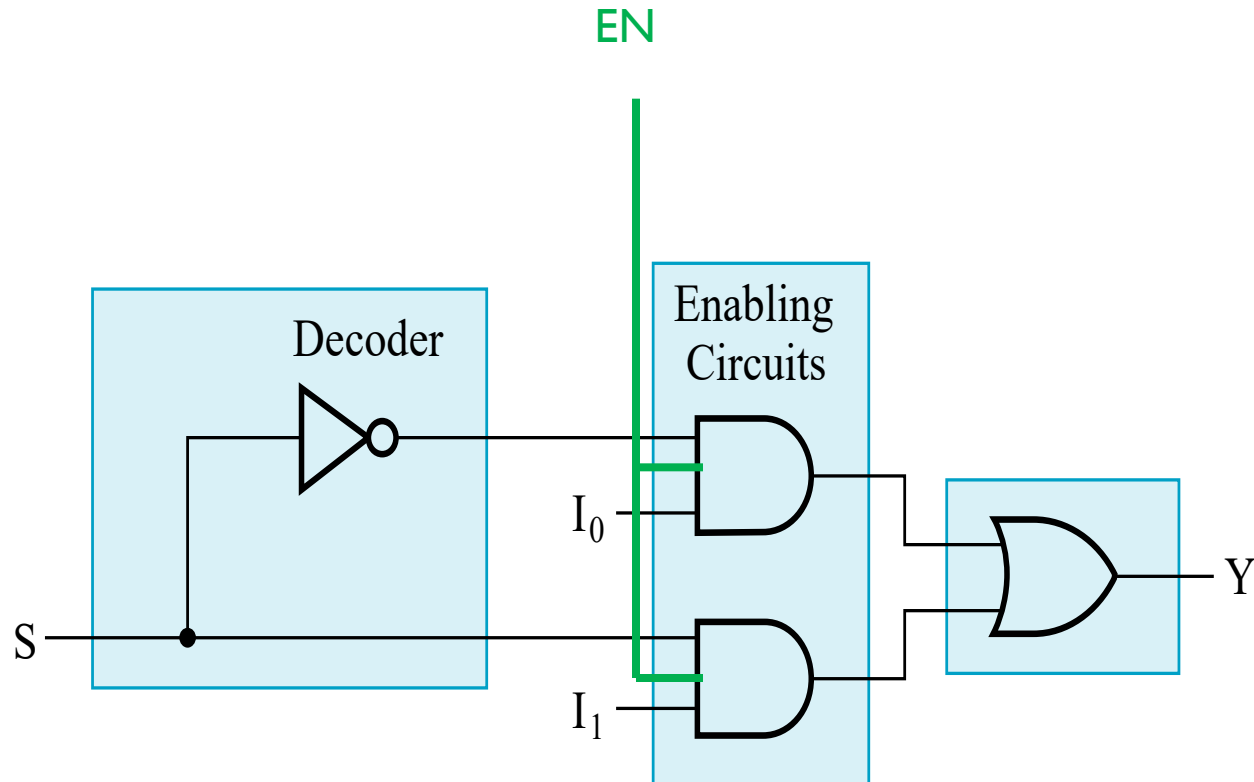
2-to-1-Line Multiplexer with Enable

As in decoders, multiplexers may have an enable input to control the operation of the unit.

When **EN=0**, the multiplexer is disabled.

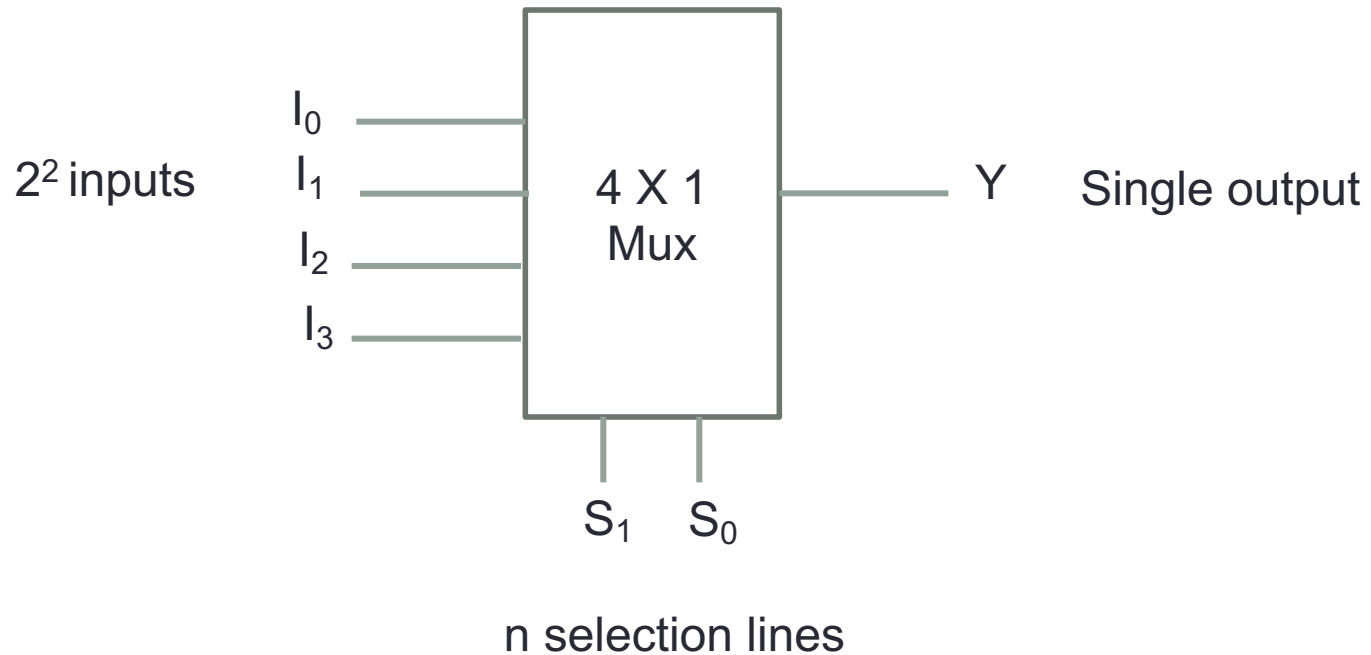
When **EN=1**, the multiplexer functions normally.

| EN | S | I ₀ | I ₁ | Y |
|----|---|----------------|----------------|---|
| 0 | X | X | X | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |



4-to-1-Line Multiplexer

Example: $n=2$

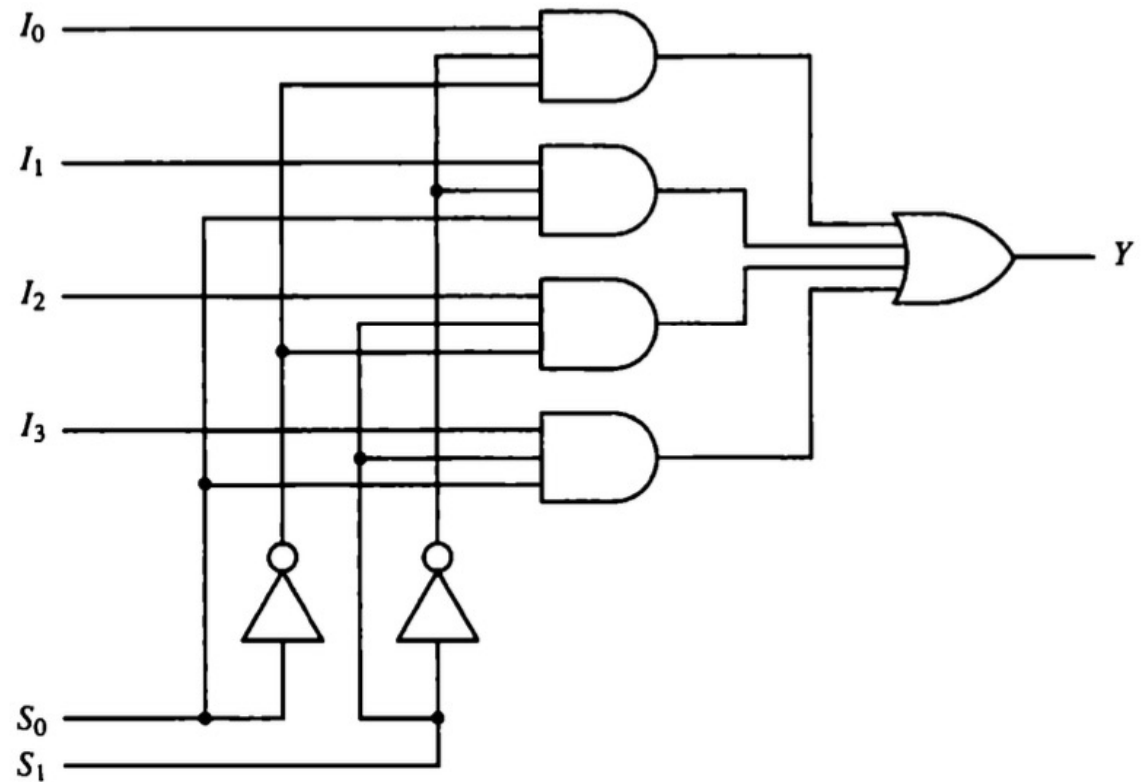


4-to-1-Line Multiplexer

Condensed Truth Table for 4-to-1-Line Multiplexer

| S_1 | S_0 | Y |
|-------|-------|-------|
| 0 | 0 | I_0 |
| 0 | 1 | I_1 |
| 1 | 0 | I_2 |
| 1 | 1 | I_3 |

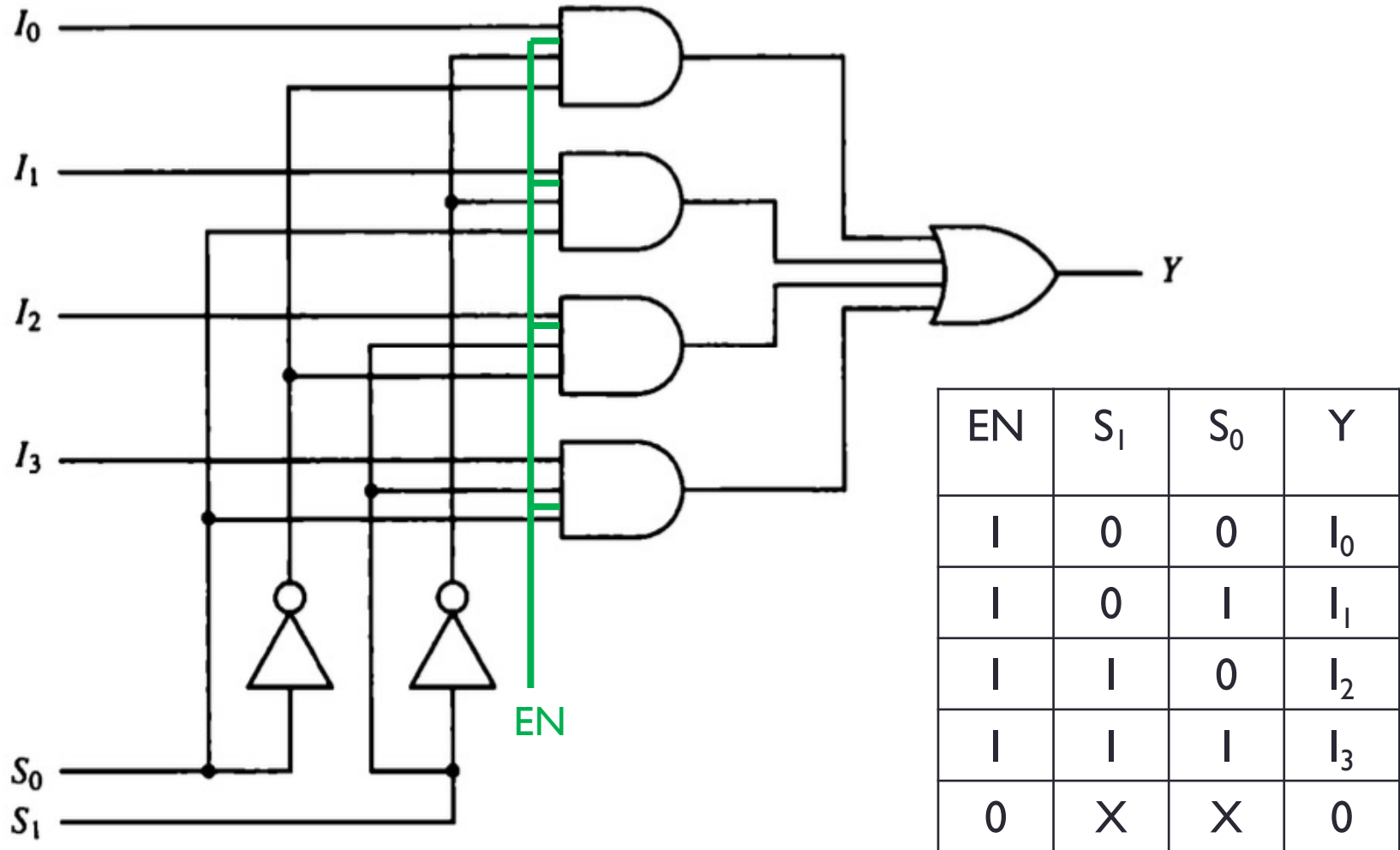
Figure 2-4 4-to-1-line multiplexer.



$$Y = (\bar{S}_1\bar{S}_0)I_0 + (\bar{S}_1S_0)I_1 + (S_1\bar{S}_0)I_2 + (S_1S_0)I_3$$

4-to-1-Line Multiplexer with Enable

Figure 2-4 4-to-1-line multiplexer.



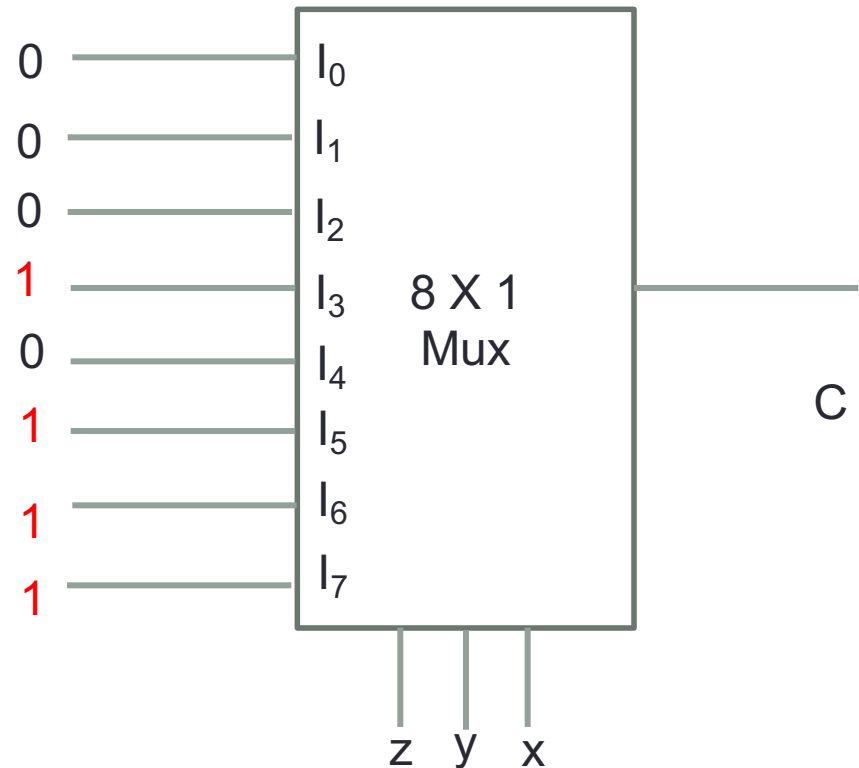
$$Y = EN (S_1' S_0') I_0 + EN (S_1' S_0) I_1 + EN (S_1 S_0') I_2 + EN (S_1 S_0) I_3$$

Mux-Based Combinational Circuits

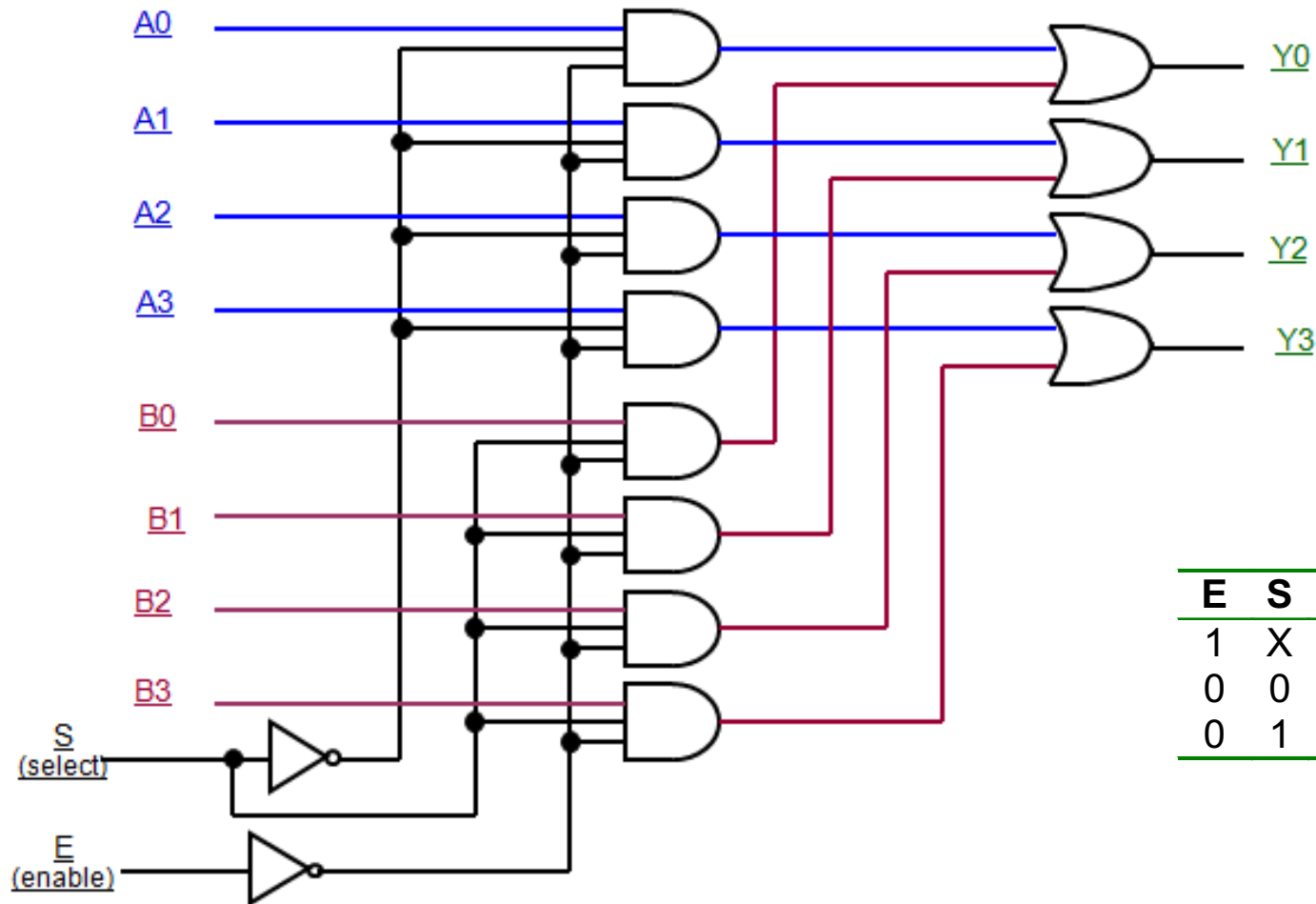
Example: Full adder with a multiplexer

| X | Y | Z | C | S |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$C = \sum(3, 5, 6, 7)$$



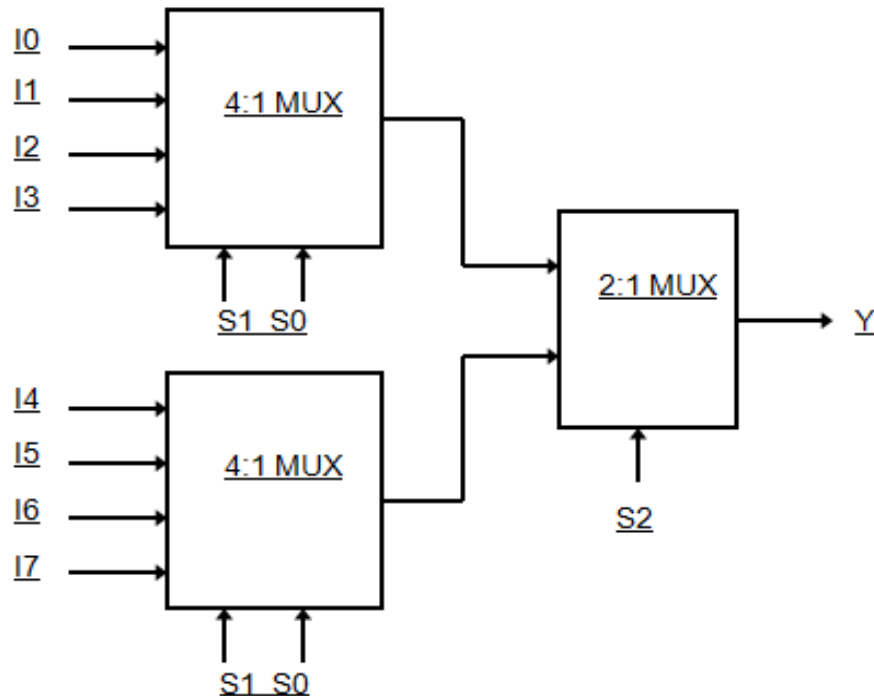
Multiplexer Width Expansion



Quadruple 2:1 multiplexer

Larger Multiplexers

- Larger multiplexers can be constructed from smaller ones.
- An 8-to-1 multiplexer can be constructed from smaller multiplexers like this (note placement of selector lines):

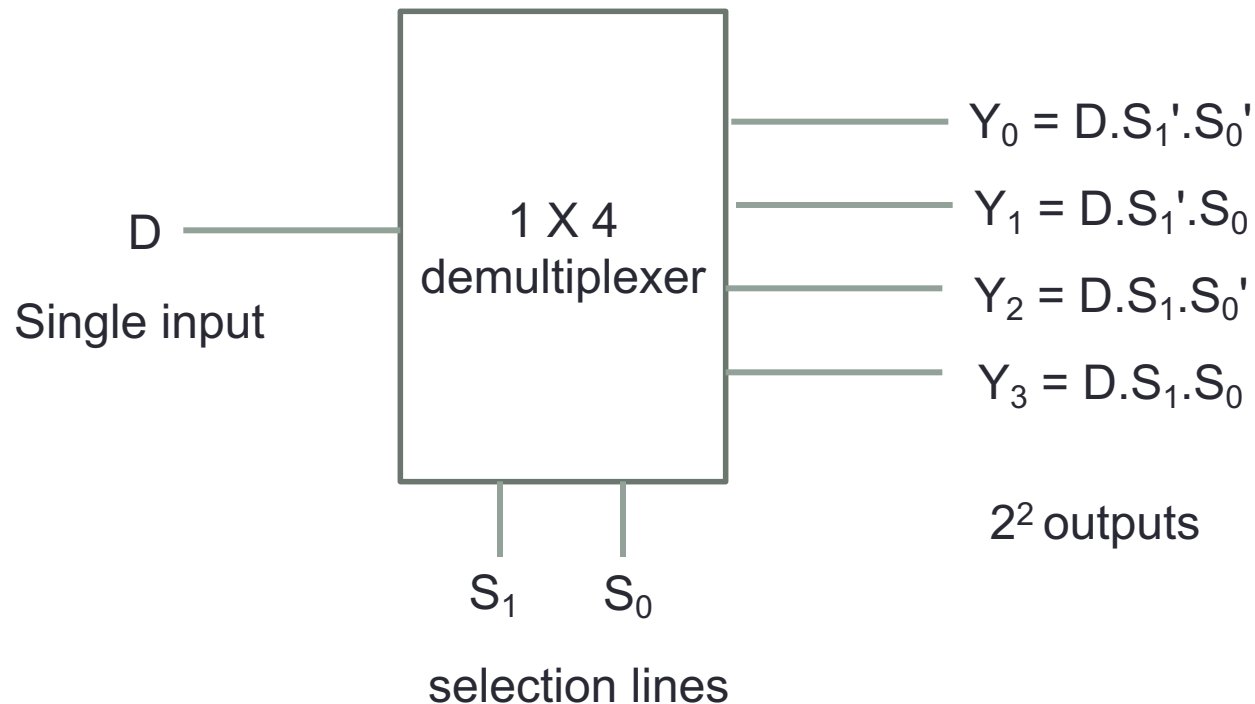


| S_2 | S_1 | S_0 | Y |
|-------|-------|-------|-------|
| 0 | 0 | 0 | I_0 |
| 0 | 0 | 1 | I_1 |
| 0 | 1 | 0 | I_2 |
| 0 | 1 | 1 | I_3 |
| 1 | 0 | 0 | I_4 |
| 1 | 0 | 1 | I_5 |
| 1 | 1 | 0 | I_6 |
| 1 | 1 | 1 | I_7 |

Demultiplexer

The **inverse** of selection is *distribution*:

- information received from a single line is transmitted to one of 2^n possible output lines.
- The circuit which implements such distribution is called a demultiplexer.



Demultiplexer

| S_1 | S_0 | Y_0 | Y_1 | Y_2 | Y_3 |
|-------|-------|-------|-------|-------|-------|
| 0 | 0 | D | 0 | 0 | 0 |
| 0 | 1 | 0 | D | 0 | 0 |
| 1 | 0 | 0 | 0 | D | 0 |
| 1 | 1 | 0 | 0 | 0 | D |

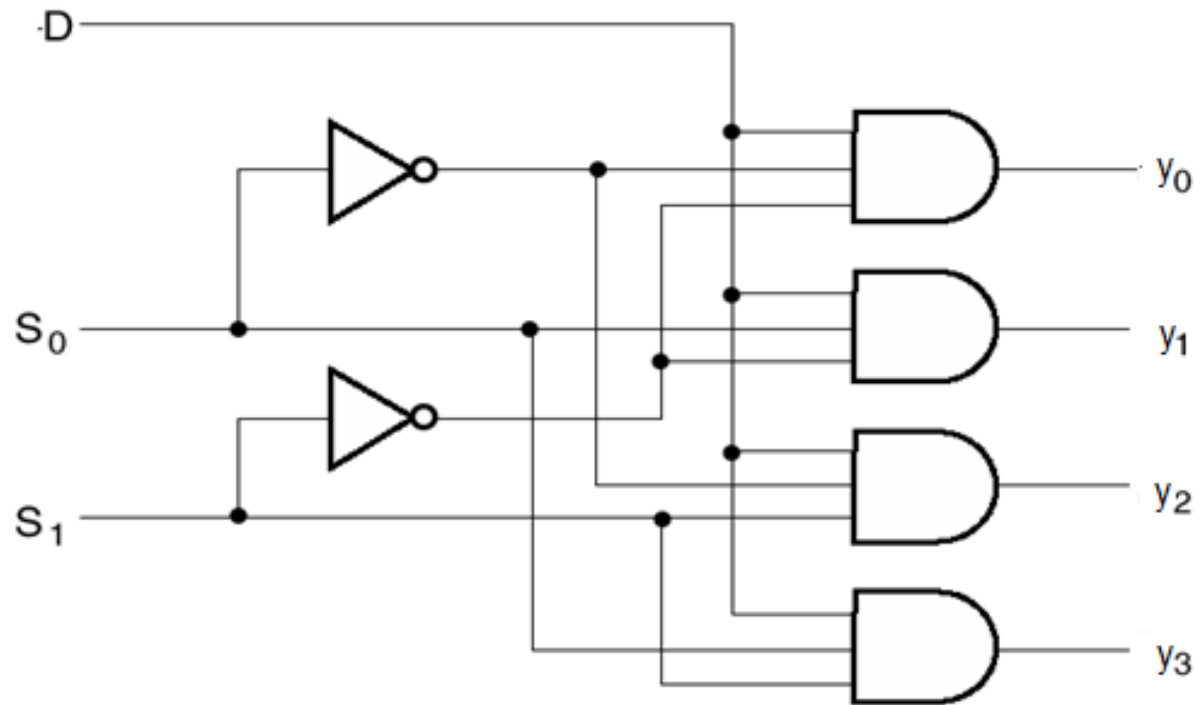
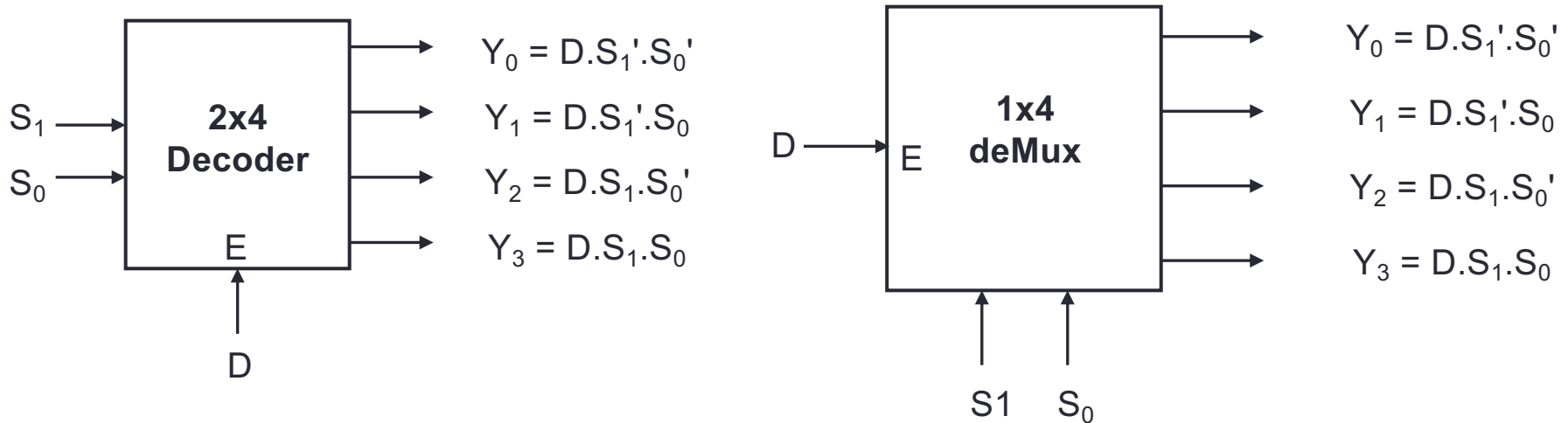


Fig. 3-24 1-to-4-Line Demultiplexer

The demultiplexer is actually identical to a decoder with enable.

Demultiplexer



- For the demultiplexer: input E provides the data, while the other inputs act as the selection variables.
- Although the two circuits have different applications, their logic diagrams are exactly the same. a decoder with enable input is referred to as a **decoder/ demultiplexer**.

Summary

Decoder:

- n -to- 2^n decoder is simply a minterm generator, with output corresponding to exactly one minterm.
- For each possible input condition, one and only one output signal will be at logic 1

Encoders:

- It has 2^n (or fewer) input lines and n output lines.
- Assign a unique output code (a binary number) for each input signal applied to the device
- Opposite of the decoder

Summary

Multiplexers:

- it has a set of inputs (2^n) and a single output
- the specific input being determined by a selection code (n lines).

Demultiplexer:

- Connects a single input line to one of 2^n output lines, the specific output being determined by a selection code (n lines)