King Saud University

Department of Mathematics

Tutorial 3- Semester 461

ACTU 371 – Financial Mathematics

Exercise 1.

An annuity pays 100 at the end of each of the next 5 years and 300 at the end of the 5 following years. At an annual effective interest rate of 6%, what is the present value of this annuity?

Exercise 2

An annual-payment annuity-immediate has a first payment of 200, and payments increase by 100 each year until they reach 600. There are then 5 additional level payments of 600. Find the present value of this annuity at a 5.5% annual effective rate.

Exercise 3

An annuity-immediate has quarterly payments of 20 for 10 years, followed by a perpetuity of quarterly payments of 25 starting in the 11^{th} year. Find its present value at an interest rate of 4% convertible quarterly.

Exercise 4

You deposit 2,000 into an investment account at the beginning of each year for 8 years. The account earns interest at an annual effective rate of 8%. But the interest received from this account will be reinvested at a 5% annual effective rate.

How much will you have at the end of 8 years.

Exercise 5

1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%. Calculate the accumulated value of Fund Y at the end of year 10.

Exercise 6

A perpetuity costs 77.1 and makes end-of-year payments. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, …., n at the end of year (n+1). After year (n+1), the payments remain constant at n. The annual effective interest rate is 10.5%.

Exercise 7

At an annual effective interest rate of 6%, the present value of a perpetuity immediate with successive annual payments of 6, 8, 10, 12, ..., is equal to X.

Calculate X.

Exercise 8

A perpetuity-due with semi-annual payments consists of two-level payments of 300, followed by a series of increasing payments. Beginning with the third payment, each payment is 200 larger than the preceding payment. Using an annual effective interest rate of i, the present value of the perpetuity is 475,000. Calculate i.

Exercise 9

Mike buys a perpetuity-immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the payment in that year is K% larger than the payment in the year immediately preceding that year, where K < 9.2. At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50. Calculate K.

Exercise 10

Payments are made to an account at a continuous rate of $(8k + tk)$, where $0 \le t \le 10$.

Interest is credited at a force of interest $\delta_t = \frac{1}{8+t}$.

After time 10, the account is worth 20,000.

Calculate k.

Exercise 11

An insurance company has an obligation to pay the medical costs for a claimant. Annual claim costs today are 5000, and medical inflation is expected to be 7% per year. The claimant will receive 20 payments. Claim payments are made at yearly intervals, with the first claim payment to be made one year from today. Calculate the present value of the obligation using an annual effective interest rate of 5%.

Exercise 12

A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of X to a fund for 25 years. The fund earns an annual nominal interest rate of 8%

compounded monthly. On his 65th birthday, each 1000 of the fund will provide 9.65 of income at the beginning of each month starting immediately and continuing as long as he survives.

Calculate X.

Exercise 13

The present value of 25-year annuity-immediate with a first payment of 2500 and decreasing by 100 each year thereafter is X. Assuming an annual effective interest rate of 10%, calculate X.

Exercise 14

Which of the following is an expression for the present value of a perpetuity with annual payments of 1, 2, 3, …, where the first payment will be made at the end of n years, using an annual effective interest rate of i?

(A) $\dfrac{\ddot{a}_{\overline{n}|} - nv^n}{i}$

(B) $\dfrac{n - a_{\overline{n}|}}{i}$

(C) $\dfrac{v^n}{d}$

(D) $\dfrac{v^n}{d^2}$

(E) $\dfrac{v^n}{di}$

Exercise 15

A perpetuity provides for continuous payments. The annual rate of payment at time t is

$$\begin{cases} 1, & \text{for } 0 \le t < 10, \\ (1.03)^{t-10}, & \text{for } t \ge 10. \end{cases}$$

Using an annual effective interest rate of 6%, the present value at time $t = 0$ of this perpetuity is x.

Calculate x.