King Saud University

Department of Mathematics

Tutorial 4 - Semester 461

ACTU 371 – Financial Mathematics

Exercise 1.

A 30-year monthly payment mortgage for 500,000 is offered at a nominal annual rate of 8.4% convertible monthly. Find:

a) The monthly payment.
b) The total principal and interest that will be paid over 30 years.
c) The balance after 5 years.
d) The amount of principal and amount of interest paid over the first 5 years.

Exercise 2

A borrower would like to borrow 50,000 at 7.5% for 5 years, but wants to pay only 6,000 in each of the first 2 years and then catch up with a higher payment amount for the final 3 years. What is the annual payment amount for the final 3 years.

Exercise 3

A loan at a 6.5% annual effective rate has an initial payment of 300 and 9 further payments. The payment amount increases by 20 each year. Find the loan balance immediately after the 6^{th} payment

Exercise 4

For a level-payment loan at an annual effective interest rate of 6,3% per payment period, the amount of principal in the third payment is 845.28. Find the amount of principle in the 7^{th} payment.

Exercise 5

A fixed-rate loan has level annual payments. The principle repaid in the 5^{th} payment is 1,489.40. The principle repaid in the 15^{th} payment is 2,795.81. What is the annual interest rate on the loan

Exercise 6

A 20-year loan of 1000 is repaid with payments at the end of each year.

Each of the first ten payments equals 150% of the amount of interest due. Each of the last ten payments is X.

The lender charges interest at an annual effective rate of 10%.

Calculate X.

Exercise 7

A 10-year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

(i) Equal annual payments at an annual effective interest rate of 8.07%.

(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective interest rate of i.

The sum of the payments under option (i) equals the sum of the payments under option (ii).

Calculate i.

Exercise 8

A loan is amortized over five years with monthly payments at an annual nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment.

Calculate the outstanding loan balance immediately after the 40^{th} payment is made.

Exercise 9

Seth, Janice, and Lori each borrow 5000 for five years at an annual nominal interest rate of 12%, compounded semi-annually.

Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years.

Lori repays her loan with 10 level payments at the end of every six-month period.

Calculate the total amount of interest paid on all three loans.

Exercise 10

Ron is repaying a loan with payments of 1 at the end of each year for n years. The annual effective interest rate on the loan is i. The amount of interest paid in year t plus the amount of principal repaid in year $t + 1$ equals X.

Determine which of the following is equal to X.

(A) $1+\frac{v^{n-t}}{i}$

(B) $1+\frac{v^{n-t}}{d}$

(C) $1+v^{n-t}i$

(D) $1+v^{n-t}d$

(E) $1+v^{n-t}$

Exercise 11

Tanner takes out a loan today and repays the loan with eight level annual payments, with the first payment one year from today. The payments are calculated based on an annual effective interest rate of 4.75%. The principal portion of the fifth payment is 699.68.

Calculate the total amount of interest paid on this loan.

Exercise 12

Turner buys a new car and finances it with a loan of 22,000. He will make n monthly payments of 450.30 starting in one month. He will make one larger payment in n+1 months to pay off the loan. Payments are calculated using an annual nominal interest rate of 8.4%, convertible monthly. Immediately after the 18th payment he refinances the loan to pay off the remaining balance with 24 monthly payments starting one month later. This refinanced loan uses an annual nominal interest rate of 4.8%, convertible monthly.

Calculate the amount of the new monthly payment.

Exercise 13

A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly.

Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j.

Exercise 14

A borrower takes out a 50-year loan, to be repaid with payments at the end of each year. The loan payment is 2500 for each of the first 26 years. Thereafter, the payments decrease by 100 per year. Interest on the loan is charged at an annual effective rate of i ($0\% < i < 10\%$).
The principal repaid in year 26 is X.

Determine the amount of interest paid in the first year.

(A) Xv^{25}
(B) $2500v^{25} - Xv^{25}$
(C) $2500 - X$
(D) $2500 - Xv^{25}$
(E) $25Xi$

Exercise 15

A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made.
The annual effective interest rate is 5%.

Calculate X.

Exercise 16

A borrower takes out a 15-year loan for 65,000, with level end-of-month payments. The annual nominal interest rate of the loan is 8%, convertible monthly.

Immediately after the 12th payment is made, the remaining loan balance is reamortized. The maturity date of the loan remains unchanged, but the annual nominal interest rate of the loan is changed to 6%, convertible monthly.

Calculate the new end-of-month payment.

Exercise 17

A company takes out a loan of 15,000,000 at an annual effective discount rate of 5.5%. You are given:

i) The loan is to be repaid with n annual payments of 1,200,000 plus a drop payment one year after the nth payment.
ii) The first payment is due three years after the loan is taken out.

Calculate the amount of the drop payment.