Tut. Session (12)

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• EX 17.20 p. 439 Textbook

Your friend selected at random one of two urns and then he pulled a ball with number 4 on it from the urn. Then he replaced the ball in the urn. One of the urns contains four balls, numbered 1–4. The other urn contains six balls, numbered 1–6. Your friend will make another random selection from the same urn.

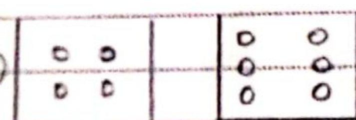
(a) Estimate the expected value of the number on the next ball using the Bayesian method.

(b) Estimate the expected value of the number on the next ball using Bühlmann credibility.

Ans:

let Θ represent the selected urn, and let X represent the selected number.

i.e $\theta = 1$ or $\theta = 2$ & $x = 1,2,3,4$ for urn ① ; $x = 1,2,3,4,5,6$ for urn ②



urn ① $\theta = 1$ urn ② $\theta = 2$

The r.v Θ is the risk parameter where $\pi(\theta) = pr(\Theta = \theta)$

So, the marginal probability function $f_X(x)$ can be written as

$$f_X(x) = \sum_{\theta} f(x|\theta)\,\pi(\theta)$$

$$\therefore f_X(4) = f(4|\theta=1)\,\pi(1) + f(4|\theta=2)\,\pi(2)$$

$$= \tfrac{1}{4}\left(\tfrac{1}{2}\right) + \tfrac{1}{6}\left(\tfrac{1}{2}\right) = \tfrac{5}{24}$$ which is the prob. of drawing 4,

* the posterior prob. for urn 1 is

$$\pi(\theta=1|X=4) = \frac{f_{X|\Theta}(4|1)\,\pi(1)}{f_X(4)} = \frac{\frac{1}{4}\left(\frac{1}{2}\right)}{5/24} = \frac{3}{5}$$

and the posterior prob. for urn 2 is

$$\pi(\theta=2|X=4) = \frac{f_{X|\Theta}(4|2)\,\pi(2)}{f_X(4)} = \frac{\frac{1}{6}\left(\frac{1}{2}\right)}{5/24} = \frac{2}{5}$$

or directly, $\pi(\theta=2|X=4) \overset{c}{=} 1 - \frac{3}{5} = \frac{2}{5}$

* Now, we want to find the expected value of the next observation

$$E(X_2|X_1=4) \quad ???$$

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(a) The expected value of the next observation by using Bayesian credibility method is

$$E(X_2|X_1=4) = \sum_{\theta} \underbrace{\mu(\theta)}_{\text{hyp. means}} \underbrace{\pi(\theta|x)}_{\text{Post. prob.}}$$

$\mu(1) = \mu(\theta=1)$ and
$\mu(2) = \mu(\theta=2)$

$$= \mu(1)\, \pi(\theta=1|X=4) + \mu(2)\, \pi(\theta=2|X=4)$$

where $\mu(1)$ and $\mu(2)$ are the hypothetical means

$$\mu(1) = E(X|\theta=1) = \frac{1+2+3+4}{4} = 2.5$$

$$\mu(2) = E(X|\theta=2) = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

$$\therefore E(X_2|X_1=4) = 2.5\left(\tfrac{3}{5}\right) + 3.5\left(\tfrac{2}{5}\right) = 2.9 \simeq \boxed{3}$$

(b) The expected value of the next observation by using Bühlmann credibility method can be estimated as follows.

* The expected value of the hypothetical means μ,

$$\mu = E[\mu(\theta)] = \sum_{\theta} \mu(\theta)\pi(\theta)$$

$\mu(\theta)$	2.5	3.5
$\pi(\theta)$	$1/2$	$1/2$

$$\therefore \mu = \tfrac{1}{2}(2.5) + \tfrac{1}{2}(3.5) = \tfrac{1}{2}(2.5+3.5) = 3$$

* The expected value of the process variance v,

$$v = E[v(\theta)] = \sum_{\theta} v(\theta)\pi(\theta) = \tfrac{1}{2} v(1) + \tfrac{1}{2} v(2)$$

$$v_1 = \tfrac{1}{4}(1^2+2^2+3^2+4^2) - (2.5)^2 = 1.25$$

$$v_2 = \tfrac{1}{6}(1^2+2^2+3^2+4^2+5^2+6^2) - (3.5)^2 = 2.917$$

$$\therefore v = \tfrac{1}{2}[v(1)+v(2)] = \tfrac{1}{2}[1.25+2.917].$$

$$\therefore v = 2.0835$$

* The variance of the hypothetical means is

$$a = Var[\mu(\theta)]$$

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$\therefore a = E\{[\mu(\Theta)]^2\} - \{E[\mu(\Theta)]\}^2$

$= [\frac{1}{2}(2.5)^2 + \frac{1}{2}(3.5)^2] - 3^2$

$\therefore a = \frac{1}{2}[(2.5)^2 + (3.5)^2] - 9 = 0.25$,

$K = \frac{\upsilon}{a} = \frac{2.0835}{0.25} = 8.334$,

and the Bühlmann credibility factor is

$$Z = \frac{n}{n+K} = \frac{1}{1+8.334} = 0.1071352$$

$\therefore$ the Bühlmann credibility permium is

$E(X_2|X_1=4) = Z\bar{X} + (1-Z)\mu$

$= 0.1071352(4) + 0.8928648(3)$

$\therefore E(X_2|X_1=4) = 3.1071 \simeq \boxed{3}$, where $\bar{X}=4$ (past number)

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