1//

Tut. Ses. (2)

*Revise Lectures (4) and (5)

• Pb (3.5) P. 26 Textbook

Determine the mean excess loss function for Models 1–4. Compare the function for Models 1, 2, and 4.

Ans:

For Model (1)

$$F_1(x) = \begin{cases} 0, & x < 0 \\ 0.01x, & 0 \le x < 100 \\ 1 & x \ge 100 \end{cases}$$

(3)' $\Rightarrow$

$$e(d) = \frac{\int_d^\infty S(x)\,dx}{S(d)}$$

$$= \frac{-1}{0.01(1-0.01d)} \left[\frac{(1-0.01x)^2}{2} \right]_d^{100}$$

$$= 0 + \frac{(1-0.01d)^{\cancel{2}}}{0.02\cancel{(1-0.01d)}}$$

$$e(d) = \frac{1-0.01d}{0.02} = \frac{100-d}{2} \quad (1)$$

For Model (2)

$$F_2(x) = \begin{cases} 0 & x < 0 \\ 1 - \left(\frac{2000}{x+2000}\right)^3, & x \ge 0 \end{cases}$$

$$e(d) = \frac{\int_d^\infty \left(\frac{\cancel{2000}}{x+2000}\right)^3 dx}{\left(\frac{\cancel{2000}}{d+2000}\right)^3}$$

$$e(d) = \frac{\left[(x+2,000)^{-2} \right]_d^\infty}{-2\left(\frac{1}{d+2,000}\right)^3} = \frac{-(d+2,000)^{\cancel{3}}}{2} \cdot \frac{-1}{\cancel{(d+2,000)^2}}$$

$$e(d) = \frac{2,000+d}{2} \quad (2)$$

2// For Model ③

$$F_3(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \leq x < 1 \\ 0.75, & 1 \leq x < 2 \\ 0.87, & 2 \leq x < 3 \\ 0.95, & 3 \leq x < 4 \\ 1, & x \geq 4 \end{cases}$$

For discrete r.v X, the mean excess loss function is

$$e_X(d) = \frac{\sum_{x_j > d} (x_j - d)\, P(x_j)}{1 - F(d)}.$$

we have to find

$$\frac{\sum_{x_j > d} (x_j - d)\, P(x_j)}{1 - F(d)} \; ; \; 0 \leq d < 1 \; , \quad \frac{\sum_{x_j > d} (x_j - d)\, P(x_j)}{1 - F(d)} \; ; \; 1 \leq d < 2$$

$$\frac{\sum_{x_j > d} (x_j - d)\, P(x_j)}{1 - F(d)} \; ; \; 2 \leq d < 3 \; , \quad \frac{\sum_{x_j > d} (x_j - d)\, P(x_j)}{1 - F(d)} \; ; \; 3 \leq d < 4$$

x_j	0	1	2	3	4	Σ
$P(x_j)$	0.5	0.25	0.12	0.08	0.05	1

3/

$$e(d) = \begin{cases} \dfrac{0.25(1-d)+0.12(2-d)+0.08(3-d)+0.05(4-d)}{1-0.5} = 1.86-d & , 0\le d<1 \\ \dfrac{0.12(2-d)+0.08(3-d)+0.05(4-d)}{1-0.75} = 2.72-d & , 1\le d<2 \\ \dfrac{0.08(3-d)+0.05(4-d)}{1-0.87} = 3.3846-d & , 2\le d<3 \\ \dfrac{0.05(4-d)}{1-0.95} = 4-d & , 3\le d<4 \end{cases}$$

(3)

For Model (4)

$$F_4(x) = \begin{cases} 0, & x<0 \\ 1-0.3\,e^{-0.00001x}, & x\ge 0 \end{cases}$$

$$e(d) = \frac{\int_d^{\infty} \cancel{0.3}\, e^{-0.00001x}}{\cancel{0.3}\, e^{-0.00001d}}$$

$$e(d) = -100{,}000\, \frac{\left[e^{-0.00001x}\right]_d^{\infty}}{e^{-0.00001d}}$$

$$\therefore\ e(d) = 100{,}000\, \frac{\cancel{e^{-0.00001d}}}{\cancel{e^{-0.00001d}}} = 100{,}000$$

(4)

* the functions are straight lines for Model 1, 2 and 4. Model 1 has negative slope, Model 2 has positive slope, and Model 4 is horizontal (Constant $f^{\underline{n}}$).

4/

pb (3.9) p. 26

Determine the limited expected value function for Models 1-4.

Ans:

$$E[(X\wedge u)^k] = \int_{-\infty}^{u} x^k f(x)\,dx + u^k[1-F(u)]$$

$$E[(X\wedge u)^k] = \sum_{x_j\leq u} x_j^k\, p(x_j) + u^k[1-F(u)]$$ (1)

&

$$E[(X\wedge u)^k] = -\int_{-\infty}^{0} k x^{k-1} F(x)\,dx + \int_0^u k x^{k-1} S(x)\,dx$$

$\Rightarrow$ at k=1, $$E[(X\wedge u)] = -\int_{-\infty}^{0} F(x)\,dx + \int_0^u S(x)\,dx$$ (2)

&

$$E(X) = e(d)\,S(d) + E(X\wedge d)$$ (3)

* For Model (1) $F_1(x) = 0.01x, \quad 0\leq x<100$

(1) $\Rightarrow$

$$E[X\wedge u] = \int_0^u x(0.01)\,dx + u(1-0.01u)$$

$$= 0.01\left[\frac{x^2}{2}\right]_0^u + u - 0.01u^2$$

$\therefore E[X\wedge u] = 0.005u^2 + u - 0.01u^2 = u(1-0.005u)$

Also, we can use (2) to get the same result.

For Model (2)

$$F_2(x) = 1-\left(\frac{2000}{x+2000}\right)^3, \quad x\geq 0$$

$\rightarrow S(x) = \left(\frac{2000}{x+2000}\right)^3$

$$\Rightarrow f_2(x) = \frac{3(2000)^3}{(x+2000)^4}$$

(1) $\Rightarrow$

$$E(X\wedge u) = \int_0^u x\cdot\frac{3(2000)^3}{(x+2000)^4}\,dx + u\left(\frac{2000}{u+2000}\right)^3$$

5

$$I = \int_0^u x \cdot \frac{3(2000)^3}{(x+2000)^4}\,dx$$

$$I = 3(2000)^3 \int_0^u \left[\frac{\cancel{(x+2000)}}{(x+2000)^{\cancel{4}\,3}} - \frac{2000}{(x+2000)^4} \right] dx$$

$$I = 3(2000)^3 \left[\frac{(x+2000)^{-2}}{-2} \right]_0^u - 3(2000)^4 \left[\frac{(x+2000)^{-3}}{-3} \right]_0^u$$

$$I = \frac{3}{2}(2000)^3 \left[\frac{1}{(2000)^2} - \frac{1}{(u+2000)^2} \right] - (2000)^4 \left[\frac{1}{(2000)^3} - \frac{1}{(u+2000)^3} \right] \quad (*)$$

$$\therefore E(X \wedge u) = 1{,}000 - \frac{3}{2}\frac{(2000)^3}{(u+2000)^2} + \frac{(2000)^4}{(u+2000)^3} + u\left(\frac{2000}{u+2000}\right)^3$$

$$= 1{,}000 - \left(\frac{2000}{u+2000}\right)^3 \left[\frac{3}{2}(u+2000) - 2000 - u \right]$$

$$= 1000 - \frac{(2000)^3}{(u+2000)^3}\left(\frac{1}{2}u + 1000\right)$$

$$= 1000 - \frac{(2000)^3}{(u+2000)^{\cancel{3}\,2}} \left(\frac{\cancel{u+2000}}{2} \right)$$

$$E(X \wedge u) = 1000 \left[1 - \frac{4000{,}000}{(u+2000)^2} \right]$$

Also, We Can use Formula (2) to get the same Result as follows:

$$E(X \wedge u) = -\int_{-\infty}^{0} F(x)\,dx + \int_0^u S(x)\,dx$$

$$E(X \wedge u) = \cancel{-\int_{-\infty}^{0} 0\,dx}_{\,0} + \int_0^u \left(\frac{2000}{x+2000}\right)^3 dx$$

6//

$$E(X \wedge u) = \int_0^u \left(\frac{2000}{x+2000}\right)^3 dx$$

$$= (2000)^3 \left[\frac{(x+2000)^{-2}}{-2}\right]_0^u$$

$$= \frac{1}{2}(2000)^3 \left[\frac{1}{(2000)^2} - \frac{1}{(u+2000)^2}\right]$$

$$E(X \wedge u) = 1000 \left[1 - \frac{4000000}{(u+2000)^2}\right]$$

Also, We can find $E(X \wedge u)$ for Model ② as follows:
By using formula ③

$$E(X \wedge u) = E(X) - e(u)\, S'(u)$$

$$E(X) = \int_0^\infty \frac{3(2000)^3 x}{(x+2000)^4} dx$$

$u \to \infty$ in (*) $\Rightarrow E(X) = 1000$

$\because S'(u)$ for Model ② is $S'(u) = \left(\frac{2000}{u+2000}\right)^3$

& $e(u) = \frac{2000+u}{2}$ revise pb 3.5 p.26

$$E(X \wedge u) = 1000 - \left(\frac{2000+u}{2}\right)\left(\frac{2000}{u+2000}\right)^3$$

$$\therefore E(X \wedge u) = 1000 \left[1 - \frac{4{,}000{,}000}{(u+2000)^2}\right]$$

* Complete the solution for Models ③ and ④

i.e Find $E(X \wedge u)$ for Models ③ and ④

H.W (Assignment #3)