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Tut. Session (12) a

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Risk 1 produces claims of amounts 100, 1000 and 20.000 with probabilities 0.5, 0.3 and 0.2, respectively. For risk 2 the probabilities are 0.7, 0.2 and 0.1. Risk 1 is twice as likely as risk 2 of being observed. A claim of 100 is observed but the observed risk is unknown.

(a) Determine the Bayesian credibility estimate of the expected value of the second claim amount from the same risk.

(b) Determine the Bühlmann credibility estimate of the expected value of the second claim amount from the same risk.

Ans: (a)

Risk	100	1000	20 000	μ ?	σ ?	$Pr(\Theta=\theta)$
1	0.5	0.3	0.2	4350	61382500	2/3
2	0.7	0.2	0.1	2270	35054100	1/3

Clearly, $\pi(\theta=1)=\frac{2}{3}$, $\pi(\theta=2)=\frac{1}{3}$

The marginal probability is

$$f_X(x)=\sum_{\theta} f(x|\theta)\,\pi(\theta)$$

$$\therefore f(100)=f(100|1)\pi(1)+f(100|2)\pi(2)$$
$$=0.5\left(\frac{2}{3}\right)+0.7\left(\frac{1}{3}\right)=\frac{17}{30}$$

The posterior probabilities are given by

$$\pi(1|100)=\frac{f(100|1)\pi(1)}{f(100)}=\frac{0.5(2/3)}{17/30}=\frac{1/3}{17/30}=\frac{10}{17},$$

and $\pi(2|100)=1-\frac{10}{17}=\frac{7}{17}$

The hypothetical means are

$$\mu(1)=100(0.5)+1000(0.3)+20.000(0.2)=4350$$
$$\mu(2)=100(0.7)+1000(0.2)+20,000(0.1)=2270$$

The expected next value through Bayesian premium is

$$E(X_2|100)=\pi(1|100)\,\mu(1)+\pi(2|100)\,\mu(2)$$
$$=\left(\frac{10}{17}\right)(4350)+\frac{7}{17}(2270)=3493.53$$

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(b) For Bühlmann credibility
The expected value of the hypothetical means is

$$\mu = E[\mu(\Theta)]$$

$$\therefore \mu = \underset{\text{risk}_1}{\pi(1)}\,\underset{\text{hypo}_1}{\mu(1)} + \underset{\text{risk}_2}{\pi(2)}\,\underset{\text{hypo}_2}{\mu(2)}$$

$$\therefore \mu = \left(\frac{2}{3}\right)(4350) + \left(\frac{1}{3}\right)(2270) = 3656.67$$

The process variance is

$$v(\theta) = Var(X_j \mid \Theta = \theta)$$

$$v(1) = 100^2(0.5) + 1000^2(0.3) + (20,000)^2(0.2) - (4350)^2$$
$$= 61,382,500$$

$$v(2) = 100^2(0.7) + 1000^2(0.2) + (20,000)^2(0.1) - (2270)^2$$
$$= 35054100$$

$\therefore$ The expected value of the process variance is

$$v = E[v(\Theta)]$$
$$= \frac{2}{3}(61,382,500) + \frac{1}{3}(35054100)$$
$$= 52,606,366.67$$

The variance of the hypothetical means is

$$a = Var[\mu(\Theta)]$$

$$= (2/3)(4350)^2 + (1/3)(2270)^2 - (3656.67)^2$$
$$= 961397.8444$$

$$k = \frac{v}{a} = \frac{52606366.67}{961397.8444} = 54.7186$$

The Bühlmann credibility factor is

$$Z = \frac{n}{n+k} = \frac{1}{1+54.7186} = 0.017947$$

The Bühlmann credibility estimate is

$$E(X_2 \mid 100) = P_c = Z\bar{X} + (1-Z)\mu$$
$$= (0.017947)(100) + (0.982053)(3656.67)$$
$$= 3592.84 \quad \text{where } X_1 = 100.$$