Tutorial 2

Chapter 11: Discrimination and Classification

Brief Summary: (you should refer to the book for all subjects)

Expected Cost of Misclassification (ECM)= $c(1|2)P(2|1)p_1 + \ ...$

The minimum **ECM** rules **(11.6)**

Assigning an observation x_0 to π_1 if : $\frac{f_1(x)}{f_2(x)} \geq \frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}$; Assigning x_0 to π_2 otherwise.

Special case : The minimum **TPM** methods are equivalent to minimizing the ECM when the costs of misclassification are equal $\frac{c(1|2)}{c(2|1)} = 1$

The Estimated minimum ECM for Tow Multivariate **Normal** Populations ($\Sigma_1 = \Sigma_2$) **(11-18)**

Allocation x_0 to π_1 if : $(\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1} X - \frac{1}{2}(\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1}\, (\bar{X}_1 + \bar{X}_2) \geq \ln\left[\frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}\right]$

Let $\hat{y}_0 = (\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1}\, X = \hat{a}^t\, x_0$

$$\hat{m} = \frac{1}{2}(\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1}\, (\bar{X}_1 + \bar{X}_2) = \frac{1}{2}(\bar{y}_1 + \bar{y}_2) \quad ; \quad \bar{y}_1 = \hat{a}^t \bar{X}_1 \quad ; \quad \bar{y}_2 = \hat{a}^t \bar{X}_2$$

$$\hat{y}_0 - \hat{m} \geq \ ln\left[\frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}\right]$$

Allocate x_0 to π_2 otherwise.

The Fisher's Classification for Two Populations with $\Sigma_1 = \Sigma_2$ **(11-25)**

Fisher's approach does not assume that the populations are normal.

Allocation x_0 to π_1 if : $\hat{y}_0 \geq \hat{m}$; Allocate x_0 to π_2 otherwise.

The Classification of **Normal** Populations When $\Sigma_1 \neq \Sigma_2$ (Quadratic Classification Rule)

(11-29) page 615

Allocation x_0 to π_1 if :

$$-\frac{1}{2}x_0'(s_1^{-1} - s_2^{-1})x_0 + (\bar{x}_1'\ s_1^{-1} - \bar{x}_2'\, s_2^{-1})x_0 - \frac{1}{2}\ln\left(\frac{|s_1|}{|s_2|}\right) - \frac{1}{2}(\bar{x}_1'\ s_1^{-1}\bar{x}_1 - \bar{x}_2'\, s_2^{-1}\bar{x}_2) \geq \ln\left[\frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}\right]$$

Allocate x_0 to π_2 otherwise.

Allocate x_0 to π_2 otherwise.

Estimated Minimum **TPM** Rule for several **Normal** Populations with **Equal-Covariance**:

Assign x to the population π_i for which $-\frac{1}{2}D_i^2(x_0) + \ln p_i$ is largest .

$$D_i^2(x) = (x - \bar{x}_i)'\ S_{pooled}^{-1}\ (x - \bar{x}_i)$$

Estimated Minimum **TPM** Rule for several **Normal** Populations with **Unequal-Covariance**:

Assign x to the population $\boldsymbol{\pi}_i$ for which $\widehat{d_i^Q}(x) = -\frac{1}{2}\ln|S_i| - \frac{1}{2}(\mathrm{x} - \bar{\mathrm{x}}_\mathrm{i})^T S_i^{-1}(\mathrm{x} - \bar{\mathrm{x}}_\mathrm{i}) + \ln p_i$ is largest .

Exercise 1: (in book Exercise 11.1)

Consider the two data sets

$$X_1 = \begin{bmatrix} 3 & 7 \\ 2 & 4 \\ 4 & 7 \end{bmatrix} \; and \; X_2 = \begin{bmatrix} 6 & 9 \\ 5 & 7 \\ 4 & 8 \end{bmatrix} \quad \text{for which } \bar{X}_1 = \begin{bmatrix} 3 \\ 6 \end{bmatrix}, \bar{X}_2 = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \; and \; S_{pooled} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

(a) Calculate the linear discriminant function $\hat{y} = (\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1} X$.

(b) Classify the observation $x_0^t = [2\;\; 7]$ as population π_1 or population π_2, using ECM [Rule (11-18)] with equal priors and equal costs.

(NOTE: Give the minimum ECM rule for two Normal population with common covariance matrix for assigning a new item to one of the two populations.)

Solution:

a) $\hat{y} = (\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1} X = \boldsymbol{\hat{a}^t X}$

$$\hat{y} = [-2 \quad -2]\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [-2 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \boldsymbol{-2x_1}$$

b) Allocation x_0 to $\boldsymbol{\pi_1}$ if :

$$(\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1} X - \frac{1}{2}(\bar{X}_1 - \bar{X}_2)^t\, S_{pooled}^{-1}(\bar{X}_1 + \bar{X}_2) \geq \ln\left[\frac{c(1|2)}{c(2|1)}\frac{P_2}{P_1}\right]$$

$$\hat{y}_0 - \hat{m} \geq \ln\left[\frac{c(1|2)}{c(2|1)}\frac{P_2}{P_1}\right]$$

$$\boldsymbol{\hat{y}_0} = \boldsymbol{\hat{a}^t x_0} = [-2 \quad 0]\begin{bmatrix} 2 \\ 7 \end{bmatrix} = \boldsymbol{-4}$$

$$\boldsymbol{\hat{m}} = \frac{\boldsymbol{1}}{\boldsymbol{2}}(\boldsymbol{\bar{y}_1} + \boldsymbol{\bar{y}_2}) = \frac{1}{2}(-6 - 10) = -8$$

Where $\boldsymbol{\bar{y}_1} = \hat{a}^t\bar{X}_1 = [-2 \quad 0]\begin{bmatrix} 3 \\ 6 \end{bmatrix} = -6 \quad and \quad \boldsymbol{\bar{y}_2} = \hat{a}^t\bar{X}_2 = [-2 \quad 0]\begin{bmatrix} 5 \\ 8 \end{bmatrix} = -10$

Since $P_1 = P_2 \;\; and \;\; C(1|2) = C(2|1) \;\rightarrow\; \ln[\boldsymbol{1}\,] = 0$

Therefore, allocation x_0 to $\boldsymbol{\pi_1}$ if : $\hat{y}_0 \geq \hat{m}$

Since $\hat{y}_0 = -4 > \hat{m} = -8$, **assign $\boldsymbol{x_0}$ to population $\boldsymbol{\pi_1}$.**

R code

```
#Exercise 1:
#the summary statistics from their example.
xbar1=c(3,6)
xbar2=c(5,8)
Spooles=matrix(c(1,1,1,2),2,2)
Spinv= solve(Spooles)
X0= c(2, 7)
```

```r
##linear discriminants, Case: equal cost and equal prior .
(at=t(xbar1-xbar2)%*% Spinv)

     [,1] [,2]
[1,]   -2    0

(y0 <- at%*%X0)

     [,1]
[1,]   -4

#ybar1=a%*%xbar1 , ybar2=a%*%xbar2
(m<- 1/2*(at%*%xbar1+at%*%xbar2))

     [,1]
[1,]   -8

ifelse(y0 >= m, "Assign X0 to First population", "Assign X0 to Second popu
lation")

     [,1]
[1,] "Assign X0 to First population "
```

Exercise 2: (in book Exercise 11.4)

A researcher wants to determine a procedure for discriminating between two multivariate populations. The researcher has enough data available to estimate the density functions $f_1(x)$ and $f_2(x)$ associated with populations π_1 and π_2, respectively. Let $c(2|1) = 50$ and $c(1|2) = 100$.

In addition, it is known that about 20% of all possible items (for which the measurements x can be recorded) belong to π_2 .

(a) Give the minimum ECM rule (in general form) for assigning a new item to one of the two populations. **(Classifying a new observation into one of the two populations)**

(b) Measurements recorded on a new item yield the density values $f_1(x) = 0.3$ and $f_2(x) = 0.5$. Given the preceding information, assign this item to population π_1 or population π_2.

Solution:

Note: $c(2|1)$: this is the cost of assigning items as π_2 , given that π_1 is true.

$c(2|1) = 50,\ c(1|2) = 100,\quad P_2 = 0.2\,,\quad P_1 = 0.8 \quad (\ P_1 + P_2 = 1\ )$

a) From Result 11.1 , The minimum ECM rule is given by assigning an observation x to π_1 if

$$\frac{f_1(x)}{f_2(x)} \geq \frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1} = \frac{100}{50}\frac{0.2}{0.8} = 0.5$$

and assigning x to π_2 if

$$\frac{f_1(x)}{f_2(x)} < \frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1} = 0.5$$

b) Since $f_1(x) = 0.3\,; f_2(x) = 0.5$

$\frac{f_1(x)}{f_2(x)} = \frac{0.3}{0.5} = \mathbf{0.6 > 0.5}$ **so, assign** $\boldsymbol{x}$ **to** $\boldsymbol{\pi_1}$

Exercise 3:

Suppose that $n_1 = 11$ *and* $n_2 = 12$ observations are made on two random variables X_1 and X_2, where X_1 and X_2 are assumed to have a bivariate normal distribution with a common covariance matrix $\sum$, but possibly different mean vectors μ_1 and μ_2 for the two samples. The sample mean vectors and pooled covariance matrix are:

$$\bar{X}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \bar{X}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{ and } S_{pooled} = \begin{bmatrix} 7.3 & -1.1 \\ -1.1 & 4.8 \end{bmatrix}$$

(a) Test for the difference in population mean vectors using Hotelling's two-sample $T^2 - statistic$. Let $\alpha = 0.10$. (for Reading)

(b) Construct Fisher's (sample) linear discriminant function.

(c) Assign the observation $x_0 = [0 \quad 1]$ to either population π_1 or π_2. Assume equal costs and equal prior probabilities.

Solution:

a) Test $H_0: \mu_1 = \mu_2 \;\; VS \;\; H_1: \mu_1 \neq \mu_2$ (from Result 6.2)

$$T^2 = (\bar{X}_1 - \bar{X}_2)^t \left[\left(\frac{1}{n_1} + \frac{1}{n_2}\right) S_{pooled}\right]^{-1} (\bar{X}_1 - \bar{X}_2)$$

$$T^2 \sim \frac{(n_1+n_2-2)P}{n_1+n_2-P-1} F_{P,n_1+n_2-P-1\,,\alpha}$$

$$[-3 \quad -2]\left[\left(\frac{23}{132}\right)\begin{bmatrix} 7.3 & -1.1 \\ -1.1 & 4.8 \end{bmatrix}\right]^{-1} \begin{bmatrix} -3 \\ -2 \end{bmatrix} = [-3 \quad -2]\begin{bmatrix} 0.8143 & 0.1866 \\ 0.1866 & 1.2384 \end{bmatrix}\begin{bmatrix} -3 \\ -2 \end{bmatrix} =$$

$$= [-2.816 \quad -3.0367]\begin{bmatrix} -3 \\ -2 \end{bmatrix}$$

$T^2 = 14.5217$ and $T_{critical} = \frac{(n_1+n_2-2)P}{n_1+n_2-P-1} F_{P,n_1+n_2-P-1\,,\alpha} = \frac{42}{20} F_{2,20.0.90} = 5.44$

Since $T^2 = 14.5217 > T_{critical}$, we reject H_0 at 0.1 level of significant .

Or by use the maximum separation as following :

$$D^2 = (\bar{X}_1 - \bar{X}_2)^t \left[S_{pooled}\right]^{-1} (\bar{X}_1 - \bar{X}_2)$$

$$\left(\frac{n_1+n_2-P-1}{(n_1+n_2-2)P}\right)\left(\frac{n_1 n_2}{n_1+n_2}\right) D^2 \sim F_{P,n_1+n_2-P-1\,,\alpha}$$

b) Fisher's linear discriminant function is $\hat{y} = (\bar{X}_1 - \bar{X}_2)^t S_{pooled}^{-1} X = \hat{a}^t X$

$$\hat{y} = [-3 \quad -2]\begin{bmatrix} 7.3 & -1.1 \\ -1.1 & 4.8 \end{bmatrix}^{-1}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = [-3 \quad -2]\begin{bmatrix} 0.142 & 0.033 \\ 0.033 & 0.216 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$= [-0.49 \quad -0.53]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\hat{y} = -0.49x_1 - 0.53x_2$$

c) $x_0 = [0 \quad 1]$

Allocation x_0 to $\boldsymbol{\pi_1}$ if : $\hat{y}_0 \geq \hat{m}$ & Allocation x_0 to $\boldsymbol{\pi_2}$ if : $\hat{y}_0 < \hat{m}$

$$\hat{y}_0 = \hat{a}^t x_0 = -0.49\,(0) - 0.53\,(1) = -0.53$$

$$\hat{m} = \frac{1}{2}(\bar{y}_1 + \bar{y}_2) = \frac{1}{2}(1.0198 - 1.5105) = -0.2453$$

Where $\bar{y}_1 = \hat{a}^t \bar{X}_1 = [-0.49 \quad -0.53]\begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1.0198$

and $\bar{y}_2 = \hat{a}^t \bar{X}_2 = [-0.49 \quad -0.53]\begin{bmatrix} 2 \\ 1 \end{bmatrix} = -1.5105$

Since $\hat{y}_0 = -0.53 < \hat{m} = -0.2453$, **assign $\boldsymbol{x_0}$ to population $\boldsymbol{\pi_2}$.**

R code

```
# Exercise 3:
xbar1=c(-1,-1)
xbar2=c(2,1)
Spooles=matrix(c(7.3,-1.1,-1.1,4.8),2,2)
n1=11
n2=12
p=2
xbar=xbar1-xbar2

# part a:
(T2<- t(xbar)%*%solve((1/n1+1/n2)*Spooles)%*%xbar)

         [,1]
[1,] 14.52171

(T2.crit <- p*(n1+n2-2)/(n1+n2-p-1)*qf(0.90,p,n1+n2-p-1))

[1] 5.437434

ifelse(T2>T2.crit,"Reject H0","Fail to Reject H0")

     [,1]
[1,] "Reject H0"

#or test by Distance (maximum separation)
D2<- t(xbar)%*%solve(Spooles)%*%xbar
(t.s<-((n1+n2-p-1)/(n1+n2-2)*p)*(n1*n2/(n1+n2)) * D2 )

         [,1]
[1,] 27.66041

(T2.crit <- qf(0.90,p,n1+n2-p-1))
```

```
[1] 2.589254

ifelse(T2>T2.crit,"Reject H0","Fail to Reject H0")

     [,1]
[1,] "Reject H0"


# part b:Fisher's linear discriminant function
(at<- t(xbar1-xbar2)%*% solve(Spooles))

           [,1]       [,2]
[1,] -0.4906887 -0.5291162

x0<-c(0,1)
(m<- 1/2*(at%*%xbar1+at%*%xbar2))

           [,1]
[1,] -0.2453444

(y0 <- at%*%x0)

           [,1]
[1,] -0.5291162

ifelse(y0 >= m,"Assign X0 to First pop ","Assign X0 to Second pop")

     [,1]
[1,] "Assign X0 to Second pop"
```

Exercise 4: Discriminating owners from nonowners of riding mowers)

Consider two groups in a city: π_1: riding-mower owners, and π_2: nonowners riding-mowers. In order to identify the best sales prospects for an intensive sales campaign, a riding-mower manufacturer is interested in classifying families as prospective owners or nonowners on the basis of $x_1 = income$ and $x_2 = lot\ size$. Random samples of $n_1 = 12$ current owners and $n_2 = 12$ current nonowners yield the values in Table 11.1. [page 599pdf]

(a) Give the minimum ECM rule for assigning a new item to one of the two **Normal** populations. Assume equal costs, equal prior probabilities, and common covariance matrix. Then classify the new observation $x_0 = (95, 19)$ into population $\pi 1, \pi 2$,

(a) $\pi_1 \equiv$ Riding-mower owners; $\pi_2 \equiv$ Nonowners

Here are some summary statistics for the data in Example 11.1:

$$\bar{x}_1 = \begin{bmatrix} 109.475 \\ 20.267 \end{bmatrix}, \quad \bar{x}_2 = \begin{bmatrix} 87.400 \\ 17.633 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} 352.644 & -11.818 \\ -11.818 & 4.082 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 200.705 & -2.589 \\ -2.589 & 4.464 \end{bmatrix}$$

$$S_{\text{pooled}} = \begin{bmatrix} 276.675 & -7.204 \\ -7.204 & 4.273 \end{bmatrix}, \quad S_{\text{pooled}}^{-1} = \begin{bmatrix} .00378 & .00637 \\ .00637 & .24475 \end{bmatrix}$$

The linear classification function for the data in Example 11.1 using (11-19) is

$$\left(\begin{bmatrix} 109.475 \\ 20.267 \end{bmatrix} - \begin{bmatrix} 87.400 \\ 17.633 \end{bmatrix} \right)' \begin{bmatrix} .00378 & .00637 \\ .00637 & .24475 \end{bmatrix} x = \begin{bmatrix} .100 & .785 \end{bmatrix} x$$

where

$$\hat{m} = \frac{1}{2}(\bar{y}_1 + \bar{y}_2) = \frac{1}{2}(\hat{a}'\bar{x}_1 + \hat{a}'\bar{x}_2) = 24.719$$

$$\hat{y}_0 = (\bar{X}_1 - \bar{X}_2)^t S_{pooled}^{-1} X \quad ; \hat{m} = \frac{1}{2}(\bar{X}_1 - \bar{X}_2)^t S_{pooled}^{-1} (\bar{X}_1 + \bar{X}_2)$$

$\hat{y}_0 - \hat{m} \geq ln\left[\frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}\right]$; Where $\hat{y}_0 = \hat{a}^t x_0$; $\hat{m} = \frac{1}{2}(\bar{y}_1 + \bar{y}_2)$

$$\bar{y}_1 = (\bar{X}_1 - \bar{X}_2)^t S_{pooled}^{-1} \bar{X}_1 = \hat{a}^t \bar{X}_1 \quad ; \quad \bar{y}_2 = (\bar{X}_1 - \bar{X}_2)^t S_{pooled}^{-1} \bar{X}_2 = \hat{a}^t \bar{X}_2$$

Allocation x_0 to π_1 if : $\hat{y}_0 = 0.1\,x_1 + 0.785\,x_2 \geq \hat{m} = 24.745$,
$\hat{y}_0 = 24.44 < \hat{m}$ so, Assign x_0 to nonowners riding-mowers population.

(b) Using the function in (a) construct the "confusion matrix" by classifying the given observations. Compare your classification results with those of Figure 11.1, where the classification regions were determined "by eye." (See Example 11.6.)

we can construct the confusion matrix:

		Predicted Membership		
		π_1	π_2	Total
Actual membership	π_1	11	1	12
	π_2	2	10	12

(c) Given the results in (b), calculate the apparent error rate (APER).
(d) State any assumptions you make to justify the use of the method in Parts (a) and (b).

(c) The apparent error rate is $\frac{1+2}{12+12} = 0.125$

(d) The assumptions are that the observations from π_1 and π_2 are from multivariate normal distributions with equal covariance matrices, $\Sigma_1 = \Sigma_2 = \Sigma$.

(e) Using the linear discriminant function from part (a), Estimate the actual error rate (cross-validation).

The actual error rate$= \frac{3+2}{12+12} = 0.20833$

(f) Assuming **different covariance** matrices of the two populations, equal costs and equal prior probabilities ,Then classify the new observation $x_0 = (95, 19)$ into population π1, π2.

The Classification of **Normal** Populations When $\Sigma_1 \neq \Sigma_2$ (Quadratic Classification Rule)

Allocation x_0 to π_1 if :

$$-\frac{1}{2}x_0'(s_1^{-1} - s_2^{-1})x_0 + (\bar{x}_1' \; s_1^{-1} - \bar{x}_2' \, s_2^{-1})x_0 - \frac{1}{2}\ln\left(\frac{|s_1|}{|s_2|}\right) - \frac{1}{2}(\bar{x}_1' \; s_1^{-1}\bar{x}_1 - \bar{x}_2' \, s_2^{-1}\bar{x}_2) \geq \ln\left[\frac{c(1|2)}{c(2|1)} \frac{P_2}{P_1}\right]$$

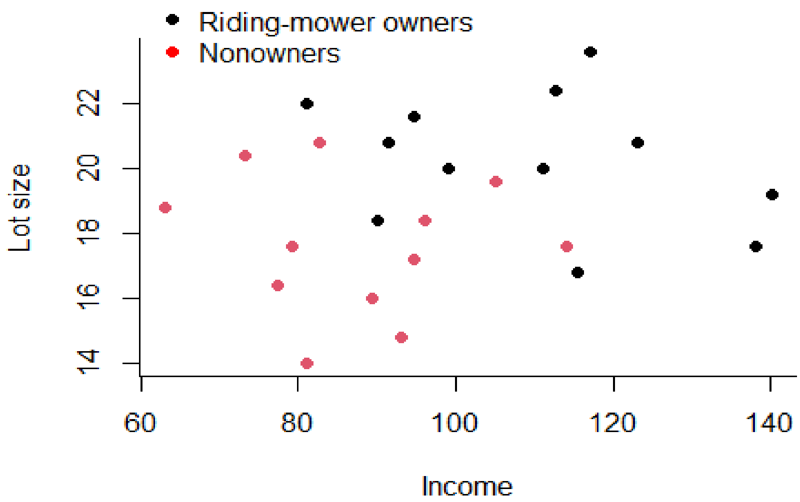
Allocate $x_0$to π_2 otherwise.

-0.5171489 $< \; ln(1) = 0$

Allocate $x_0$to π_2 nonowners riding-mowers

R code

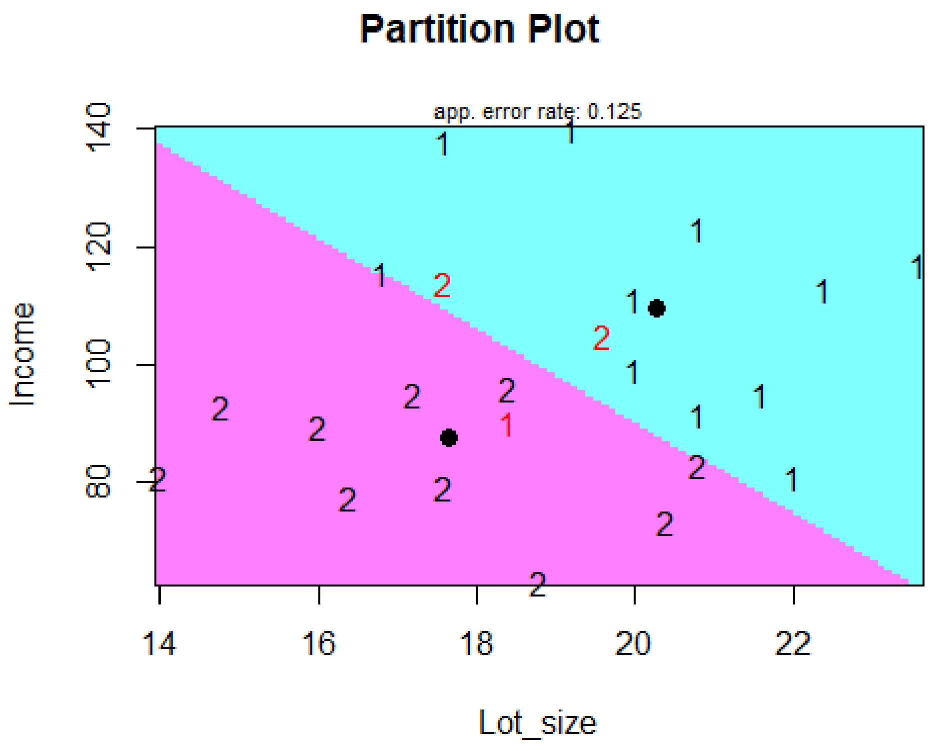
```
rm(list=ls())
# Exercise 4 (11.2) :
#install.packages("MASS")
library("MASS") #for classification
data1 <- read.table(choose.files()) #T11-1.DAT
names(data1) <- c("Income","Lot_size","groups")
data1[,3]<-as.factor(data1[,3])  #or as.factor(data$groups)
## plot
par(xpd=TRUE) # allows you to plot elements outside figure region
plot(data1$Income,data1$Lot_size,xlab="Income",ylab="Lot size",col=c(data1
$groups),pch=16,bty='L')
#Add Legends to Plots.
legend("topleft",inset= c(0.01,-0.15),legend = c("Riding-mower owners","No
nowners"),col=c("black","red") ,pch=16,bty='n')
```




```
#pch: symbol to use. #bty: determined the type of box.
## part a:
(xbar1<-as.matrix(colMeans(data1[data1$groups== "1",-3]),2))

              [,1]
Income   109.47500
Lot_size  20.26667

(xbar2<-as.matrix(colMeans(data1[data1$groups== "2" ,-3]),2))

             [,1]
Income   87.40000
Lot_size 17.63333

(S1<-var(data1[data1$groups== "1",-3]))
```

```
             Income   Lot_size
Income   352.64386 -11.818182
Lot_size -11.81818   4.082424

(S2<-var(data1[data1$groups== "2" ,-3]))

              Income  Lot_size
Income   200.705455 -2.589091
Lot_size  -2.589091  4.464242

table(data1$groups)


 1  2 
12 12 

n1=12 ; n2=12
# or
n1<-nrow(data1[data1$groups== "1",])
n2<-nrow(data1[data1$groups== "2" ,])

(Spooled<- ((n1-1)*S1+(n2-1)*S2)/(n1+n2-2))

              Income  Lot_size
Income   276.674659 -7.203636
Lot_size  -7.203636  4.273333

(ta<-t(xbar1-xbar2)%*% solve(Spooled))

         Income  Lot_size
[1,] 0.1002303 0.7851847

(m<- 1/2*(ta%*%xbar1+ta%*%xbar2))

         [,1]
[1,] 24.74567

x0<-matrix(c(95,19),nrow=2) #new observation.
(y0= ta%*%x0)
         [,1]

[1,] 24.44039
ifelse(y0 >= m,"Assign X0 to riding-mower owners pop ","Assign X0 to nonow
ners riding-mowers pop")

     [,1]                                  
[1,] "Assign X0 to nonowners riding-mowers pop"

#Classify the new observation using built function of Linear Discriminant
Analysis (lda).

model<-lda(groups~., data=data1 ,prior= c(0.5,0.5))
predict(model,newdata=data.frame(Income=95,Lot_size=19))

$class
[1] 2
Levels: 1 2
```

```
$posterior
          1         2
1 0.4242667 0.5757333

$x
        LD1
1 0.1475597

## part b & c: make confusion matrix and Calculate APER (page 619 and 619)
(confusion<-table(truth=data1$groups,fitted=predict(model)$class) )

     fitted
truth  1  2
    1 11  1
    2  2 10

(n<-sum(confusion))

[1] 24

(Eaper<- (n-sum(diag(confusion)))/n)

[1] 0.125

library("klaR") #for partition plot function
partimat(groups~ ., data=data1,method="lda",prec=100)
```




```
## part e:leave-one-out cross validation for linear discriminant analysis.
# cannot run the predict function using the object with CV = TRUE.
modelcv<-lda(groups~ ., data=data1,CV=TRUE ,prior= c(0.5,0.5))
```

```
# confusion matrix of cross validation
(confusion2<-table(truth=data1$groups,fitted=modelcv$class) )

     fitted
truth  1  2
    1 10  2
    2  3  9

(Eaper<- (n-sum(diag(confusion2)))/n)

[1] 0.2083333

## part f: Σ1 ≠ Σ2 (Quadratic Classification Rule)
-0.5*t(x0)%*%(solve(S1)-solve(S2))%*%x0 + (t(xbar1)%*%solve(S1)-t(xbar2)
%*%solve(S2))%*%x0 - 0.5*log(det(S1)/det(S2))- 0.5*(t(xbar1)%*%solve(S1)
%*%xbar1-t(xbar2)%*%solve(S2)%*%xbar2)

         [,1]
[1,] -0.5171489

#since -0.5171489 < log(1)=0, Allocate x_0 to π_2 nonowners riding-mowers.


modelq<-qda(groups~ ., data=data1,prior= c(0.5,0.5),method="mle")
predict(modelq,newdata=data.frame(Income=95,Lot_size=19))

$class
[1] 2
Levels: 1 2

$posterior
          1         2
1 0.3665828 0.6334172

#Allocate x_0 to π_2 nonowners riding-mowers
```