Tut. Sess. 9

Ch 11 - Math. Stat.

* pb. 11.24 p. 241

Five hundred losses are observed. Five of the losses are 1,100, 3,200, 3,300, 3,500, and 3,900. All that is known about the other 495 losses is that they exceed 4,000. Determine the maximum likelihood estimate of the mean of an exponential model.

Ans:

$$L(\theta) = f(1100)\,f(3200)\,f(3300)\,f(3500)\,f(3900)\cdot[S(4000)]^{495}$$

For exp. distn $X \sim \exp(\theta)$

p.d.f

$$f(x) = \frac{e^{-x/\theta}}{\theta}$$

$$F(x) = 1 - e^{-x/\theta}$$

$$= \theta^{-1} e^{-1100/\theta}\,\theta^{-1} e^{-3200/\theta}\,\theta^{-1} e^{-3300/\theta}\cdot\theta^{-1} e^{-3500/\theta}\,\theta^{-1} e^{-3900/\theta}\left[e^{-4000/\theta}\right]^{495}$$

$$\therefore L(\theta) = \theta^{-5} e^{-1995000/\theta}$$

$$\Rightarrow \ell(\theta) = -5\ln\theta - \frac{1995,000}{\theta}$$

To get $\hat{\theta}$, let $\ell'(\theta) = 0$

$$\frac{\partial \ell}{\partial \theta} = \frac{-5}{\theta} + \frac{1,995,000}{\theta^2} = 0$$

and the solution is $\hat{\theta} = \frac{1,995,000}{5} = 399,000$ #

* pb. 11.26 p. 241

A random sample of size 5 is taken from a Weibull distribution with $\tau = 2$. Two of the sample observations are known to exceed 50 and the three remaining observations are 20, 30, and 45. Determine the maximum likelihood estimate of θ.

Ans:

$$F(x) = 1 - e^{-(x/\theta)^2} \quad \text{(distn fn)}$$

$$f(x) = \frac{2x}{\theta^2} e^{-(x/\theta)^2} \quad \text{(p.d.f)}$$

For Weibull distn $-\theta, \tau$

p.d.f

$$f(x) = \frac{\tau (x/\theta)^{\tau} e^{-(x/\theta)^{\tau}}}{x}$$

c.d.f

$$F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

The likelihood fn is

$$L(\theta) = f(20)\,f(30)\,f(45)\,[1 - F(50)]^2$$

$$L(\theta) = 40\,\theta^{-2} e^{-(20/\theta)^2}\,60\,\theta^{-2} e^{-(30/\theta)^2}\,90\,\theta^{-2} e^{-(45/\theta)^2}\cdot\left[e^{-(50/\theta)^2}\right]^2$$

$$\therefore L(\theta) = 216000\,\theta^{-6} e^{-8325/\theta^2}$$

$$\Rightarrow \ell(\theta) = -6\ln\theta - 8,325\,\theta^{-2}$$, by neglecting the constant term.

To get $\hat{\theta}$, let $\ell'(\theta) = 0$

$$\frac{-6}{\theta} + 2(8325)\theta^{-3} = 0 \qquad \times \theta^3$$

$$\Rightarrow 6\theta^2 = 16650 \quad \therefore \hat{\theta} = \sqrt{\frac{16650}{6}} \simeq 52.68$$

* pb. 11.28 p. 241

A sample of 100 losses revealed that 62 were below 1000 and 38 were above 1000. An exponential distribution with mean θ is considered. Using only the given information, determine the maximum likelihood estimate of θ. Now given that the 62 losses that were below 1000 totaled 28140 while the total for the 38 above 1000 remains unknown. Using this additional information, determine the maximum likelihood estimate of θ.

Ans:

* for the 1st part of the pb

$$L(\theta) = [F(1000)]^{62}[1 - F(1000)]^{38}$$

$$L(\theta) = [1 - e^{-1000/\theta}]^{62}[e^{-1000/\theta}]^{38}$$

let $x = e^{-1000/\theta}$, Then

$$L(x) = (1-x)^{62} x^{38}$$

$$\Rightarrow \ell(x) = 62 \ln(1-x) + 38 \ln x$$

$$\Rightarrow \ell'(x) = \frac{-62}{1-x} + \frac{38}{x}$$

Set $\ell'(x) = 0 \Rightarrow \frac{-62x + 38(1-x)}{x(1-x)} = 0$

$\therefore 100x = 38 \quad \therefore x = 0.38$

$\therefore 0.38 = e^{-1000/\theta}$

$\therefore \hat{\theta} = -1000/\ln 0.38 = 1033.50$

لا يكتب في هذا الهامش

* for second part (with additional information)

$$L(\theta) = \left[\prod_{j=1}^{62} f(x_j)\right]\left[S(1000)\right]^{38}$$

$$= \theta^{-62} e^{-28,140/\theta} e^{-38,000/\theta}$$

$f(x) = \frac{e^{-x/\theta}}{\theta}$

$S(x) = e^{-x/\theta}$

$$L(\theta) = \theta^{-62} e^{-66,140/\theta}$$

$$\ell(\theta) = -62 \ln \theta - 66,140/\theta$$

$$\ell'(\theta) = \frac{-62}{\theta} + \frac{66,140}{\theta^2}$$

$$\Rightarrow -62\theta^{\cancel{2}} + 66,140\,\cancel{\theta} = 0$$

$$\therefore \hat{\theta} = 66,140/62 = 1066.77$$

H.W "Assignment*" #(3) Mid II ... last week (pb.11.1 p.232 Textbook)

H.W "Assignment*" # (4) Mid II

pb 11.20 p.241 Textbook

Hint for Exercise 11.20

Model	Data Set B "Complete Data"	Data Set B that Censored at 250
Exponential	$\hat{\theta} = 1424.4$	$\hat{\theta} = 594.14$
Gamma	$\hat{\alpha} = 0.55616$, $\hat{\theta} = 2,561.1$	$\hat{\alpha} = 1.5183$, $\hat{\theta} = 295.69$
Inverse exponential	$\hat{\theta} = 197.72$	$\hat{\theta} = 189.78$
Inverse gamma	$\hat{\alpha} = 0.70888$, $\hat{\theta} = 140.16$	$\hat{\alpha} = 0.41612$, $\hat{\theta} = 86.290$

Note: All pbs discussed in this session were including right censoring.