

King Saud University

College of Engineering

IE – 341: “Human Factors Engineering”

Spring – 2025 (2<sup>nd</sup> Sem. 1445H)

***Chapter 3. Information Input and Processing***

***Part – 4: Signal Detection Theory - TUTORIAL***

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# **Signal Detection Theory:**

## **PROBLEM 1 (SOLVED)**

**1)** Compare the inspection capability of Inspector A and Inspector B, determining  $d'$  and decision criterion. Inspector A located 26 of 28 defective parts, but also called 2 of 15 good parts defective. Inspector B found 29 of 30 defects, but called 6 of 20 good parts defective. In this case, the value of a 'hit' was greater than the cost of a 'false alarm'.

Q1

Given: • 2 inspectors

• assume → 'signal' = "defective" part

↳ 'no signal' = "good" part (non-defective)

\* Inspector A: • SPT outcomes for every inspector:

Hit Rate

$$= HR = 26/28$$

$$= 0.9286$$

False Alarm Rate

$$= FAR = 2/15$$

$$= 0.1333$$

Miss Rate =  
 $MR = 2/28$

Correct Rej. Rate  
 $CRR = 13/15$

A says/Finds ↓	Actually Defective Part	Actually Good Part
"Defective"	② 26 HIT	⑤ 2 FA
"Good"	③ 28-26 = 2 MISS	⑥ 15-2 = 13 CR
Total	① 28	④ 15

↳ should be = 1.0 (E should be = 1.0)

\* Inspector B:

$$HR = 29/30$$

$$= 0.9667$$

$$MR = 1/30$$

$$FAR = 6/20$$

$$= 0.3000$$

$$CRR = 14/20$$

A says/Finds ↓	Actually Defective	Actually Good
"Defective"	29 (H)	6 (FA)
"Good"	30-29 = 1 (M)	20-6 = 14 (CR)
Total	30	20

• Also give: Value (H) > |Value (FA)

Req: a)  $d'$  (both cases)  
b) decision (aka response) criterion (RC)  
c) which inspector is better

Sol<sup>n</sup>: first, to get a good visualization/image of the situation, we draw and compare both results (using the outcomes generated) with the help of the  $d'$  Calculator:

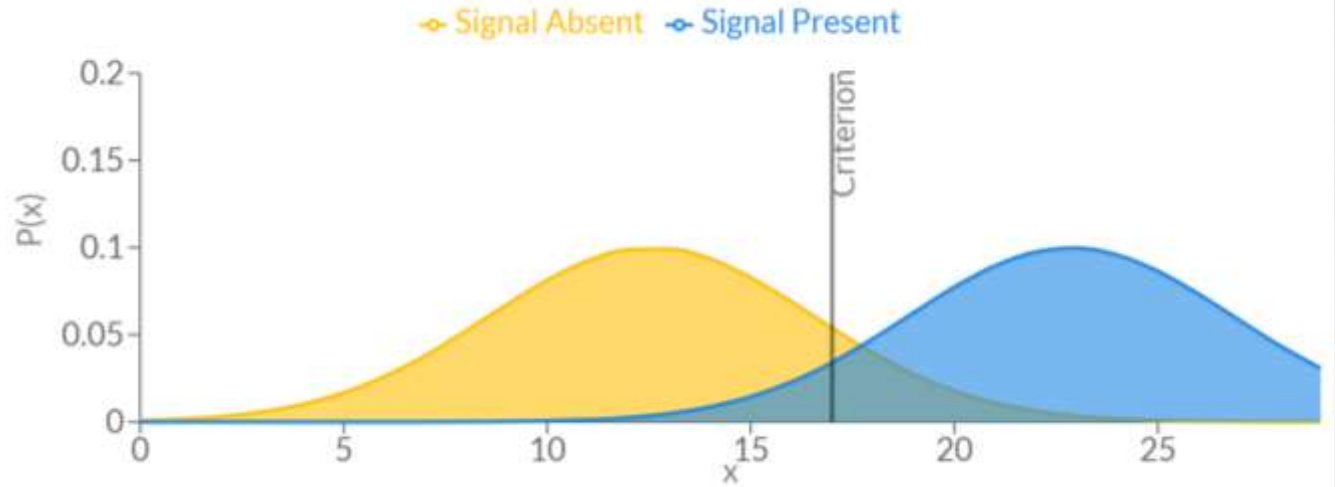
$d'$  calculator:

[https://eshedmargalit.com/dprime\\_calculator/](https://eshedmargalit.com/dprime_calculator/)



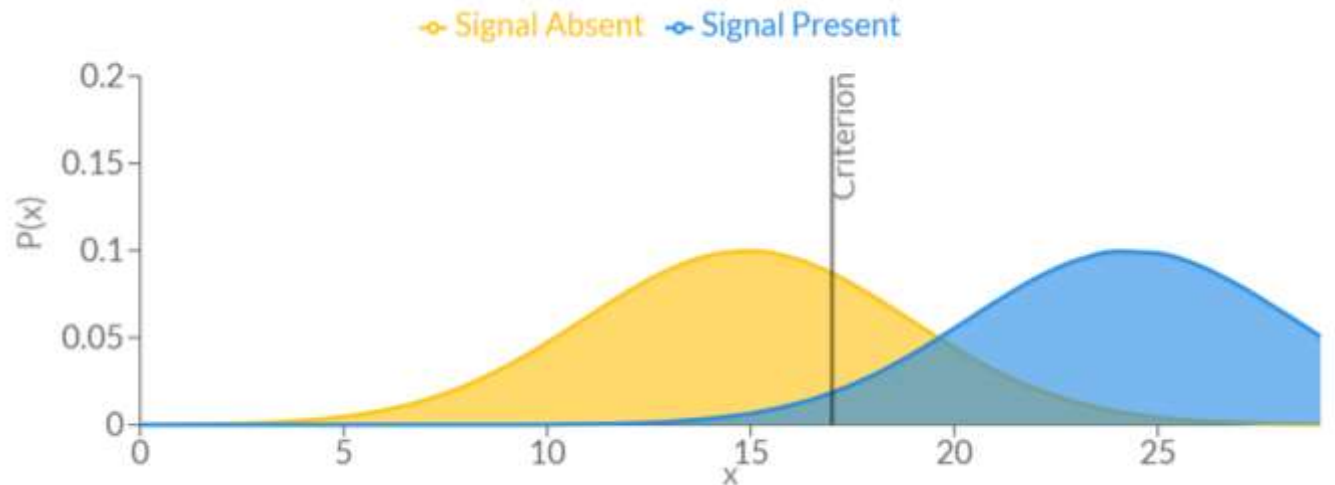
## Inspector A

- $d' = 2.58$
- slightly liberal (criterion is bet. the  $\mu$  of the 2 distributions; little to the left of the crossing/neutral point)



## Inspector B

- $d' = 2.36$
- more liberal/risky than A



a)

• Now, let's show these results using calculations:

$$d' = Z_{S+N} - Z_N$$

\*Ins. A

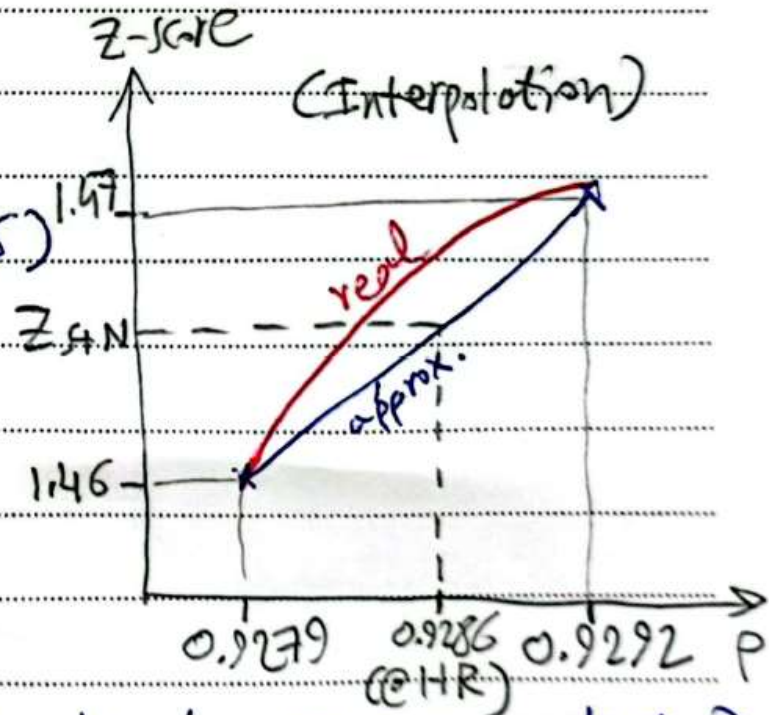
$$Z_{S+N} = Z (@p = HR = 0.9286)$$

using Z-score table,

and applying interpolation (linear)

$$\frac{Z_{S+N} - 1.46}{0.9286 - 0.9279} = \frac{1.47 - 1.46}{0.9292 - 0.9279}$$

$\Rightarrow Z_{S+N} = 1.465$  (check value also from Z-score website)  
 $\rightarrow$  ie to the right of the criterion ( $\because +ve$ )





z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998



## Z-score and Probability Converter

Please provide any one value to convert between z-score and probability. This is the equivalent of referencing a z-table.

Result



Given  $P(x < Z) = 0.9286$ ,

$Z = 1.465$

$P(x < Z) = 0.9286$



$P(x > Z) = 0.0714$



$P(0 < x < Z) = 0.4286$



$P(-Z < x < Z) = 0.8572$



$P(x < -Z \text{ or } x > Z) = 0.1428$



Z-score, Z	<input type="text"/>
Probability, $P(x < Z)$	<input type="text" value="0.9286"/>
Probability, $P(x > Z)$	<input type="text"/>
Probability, $P(0 \text{ to } Z \text{ or } Z \text{ to } 0)$	<input type="text"/>
Probability, $P(-Z < x < Z)$	<input type="text"/>
Probability, $P(x < -Z \text{ or } x > Z)$	<input type="text"/>

<https://www.calculator.net/z-score-calculator.html?c2z=&c2p=0.9286&c2pg=&c2p0=&c2pin=&c2pout=&calctype=converter&x=Calculate#converter>

• Similarly,  $Z_N = Z(\text{CP}=\text{FAR}=0.1333) = -1.111$   
(ie to left of criterion)

$$\Rightarrow d' = Z_{S+N} - Z_N = 1.465 - (-1.111)$$

$$\Rightarrow d'_A = 2.576 \blacktriangleleft$$

+ **Insp. B** (follow some steps as above):

$$d' = Z_{S+N} - Z_N$$

$$Z_{S+N} = Z(\text{CP}=\text{HR}=0.9667) = 1.834$$
  
(ie right of criterion)

$$Z_N = Z(\text{CP}=\text{FAR}=0.3000) = -0.524$$
  
(ie. left of criterion)

$$\Rightarrow d' = 1.834 - (-0.524)$$

$$\Rightarrow d'_B = 2.358 \blacktriangleleft$$

compare the values of  $d'_A$ ,  $d'_B$  to the ones obtained from the  $d'$  calculator (website) ✓



b) which inspector is more liberal/risky?  
 we determined (by inspection) from "d' calculator"  
 that (A) was slightly liberal, and (B) was  
 more liberal.

Now, we want to perform this manually;  
 we can use generated Z values for this:

\* for (A)  $Z_{s+N} = 1.465$ ,  $Z_N = -1.111$ ,  $d' = 2.576$   
 (B)  $Z_{s+N} = 1.834$ ,  $Z_N = -0.524$ ,  $d' = 2.358$

\* we draw  
 distribution for (A)

@ neutral point (0)  
 (mid-way bet. the 2 distributions)

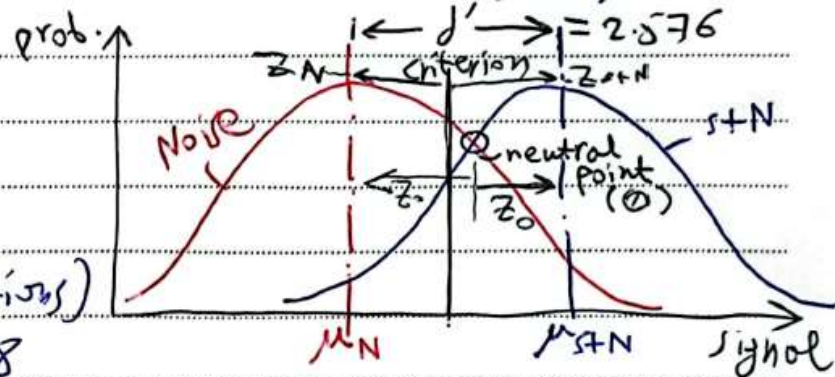
$Z_0 = d'/2 = 1.288$   
 (from either side)

and since  $Z_{s+N} > Z_0 \Rightarrow$  criterion is further away from  
 $\mu_{s+N}$  than the neutral point  $\Rightarrow RC < 1$

and  $Z_{s+N} - Z_0 =$

$$1.465 - 1.288 = 0.1777$$

so criterion\_A is only slightly liberal/risky



**Inspector A is  
 liberal/risky**  $\triangleleft$



Cont b)

\* we repeat the same process for Inspector B:


$$z_0 = d'/2 = 2.358/2 = 1.179$$

and since  $z_{stn} (= 1.834)$  is further away from  $\mu_{stn}$  than the neutral point

$$\begin{aligned} \text{and } z_{stn} - z_0 &= \\ 1.834 - 1.179 &= \\ &= 0.655 \end{aligned}$$

$$\Rightarrow RC < 1$$


$$(\text{beta} = b/a)$$

$\Rightarrow$  Inspector B  
is also liberal/risky 

now compare  $(z_{stn} - z_0)_{(B)}$  vs.  $(z_{stn} - z_0)_{(A)}$

$$= 0.655 \quad \text{vs.} \quad 0.1777$$

ie in the case of (B) the criterion is further away from the neutral point than for (A)

$\Rightarrow$  Inspector (B)  
is more risky/liberal  
than Inspector (A) 

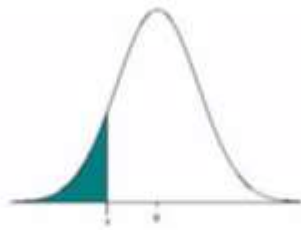
c) Finally, which inspector is better (A) or (B)?

- since  $\text{Value}(A) > \text{Value}(FA)$
- ie we are more concerned with hits than FA's
- $\Rightarrow$  we choose the inspector with more hits
- ie the more liberal/risky inspector:

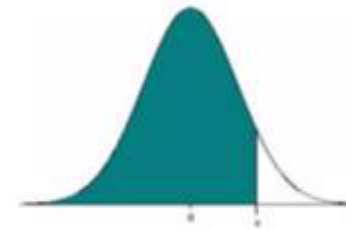
$\Rightarrow$  Inspector (B) is a better choice



**Table of Standard Normal Probabilities for Negative Z-scores**



**Table of Standard Normal Probabilities for Positive Z-scores**



<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0015	0.0015	0.0014	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

<i>z</i>	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

**Note that the probabilities given in this table represent the area to the LEFT of the z-score.  
The area to the RIGHT of a z-score = 1 – the area to the LEFT of the z-score**



# References

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