

# King Saud University

## Department of Mathematics

### Tutorial 5 - Semester 452

#### ACTU 371 – Financial Mathematics

##### Exercise 1.

- a) A 15-year 1,000 par value bond with 7% semi-annual coupon is priced to yield 6% convertible semi-annually. Find the price.
- b) Suppose the bond in a) is offered at a price of 975. What is the nominal yield convertible semi-annually?
- c) A 10-year bond with a face value of 1,000 and 5% semi-annual coupon is sold for 980. What should the redemption value be if the bond is to yield 5.4% convertible semi-annually.

##### Exercise 2

A 5-year 1,000 par value bond with 7% semi-annual coupon is purchased to yield 6.4% convertible semi-annually. The coupon payments are reinvested in a fund that earns 7.2% convertible semi-annually. What is the annual effective yield on the total investment at the end of the 5-year period.

##### Exercise 3

A 1000 par value 20-year bond sells for  $P$  and yields a nominal interest rate of 10% convertible semiannually. The bond has 9% coupons payable semiannually and a redemption value of 1200.

Calculate  $P$ .

##### Exercise 4

A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%.

Calculate the interest portion of the 7th coupon.

### Exercise 5

You have decided to invest in Bond X, an  $n$ -year bond with semi-annual coupons and the following characteristics:

- (i) Par value is 1000.
- (ii) The ratio of the semi-annual coupon rate,  $r$ , to the desired semi-annual yield rate,  $i$ , is 1.03125.
- (iii) The present value of the redemption value is 381.50.

Given  $(1+i)^{-n} = 0.5889$ , calculate the price of bond X.

### Exercise 6

Bill buys a 10-year 1000 par value bond with semi-annual coupons paid at an annual rate of 6%. The price assumes an annual nominal yield of 6%, compounded semi-annually.

As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of  $i$ .

At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond.

Calculate  $i$ .

### Exercise 7

Matt purchased a 20-year par value bond with an annual nominal coupon rate of 8% payable semiannually at a price of 1722.25. The bond can be called at par value  $X$  on any coupon date starting at the end of year 15 after the coupon is paid. The lowest yield rate that Matt can possibly receive is a nominal annual interest rate of 6% convertible semiannually.

### Exercise 8

Toby purchased a 20-year par value bond with semiannual coupons of 40 and a redemption value of 1100. The bond can be called at 1200 on any coupon date prior to maturity, starting at the end of year 15.

Calculate the maximum price of the bond to guarantee that Toby will earn an annual nominal interest rate of at least 6% convertible semiannually.

### Exercise 9

Sue purchased a 10-year par value bond with an annual nominal coupon rate of 4% payable semiannually at a price of 1021.50. The bond can be called at par value  $X$  on any coupon date starting at the end of year 5. The lowest yield rate that Sue can possibly receive is an annual nominal rate of 6% convertible semiannually.

Calculate  $X$ .

### Exercise 10

A 40-year bond is purchased at a discount. The bond pays annual coupons. The amount for accumulation of discount in the 15th coupon is 194.82. The amount for accumulation of discount in the 20th coupon is 306.69.

Calculate the amount of discount in the purchase price of this bond.

### Exercise 11

Consider two 30-year bonds with the same purchase price. Each has an annual coupon rate of 5% paid semiannually and a par value of 1000.

The first bond has an annual nominal yield rate of 5% compounded semiannually, and a redemption value of 1200.

The second bond has an annual nominal yield rate of  $j$  compounded semiannually, and a redemption value of 800.

Calculate  $j$ .

### Exercise 12

An investor owns a bond that is redeemable for 300 in seven years. The investor has just received a coupon of 22.50 and each subsequent semiannual coupon will be  $X$  more than the preceding coupon. The present value of this bond immediately after the payment of the coupon is 1050.50 assuming an annual nominal yield rate of 6% convertible semiannually.

Calculate  $X$ .

### Exercise 13

A bank issues three annual coupon bonds redeemable at par, all with the same term, price, and annual effective yield rate.

The first bond has face value 1000 and annual coupon rate 5.28%.

The second bond has face value 1100 and annual coupon rate 4.40%.

The third bond has face value 1320 and annual coupon rate  $r$ .

Calculate  $r$ .

### Exercise 14

An  $n$ -year bond with annual coupons has the following characteristics:

- i) The redemption value at maturity is 1890;
- ii) The annual effective yield rate is 6%;
- iii) The book value immediately after the third coupon is 1254.87; and
- iv) The book value immediately after the fourth coupon is 1277.38.

Calculate  $n$ .

### Exercise 15

A 1000-par value 30-year bond has an annual coupon rate of 7% paid semiannually. After an initial 10-year period of call protection, the bond is callable immediately following the payment of any of the 20th through the 59th coupons.

- i) If the bond is called before payment of the 40th coupon, the redemption value is 1250.
- ii) If the bond is called immediately after the payment of any of the 40th through the 59th coupons, the redemption value is 1125.
- iii) If the bond is not called, it will be redeemed at par.

To ensure that the bond will provide at least an annual nominal yield rate of 5% convertible semiannually, it must be assumed that the bond will be called or redeemed immediately after the payment of the  $n$ th coupon.

Calculate  $n$ .

### Exercise 16

You are given the following information about a 20-year bond with face amount 7500:

- i) The bond has an annual coupon rate of 7.4% paid semiannually.
- ii) The purchase price results in an annual nominal yield rate to the investor of 5.3% convertible semiannually.
- iii) The amount for amortization of premium in the fourth coupon payment is 28.31.

Calculate the redemption value of the bond.

### Exercise 17

The price of a 36-year zero-coupon bond is 80% of its face value.

A second bond, with the same price, same face value, and same annual effective yield rate, offers annual coupons with the coupon rate equal to  $\frac{4}{9}$  of the annual effective yield rate.

Calculate the number of years until maturity for the second bond.

### Exercise 18

Kate buys a five-year 1000 face amount bond today with a 100 discount. The annual nominal coupon rate is 5% convertible semiannually.

One year later, Wallace buys a four-year bond. It has the same face amount and coupon values as Kate's and is priced to yield an annual nominal interest rate of 10% convertible semiannually. The discount on Wallace's bond is  $D$ .

The book value of Kate's bond at the time Wallace buys his bond is  $B$ .

Calculate  $B - D$ .