STAT 216: Actuarial Probability **First Midterm** **Second Semester 1447/2026**

Question (1): Let $P(A \cap B) = 0.2$, $P(A) = 0.6$, and $P(B) = 0.5$. Find

(a) $P(A^c \cap B) =$

1.5 $= P(B) - P(A \cap B) = 0.5 - 0.2 = 0.3$

(b) $P(A|B) =$

1.5 $= \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.2}{0.5} = 0.4$

(c) $P(A^c \cup B^c) =$

1.5 $= P(\overline{A \cap B}) = 1 - P(A \cap B) = 1 - 0.2 = 0.8$

Question (2): A calculator has a random number generator button which, when pressed, displays a random digit 0, 1, …, 9. The button is pressed four times. Assuming that the numbers generated are independent of one another, calculate

(a) Assuming that the numbers generated are independent of one another, how many ways for obtaining any 4 random digits in any order.

1.5 $10^4 = 10000$

(b) Assuming that the numbers generated are independent of one another, how many ways for obtaining one "0", one "5", and two "9"'s in any order.

1.5 $\dfrac{4!}{1!\,1!\,2!} = 12$

12

Question (4): An insurance agent meets twelve potential customers independently, each of whom is equally likely to purchase an insurance product. Six are interested only in auto insurance, four are interested only in homeowners insurance, and two are interested only in life insurance. The agent makes six sales. Calculate

(a) the probability that two are for auto insurance, two are for homeowners insurance, and two are for life insurance.

1.5 $\frac{\binom{6}{2}\binom{4}{2}\binom{2}{2}}{\binom{12}{6}} = 0.0974$

(b) the probability that six are for life insurance.

1 $\boxed{\text{zero}}$ because there are just 2 interested in life insurance so this is imposible there are 6 interested in life insurance

(c) the probability that three are for auto insurance.

1.5 $\frac{\binom{6}{3}\binom{6}{3}}{\binom{12}{6}} = 0.4329$

Question (5): A survey of 100 TV viewers revealed that over the last year:

i) 34 watched CBS. = C
ii) 15 watched NBC. = N
iii) 10 watched ABC. = A
iv) 7 watched CBS and NBC.
v) 6 watched CBS and ABC.
vi) 5 watched NBC and ABC.
vii) 4 watched CBS, NBC, and ABC.
viii) 18 watched HGTV, and of these, none watched CBS, NBC, or ABC.

H = is mutully exclusive

Question (7): An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06 $= P(A\mid B_1)$	0.08 $= P(B_1)$
21-30	0.03 $= P(A\mid B_2)$	0.15 $= P(B_2)$
31-65	0.02 $= P(A\mid B_3)$	0.49 $= P(B_3)$
66-99	0.04 $= P(A\mid B_4)$	0.28 $= P(B_4)$

A randomly selected driver that the company insures has an accident,

(a) calculate the probability that the driver was age 16-20.

$$P(B_1\mid A) = \frac{P(B_1 \cap A)}{P(A)} = \frac{P(A\mid B_1)\,P(B_1)}{P(A\mid B_1)\,P(B_1) + P(A\mid B_2)\,P(B_2) + P(A\mid B_3)\,P(B_3) + P(A\mid B_4)\,P(B_4)}$$

$$= \frac{(0.06)(0.08)}{(0.06)(0.08) + (0.03)(0.15) + (0.02)(0.49) + (0.04)(0.28)} = 0.1584$$

1.5

End of Questions

(c) If the numbers are generated without replacement, how many ways for obtaining any 4 random digits.

1.5

$10^4 = 10000$ $\binom{10}{4} = 210$

$_{10}P_4 = 5040$

Question (3): In a group of health insurance policyholders, 20% have high blood pressure (B) and 30% have high cholesterol (C). Of the policyholders with high blood pressure, 25% have high cholesterol $(C|B)$. A policyholder is randomly selected from the group.

(a) Calculate the probability that a policyholder has high blood pressure and low cholesterol. $P(\bar{C}|B) = 1 - P(C|B) = 1 - 0.25 = 0.75$

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1.5

$P(\bar{C}|B) = \frac{\boxed{P(B \cap \bar{C})}}{P(B)} \longrightarrow 0.75 = \frac{P(B \cap \bar{C})}{0.2}$

$P(B \cap \bar{C}) = (0.75)(0.2) = 0.15$

(b) Calculate the probability that a policyholder has high blood pressure, given that the policyholder has high cholesterol. $P(\bar{B}) = 1 - P(B) = 1 - 0.2 = 0.8$

1.5

$P(B|C) = \frac{P(B \cap C)}{P(C)} = \frac{P(C|B)\,P(B)}{P(C)}$

$= \frac{(0.25)(0.2)}{0.3} = \frac{1}{6} = 0.1\bar{6}7$

$P(\bar{C}) = 1 - P(C) = 1 - 0.3 = 0.7$

(c) Calculate the probability that a policyholder that a policyholder has low cholesterol, given that a policyholder has low blood pressure. $P(\bar{B}) = 1 - P(B) = 1 - 0.2 = 0.8$

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$P(\bar{C}|\bar{B}) = \frac{P(\bar{C} \cap \bar{B})}{P(\bar{B})} = \frac{\boxed{P(\bar{B}|\bar{C})}\,P(\bar{C})}{P(\bar{B})}$

$\boxed{P(\bar{B}|\bar{C})} = 1 - P(B|\bar{C}) = 1 - \left[\frac{P(B \cap \bar{C})}{P(\bar{C})}\right] = 1 - \left[\frac{0.15}{0.7}\right] = 0.7857$

$P(\bar{C}|\bar{B}) = \frac{(0.7857)(0.7)}{0.8} = 0.6875$

1.5

(a) Calculate how many of the 100 TV viewers **did not** watch any of the four channels (CBS, NBC, ABC or HGTV).

1.5

$100-[(C\cup N\cup A)+(H)] = 100-[C+N+A-(C\cap N)-(C\cap A)-(N\cap A)+(C\cap N\cap A)+H]$

$= 100-[34+15+10-7-6-5+4+18] = 37$

(b) Calculate the probability of the viewers who **did** watch NBC or ABC.

$P(N\cup A) = P(N)+P(A)-P(N\cap A)$

1.5

$= 0.15+0.1-0.05 = 0.2$

(c) Calculate the probability of the viewers who **did not** watch ABC.

$P(\bar{A}) = 1-P(A) = 1-0.1 = 0.9$

1.5

$P(B_1)=$ transferred black

Question (6): Urn I contains 7 red and 3 black balls, and Urn II contains 4 red and 5 black balls. After a randomly selected ball is transferred from Urn I to Urn II, 2 balls are randomly drawn from Urn II without replacement. $P(R_1)=$ transferred red

(a) Calculate the probability that both balls drawn from Urn II are red. $\boxed{P(R_2)}$

$P(R_2) = P(R_2\cap R_1)+P(R_2\cap B_1)$

1.5

$= P(R_2\mid R_1)P(R_1)+P(R_2\mid B_1)P(B_1)$

$= \dfrac{\binom{5}{2}}{\binom{10}{2}}\cdot\dfrac{7}{10}+\dfrac{\binom{4}{2}}{\binom{10}{2}}\cdot\dfrac{3}{10} = 0.19\overline{5}6$
