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Tut. Sess. 8

- pb. 11.2 p. 233

You are given the five observations 521, 658, 702, 819 and 1,217. Your model is the single-parameter Pareto distribution with distribution function

$$F(x) = 1 - \left(\frac{500}{x}\right)^{\alpha}, \quad x > 500, \quad \alpha > 0.$$

Determine the maximum likelihood estimate of α. $\hat{\alpha}$?? أطلب

Ans: $F(x) = 1 - \left(\frac{500}{x}\right)^{\alpha}$, $x > 500$, $\alpha > 0$

For $X \sim$ single-parameter Pareto α, θ

$f(x) = \dfrac{\alpha\theta^{\alpha}}{x^{\alpha+1}}$, $x > \theta$

See p. 502

$F(x) = 1 - \left(\frac{\theta}{x}\right)^{\alpha}$, $x > \theta$

$\Rightarrow \theta = 500$

$\Rightarrow f(x) = \dfrac{\alpha (500)^{\alpha}}{x^{\alpha+1}}, \quad x > 500$

$$\ln f(x|\alpha) = \ln\alpha + \alpha\ln 500 - (\alpha+1)\ln x$$

$$l(\alpha) = \sum_{j=1}^{n} \ln f_{X_j}(x_j|\alpha)$$

Revise lecture 20

$$\therefore l(\alpha) = \sum_{j=1}^{n}\left[\ln\alpha + \alpha\ln 500 - (\alpha+1)\ln x_j\right]$$

$$l(\alpha) = n\ln\alpha + n\alpha\ln 500 - (\alpha+1)\sum_{j=1}^{n}\ln x_j$$

To get $\hat{\alpha}$, set $l'(\alpha) = 0$

$$\Rightarrow \frac{n}{\alpha} + n\ln 500 - \sum_{j=1}^{n}\ln x_j = 0$$

For $n = 5$, and by calculating $\sum_{j=1}^{5}\ln x_j$ for the given data.

$$5\alpha^{-1} + 5\ln 500 - 33.1111 = 0$$

$$5\alpha^{-1} - 2.0381 = 0$$

$$\therefore \hat{\alpha} = \frac{5}{2.0381} \simeq 2.45$$

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• pb 11.3 p. 233

You have observed the following five claim severities: 11.0, 15.2, 18.0, 21.0 and 25.8. Determine the maximum likelihood estimate of μ for the following model (which is the reciprocal inverse Gaussian distribution; see Exercise 5.20a of [74]:

$$f(x) = \frac{1}{\sqrt{2\pi x}} \exp\left[-\frac{1}{2x}(x-\mu)^2\right], \quad x, \mu > 0$$

Ans:

$$\text{Data} = [11 \quad 15.2 \quad 18 \quad 21 \quad 25.8]$$

For reciprocal inverse Gaussian dist$\underline{n}$

$$f(x) = \frac{1}{\sqrt{2\pi x}} \exp\left[-\frac{1}{2x}(x-\mu)^2\right], \quad x, \mu > 0$$

$$\Rightarrow \ln f(x|\mu) = \ln\frac{1}{\sqrt{2\pi}} + \ln\frac{1}{\sqrt{x}} - \frac{1}{2x}(x-\mu)^2$$

$$= -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln x - \frac{1}{2x}(x^2 - 2x\mu + \mu^2)$$

$$\ln f(x|\mu) = \frac{-1}{2}\ln 2\pi - \frac{1}{2}\ln x - \frac{1}{2}x + \mu - \frac{\mu^2}{2x}$$

$$\therefore \ell(\mu) = \sum_{j=1}^{n} \ln f_{X_j}(x_j|\mu)$$

$$\therefore \ell(\mu) = \underbrace{-\frac{n}{2}\ln 2\pi - \frac{1}{2}\sum_{j=1}^{n}\ln x_j - \frac{1}{2}\sum_{j=1}^{n} x_j}_{\mu \text{ [illegible] } 3} + \mu n - \frac{\mu^2}{2}\sum_{j=1}^{n}\frac{1}{x_j}$$

To get $\hat{\mu}$, let $\ell'(\mu) = 0$

$$\therefore \ell'(\mu) = n - \mu\sum_{j=1}^{n}\frac{1}{x_j} = 0$$

$$\Rightarrow \ell'(\mu) = \cancel{n} - \cancel{n}\mu y = 0, \quad y = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{x_j}$$

$$\therefore \hat{\mu} = \frac{1}{y}, \quad y = \frac{1}{n}\sum_{j=1}^{n}\frac{1}{x_j}$$

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By using MATLAB

```
>> data = [11   15.2   18   21   25.8]
>> y = 1/5 * sum (1./data)
```
بدون مسافات

```
>> muo = 1/y
=> y = 0.0597
   muo = 16.7430
```

$\therefore \hat{\mu} = 16.7430$ #

- Pb 11.4 P. 233

The following values were calculated from a random sample of 10 losses:

$$\sum_{j=1}^{10} x_j^{-2} = 0.00033674, \quad \sum_{j=1}^{10} x_j^{-1} = 0.023999$$

$$\sum_{j=1}^{10} x_j^{-0.5} = 0.34445, \quad \sum_{j=1}^{10} x_j^{0.5} = 488.97$$

$$\sum_{j=1}^{10} x_j = 31{,}939, \quad \sum_{j=1}^{10} x_j^2 = 211{,}498{,}983$$

Losses come from a Weibull distribution with $\tau = 0.5$ [So $F(x) = 1 - e^{-(x/\theta)^{0.5}}$]. Determine the maximum likelihood estimate of θ.

Ans:

$F(x) = 1 - e^{-(x/\theta)^{0.5}}$

For $X \sim$ Weibull $\underset{\text{scale}}{\theta}, \tau \rightarrow$ shape

$$f(x) = \frac{\tau (x/\theta)^{\tau} e^{-(x/\theta)^{\tau}}}{x}$$

$$F(x) = 1 - e^{-(x/\theta)^{\tau}}$$

See p. 498

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$$\Rightarrow \quad f(x) = \frac{0.5\,(x/\theta)^{0.5}\, e^{-(x/\theta)^{0.5}}}{x}$$

$$\ln(f(x)/\theta) = \ln\left(\tfrac{1}{2}\right) + 0.5\ln(x/\theta) - (x/\theta)^{0.5} - \ln x$$

$$\therefore \quad \ell(\theta) = \sum_{j=1}^{n} \ln f_{X_j}(x_j/\theta)$$

$$\therefore \ell(\theta) = -n\ln 2 + 0.5\sum_{j=1}^{n}\ln(x_j/\theta) - \sum_{j=1}^{n}\left(\frac{x_j}{\theta}\right)^{0.5} - \sum_{j=1}^{n}\ln x_j$$

Set $n=10$ and rearrange the terms as follows

$$\ell(\theta) = \underline{-10\ln 2} + \underline{0.5\sum_{j=1}^{10}\ln x_j} - 0.5\sum_{j=1}^{10}\ln\theta$$

$$-\sum_{j=1}^{10}\left(\frac{x_j}{\theta}\right)^{0.5} - \underline{\sum_{j=1}^{10}\ln x_j}$$

الحد 3 بعده لا يتغير بدل θ

$$\therefore \ell'(\theta) = -0.5\sum_{j=1}^{10}\frac{1}{\theta} + \frac{1}{2}\sum_{j=1}^{10} x_j^{0.5}\,\theta^{-1.5}$$

$-\theta^{-0.5} \Rightarrow \frac{1}{2}\theta^{-1.5}$

$$\ell'(\theta) = \frac{-5}{\theta} + \frac{\theta^{-1.5}}{2}\sum_{j=1}^{10} x_j^{0.5}$$

$$\therefore \ell'(\theta) = \frac{-5}{\theta} + \frac{488.97}{2}\,\theta^{-1.5} = \frac{-5}{\theta} + 244.485\,\theta^{-1.5}$$

Set $\ell'(\theta) = 0$ to get $\hat{\theta}$

$$\frac{-5}{\theta} + 244.485\,\theta^{-1.5} = 0 \qquad \times\theta$$

$$\Rightarrow \quad 244.485\,\theta^{-0.5} = 5$$

$$\sqrt{\theta} = \frac{244.485}{5}$$

$$\therefore \hat{\theta} = (48.897)^2 \simeq 2390.9166$$

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