Q(1): An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	$P(A)$ 0.06	$P(i \mid A_1)$ 0.08
21-30	$P(A_2)$ 0.03	$P(i \mid A_2)$ 0.15
31-65	$P(A_3)$ 0.02	$P(i \mid A)$ 0.49
66-99	$P(A_4)$ 0.04	$P(i \mid A_4)$ 0.28

a) The probability of a driver that the company insures has an accident.

1.25

$P(i) = P(i \mid A)P(A) + P(i \mid A_2) \cdot P(A_2) + P(i \mid A_3)\, P(3) + P(i \mid A_4) P(4)$

$(0.08)(0.06) + (0.15)(0.03) + (0.49)(0.02) + (0.28)(0.04)$

$P(i) = 0.030$

b) If a randomly selected driver that the company insures has an accident, the probability that the driver was age 16-20 is

1.25

$P(A_1 \mid i) = \dfrac{P(i \mid A_1) \cdot P(A_1)}{P(i)} = \dfrac{(0.08)(0.06)}{0.030} = 0.16$

$P(i) = P(i \mid A_1) \cdot P(A_1) + P(i \mid A_2) \cdot P(A_2) + P(i \mid A_3) \cdot P(A_3) + P(i \mid A_4) \cdot P(A_4)$

Q(2): An insurer's annual weather-related loss, X, is a random variable with probability function:

$$f(x) = \begin{cases} 0.5, & x = 1 \\ x, & 0 < x < 1 \end{cases}$$

a) $P(0.8 < X \leq 1) =$

1.25

$\int_{0.8}^{1} x\,dx + P(x = 1)$

$\left.\frac{x^2}{2}\right|_{0.8}^{1} + 0.5$

$0.18 + 0.5 = 0.68$

b) $E(X^3) =$

1.25

$\int_0^1 x^3 \, x\,dx + (1)^3 P(x=1)$

$\int_0^1 x^4\,dx + 1 \cdot P(x=1)$

$\left.\frac{x^5}{5}\right|_0^1 + 0.5$

$\frac{1}{5} + 0.5 = \frac{7}{10}$

$P(A \cap d) = 0.2$
$P(B \cap d) = 0.4$

Q(3): A store has 50 modems in its inventory, 30 coming from Source A and the remainder from Source B. Of the modems from Source A, 20% are defective. Of the modems from Source B, 40% are defective.

a) The probability that exactly two out of a sample of five modems selected without replacement from the store's inventory are defective.

1.5

$n(A) = 30$
$n(B) = 20$
defective A $30 \times 0.2 = 6$
defective B $20 \times 0.4 = 8$
$\boxed{\text{defective} = 14}$

$$\frac{\binom{14}{2}\binom{36}{3}}{\binom{50}{5}} = 0.30$$

b) If it known that a selected modem is defective, what is the probability that the modem is from source B.

Q(4): An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.

(ii) 70% of the customers insure more than one car. $P(M) = 0.70$

(iii) 20% of the customers insure a sports car. $P(S) = 0.2$

(iv) Of those customers who insure more than one car, 15% insure a sports car.

a) The probability that a randomly selected customer insures exactly one car

1.25

$P(\bar{M}) = 1 - P(M)$
$1 - 0.7 = 0.3$

b) **The probability that a randomly selected customer insures exactly one car and that car is not a sports car is**

c) **The probability that a randomly selected customer insures more than one car and that car is not a sports car**

Q(5): **An actuary determines that the claim size for a certain class of accidents is a random variable, *X*, with moment generating function**

$$M_X(t) = \frac{1}{(1-5t)^3}$$

$E(X) = M'_X(0) = \frac{3(1-5t)^2(5)}{(1-5t)^6} = 15$

$E(X^2) = M''_X(0) = 300$

1.5

a) $E(2X + 4X^2 + 1) =$

$E(2x + 4x^2 + 1) =$

$2E(x) + 4E(x^2) + 1$

$2(15) + 4(300) + 1 = 1231$

$Var\,X = E(x^2) - (E(x))^2$

$300 - (15)^2 = 75$

1.25

b) $Var(3X - 1) =$

$9\,var(x) - 0$

$9(75) = 675$

Q(6): A random variable X has the cumulative distribution function

cont

$$F(x) = \begin{cases} 0, & x < 1 \\ x^3, & 0 \le x < 1 \\ 1, & x \ge 1 \end{cases}$$

a) The probability density function of X is

$f(x) = F'(x)$

1.25

$f(x) = 3x^2$ $\quad 0 < x < 1$

b) The standard deviation of X is

$\sqrt{var(X)} = \sqrt{E(x^2) - (E(x))^2}$

$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

1.5

$E(x) = \int_0^1 x \cdot 3x^2\,dx = \frac{3}{4}$

$E(x^2) = \int_0^1 x^2 \cdot 3x^2\,dx = \frac{3}{5}$

$var\,x = E(x^2) - (E(x))^2$

$\left(\frac{3}{5}\right) - \left(\frac{3}{4}\right)^2 = \frac{3}{80}$

$stand = \sqrt{var\,x} = \sqrt{3/80} = 0.193$

c) The 30% percentile of X is

$P(X \le c_p) = 0.3$

$F(c_p) = 0.3$

$c_p^3 = 0.3$

1.25

$\int_0^t 3x^2\,dx = 0.3$

$\frac{3x^3}{3}\Big|_0^t$

$t^3 - 0 = 0.3$

$t = \sqrt[3]{0.3}$

$t = 0.669$

Q(7): Let X represent the number of policies sold by an agent in a day. The probability function is given by

$$f(x) = \begin{cases} 0.45, & x = 1 \\ 0.35, & x = 2 \\ 0.15, & x = 3 \\ 0.05, & x = 4 \end{cases}$$

a) The probability of number of policies will exceed 2

1.25

$P(X=3)+P(X=4)$

$0.15+0.05=0.2$

b) The mode is

1.25

mode اكبر قيمة احتمال هي

$x=1$

1.5 c) $E\left(\frac{X}{X+1}\right)=$

$$E\left(\frac{x}{x+1}\right)=\sum_{k=1}^{4}\frac{x}{x+1}\cdot P(X=x)$$

$(0.45)(\frac{1}{2})+(0.35)(\frac{2}{3})+(0.15)(\frac{3}{4})+(0.05)(\frac{4}{5})$

$=0.61$

$X=$	1	2	3	4
$P(X=x)$	0.45	0.35	0.15	0.05
$\frac{x}{x+1}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

Q(8):

a) In designing a study of the effectiveness of migraine medicines, 3 factors were

(i) Medicine (A, B, C, D, Placebo)

(ii) Dosage Level (Low, Medium, High)

(iii) Dosage Frequency (1,2,3,4 times/day)

how many possible ways can a migraine patient be given medicine?

$C^{5}_{1}\times C^{3}_{1}\times C^{4}_{1}=60$

1.25

b) How many strings can be made using 4 A's, 3 B's, 7 C's and 1 D?

1.25

$$\frac{N!}{K_1!\,K_2!\,K_3!\,K_4!}$$

$$\frac{15!}{4!\,3!\,7!\,1!}=1801800$$

End of Questions