d) The mode is $\mu = 10 \Rightarrow$ mode = mean = median $= 10$

Question (4): Let X represent the number of policies sold by an agent in a day. The moment generating function of X is

$$M(t) = 0.45e^{t} + 0.35e^{2t} + 0.15e^{3t} + 0.05e^{4t}, for -\infty < t < \infty.$$

a) The probability function of X is

$M_X(t) = E(e^{tx})$

$e^{t} \Rightarrow x = 1$
$e^{2t} \Rightarrow x = 2$ and so on

$P(X=1) = 0.45$, $P(X=2) = 0.35$
$P(X=3) = 0.15$, $P(X=4) = 0.05$

X is a discrete r.v

b) The standard deviation of X is

$* E(X) = \sum_{1}^{4} x \cdot f(x) = (1 \times 0.45) + (2 \times 0.35) + (3 \times 0.15) + (4 \times 0.05)$
$= \frac{9}{5}$

$* E(x^2) = \sum_{1}^{4} x^2 \cdot f(x) = (1^2 \times 0.45) + (2^2 \times 0.35) + (3^2 \times 0.15) + (4^2 \times 0.05)$
$= 4$

$S.d = \sqrt{E(x^2) - (E(x))^2} = \sqrt{4 - (\frac{9}{5})^2} = 0.872$

Question (5): Let X equal the number of tosses of a fair die until the first "1" appears.

a) What is the distribution of X.

$X \sim$ geometric$(p = \frac{1}{6})$

$f(x) = (\frac{1}{6})(\frac{5}{6})^{x-1}$, $x = 1, 2, \ldots$

Question (1): The distribution of the size of claims paid under an insurance policy has probability density function

$$f(x) = cx^{1.5}, \qquad 0 < x < 5.$$

a) The value of c is

$$\int_0^5 c x^{1.5}\, dx = 1 \Rightarrow c \cdot \frac{x^{2.5}}{2.5}\Big]_0^5 = 1 \Rightarrow c\left[\frac{5^{2.5}}{2.5} - 0\right] = 1$$

$$\Rightarrow c = 0{,}04472$$

b) $P(4 < X < 10) = \int_4^5 0{,}04472\, x^{1.5}\, dx = 0{,}4275$

c) $E(X^3) = \int_0^5 x^3 \cdot 0{,}04472\, x^{1.5}\, dx = 56.81$

Question (2): The number of traffic accidents per week at intersection Q has a Poisson distribution with mean 3. The number of traffic accidents per week at intersection R has a Poisson distribution with mean 1.5.

a) The probability that the number of accidents at intersection R exceeds its mean is $R \sim P(1.5)$

$$P(R > 1.5) = 1 - P(R \le 1.5) = 1 - [P(R=0) + P(R=1)]$$

$$= 1 - \sum_{R=0}^{1} e^{-1.5} \cdot \frac{(1.5)^R}{R!}$$

$$= 0{,}4422$$

b) $V\left(\frac{1}{2}X+1\right) = \frac{1}{4}Var(x)$

$= \frac{1}{4} \cdot \frac{1-p}{p^2}$

$= \frac{1}{4} \cdot \frac{1-\frac{1}{6}}{(\frac{1}{6})^2}$

$= \frac{15}{2}$

c) $P(X=4) = (\frac{1}{6})(\frac{5}{6})^{4-1}$

$= 0,09645$

Question (6): The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident (T) is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year.

a) The value of parameter λ is

$P(T \leq 50) = 0.3 \Rightarrow 1-e^{-50\lambda} = 0.3$

$e^{-50\lambda} = 0.7$

$\ln(e^{-50\lambda}) = \ln(0.7)$

$-50\lambda = \ln(0.7)$

$\lambda = \frac{\ln(0.7)}{-50}$

b) Calculate the portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year.

$P(T \leq 80) = 1-e^{-80 \times \frac{\ln(0.7)}{-50}}$

$= 0,435$

c) $P(T > 80|T > 50) =$ $P(T>50+30|T>50)$

$= P(T>30)$

$= e^{-30 \times \frac{\ln(.7)}{-50}}$

$= 0.8073$

Question (7): In a group of 25 factory workers, 20 are low-risk and five are high-risk. Two of the 25 factory workers are randomly selected without replacement.

a) Calculate the probability that exactly one of the two selected factory workers are low-risk.

$X=1$

$M=25, K=20, n=2$

$X \sim hypergeometric(M=25, n=2, K=20)$

$P(X=1) = \frac{\binom{20}{1} \cdot \binom{25-20}{2-1}}{\binom{25}{2}}$

$= \frac{1}{3}$

b) The expected number of selected workers who are low- risk is.

$E(X) = n \cdot \frac{K}{M} = 2 \cdot \frac{20}{25} = \frac{8}{5}$

c) The variance of the number of selected workers who are low risk is.

$Var(X) = \frac{n \cdot K(M-K) \cdot (M-n)}{M^2(M-1)}$

$= \frac{2 \cdot 20(5)(23)}{25^2(24)}$

$= \frac{23}{3}$

End of Questions

$Var(Q) = E(Q^2) - [E(Q)]^2$

b) $E(Q^2) =$ $Var(Q) = 3$, $E(Q) = 3$

$E(Q^2) = Var(Q) + (E(Q))^2$

$= 3 + (3)^2 = 12$

c) $E(3R - 2Q) =$

$3E(R) - 2E(Q)$

$= 3(1.5) - 2(3)$

$= -\frac{3}{2}$

Question (3): The working lifetime, in years (X), of a particular model of bread maker is normally distributed with mean 10 and variance 4.

$\mu = 10, \quad \sigma = \sqrt{4}$

a) Calculate the 12th percentile of the working lifetime, in years.

$X \sim N(10, 4)$

$F(x) = 0.12 \Rightarrow P(X \le x) = 0.12 \Rightarrow P\left(\frac{X-\mu}{\sigma} \le \frac{x-10}{\sqrt{4}}\right) = 0.12$

$\Rightarrow \frac{x-10}{\sqrt{4}} = -1.175 \Rightarrow x = 7.65$

b) Calculate the probability of the working lifetime will be more than 6, in years.

$P(X > 6) = P\left(\frac{X-\mu}{\sigma} > \frac{6-10}{\sqrt{4}}\right)$

$= P(Z > -2) = 1 - P(Z \le -2)$

$= 1 - 0.0228 = 0.9772$

10 12

c) $P(X > 10 | X < 12) =$

$\frac{P(10 < X < 12)}{P(X < 12)} = \frac{P\left(\frac{10-10}{\sqrt{4}} < \frac{X-\mu}{\sigma} < \frac{12-10}{\sqrt{4}}\right)}{P\left(\frac{X-\mu}{\sigma} < \frac{12-10}{\sqrt{4}}\right)}$

$= \frac{P(0 < Z < 1)}{P(Z < 1)} = \frac{P(Z \le 1) - P(Z \le 0)}{P(Z < 1)}$

$= \frac{0.8413 - 0.5}{0.8413} = 0.4057$