الاختبار الفصلى ألأول		جامعة الملك سعود
	الفصل الدراسى الأول 1436	كلية العلوم
	مقرر 215احص	قسم الإحصاء وبحوث العمليات

الساعة 10:00 – 11:00	الاحد ١٧/ ٥ / ١٤٣٦ هـ

اسم الطالبة ....

رقم الطالبة: ....

رقم الشعبة:...

رقم التسلسل:...

But your answer below

Passage	1	2	3	4	5	6	7	8	9	10
Answer	c	a	d	c	c	b	b	b	c	a

Passage	11	12	13	14	15	16	17	18	19	20
Answer	a	a	a	c	c		c	d	a	

Passage	21	22	23	24	25	26	27
Answer			d	d	a	a	b

Good luck

Dr. Saba

* الحل يحتمل الصح و الخطأ *

Question (1):

The cumulative distribution function of a continuous r.v. Y is given by:

$$F(y)=\begin{cases}0, & y\le 2\\ 1-\dfrac{4}{y^2}, & y>2\end{cases}$$

1. $P(Y\le 5)=$

(a) 0 (b) 4/25 (c) 21/25 (d) 19/25 (e) None of these

2. **The p.d.f of Y is given by:**

(a) $f(y)=\begin{cases}\frac{8}{y^3}, & y>2\\ 0 & o.w\end{cases}$ **(b)** $f(y)=\begin{cases}1-\frac{8}{y^3}, & y>2\\ 0 & o.w\end{cases}$ **(c)** $f(y)=\begin{cases}\frac{8}{y^3}, & y\le 2\\ 0 & o.w\end{cases}$ (d) $f(y)=\begin{cases}1-\frac{8}{y^3}, & y<2\\ 0 & o.w\end{cases}$

3. **Let $f(x)=\frac{cx}{3}, 0<x<3$ is the p.d.f of X then, the value of c must to be equal**

(a) 3 (b) 9 (c) 1/3 (d)2/ 3 (e) None of these

For the following probability mass function of the discrete random variable X,

x	$f(x)$
1	**0.20**
2	**0.25**
3	**0.25**
4	**0.30**

4. $P(2<X\le 4)$

(a) 0.25 b) 0. 00 c) 0.55 d) 1.00 e) none of these

5. **The expected value of X =**

a) 4.25 b) 15.10 c) 2.65 d) 10.23 e) none of these

Question (2):

Let the moment generating function of a r.v. X in the expanded form is given by:

$$M_X(t)=1+\frac{6t}{4}+\frac{14t^2}{8}+\frac{36t^3}{24}+\frac{98t^4}{96}+\ \ldots.$$

6. **The variance of X,**

(a) 0 (b) 5/4 (c) 1/2 (d) 3 (e) None of these

If $M_{X+Y}(t)=\left(\frac{2}{2-t}\right)^3, t<2$ and $M_X(t)=\left(\frac{2}{2-t}\right), t<2$ and X and Y independent, then

7. $M_Y(t)=$

(a) $(\frac{2}{2-t}), t<2$ (b) $(\frac{2}{2-t})^2, t<2$ **(c)** $(\frac{1}{1-t}), t<1$ **(d)None of these**

8. $M_{-3X+2}(t)=$

(a) $\frac{2e^t}{2-3t}, t<2/3$ (b) $\frac{2e^{2t}}{2+3t}$ **(c)** $\frac{2e^{2t}}{1-t}, t<1$ **(d)None of these**

Question (3):

Suppose that the continuous random variable Y has the following probability density function

$$f(y)=\begin{cases}2\,e^{-2y}\ ;\ y>0\\ 0\ ;\ o.w\end{cases}$$

9. $\boldsymbol{P(Y=3)}=$

(a)1 (b) 0.5 (c) 0 (d)0.11 (e) None of these

10. $\boldsymbol{F(Y)}$,

(a) $(1-e^{-2y})$ (b) $(e^{-2y}-1)$ (c) e^{-2y} (d) $(1+e^{-2y})$ (e) None of these

11. $\boldsymbol{P(2<Y<3)}=$

(a) $(e^{-4}+e^{-6})$ (b) $(e^{-4}-e^{-6})$ (c) $(e^{-10}$ (d) $(1-e^{-10})$ (e) None of these

12. **The mean of Y is**

(a) 1/2 (b) 2 (c) 1 (d) e^{-2} (e) None of these

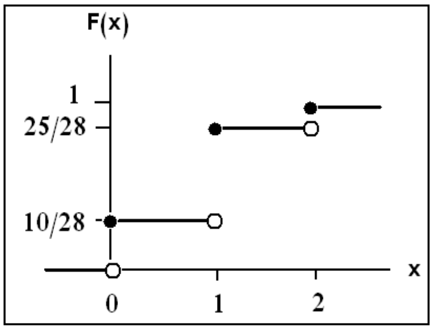
13. **The moment generating function** $M_y(t)$ **is given by**

(a) $\frac{2}{2-t}, t\neq 2$ (b) $\frac{2}{t-2}, t\neq 2$ (c) e^{-t} (d) $3-t$ (e) None of these

14. **If X is a r.v. independent of Y, then** $\boldsymbol{Var(X^3-3e^Y-3)}=$

a) $Var(X^3)-3Var(e^Y)$ b) $Var(X^3)-9Var(e^Y)$ c) $Var(X^3)+9Var(e^Y)$

<u>According the following graph of CDF of a disrete r.v</u>



15. $\boldsymbol{P(X\leq 1)}=$

(a) 3/28 (b) 10/28 (c) 25/28
(d) 1 (e) None of these

16. $\mathbf{P(X=2)}=$

(a) 3/28 (b) 10/28 (c) 25/28 (d) 1
(e) None of these

Question (4)

In a certain population, it is known that **30%** of the them have two jobs. Let X= number of people who have two jobs. If **a sample of 15 people** of this population are selected randomly, then

17. **The <u>probability distribution</u> formula of the random variable X,** $P(X=x)$ **is given by**

(a) $\binom{15}{x}\,0.3^{x}\ 0.7^{\,15-x}, x=1,\dots.15$ (b) $\binom{15}{x}\,0.3^{x}\ 0.7^{\,15-x}, x=0,1,\dots.15$

(c) $\binom{15}{x}\,0.3^{\,15-x}\ 0.7^{\,x}, x=0,1,\dots.15$ (d) $0.3^{x}\ 0.7^{\,1-x}, x=0,1,$

(e) $0.7^{x}\ 0.3^{\,1-x}, x=0,1,$

18. **The probability that <u>5 people</u> have two jobs**

(a) 0.876 (b) 0.995 (c) 0.667 (d) 0.206 (e) None of these

19. **The <u>variance</u> of** X **is**

(a) 3.15 (b) 5.14 (c) 15 (d) 4.50 (e) None of these

Question (5):

The **average** of traffic accidents in a certain street is **6** accidents per **month**. Let the random variable X=the number of traffic accidents per **month**.

(*) For any given **month** in this street:

20. P(X=3)

(a) 0.09 (b) 0.94 (c) 0.68 (d) 0.43 (e) None of these

21. The **mean** of the random variable is

(a) 1/30 (b) 30 (c) 6 (d) 1/6 (e) None of these

(**) For any given **2 months** in this street:

22. The probability that **exactly 6** accidents will occur is

(a) 0.23 (b) 0.21 (c) 0.03 (d) 0.12 (e) None of these

23. The **variance** of the random variable is

(a) 5 (b) 6 (c) 2 (d) 12 (e) None of these

24. If the probability is 0.4 that an application for a driver's license will pass the road test on any given try, what is the probability that an application will finally pass the test on the fifth try

(a) 0.15 (b) 0.16 (c) 0.08 (d) 0.05 (e) None of these

25. If it is assumed that **3%** of the units of a specific process will be rejected, then what is the probability that the 20^{th} unit observed will be the 3^{rd} rejected unit?

(a) 0.0028 (b) 0.1463 (c) 0.2340 (d) 0.0540 (e) None of these

A random committee of size 4 is selected from 3 men and 4women's. Let the random variable X representing the number of women's on the committee, then

26. The range of the r.v. X is

(a) 0,1,,..4 (b) 1,2,…4 (c) 1,2, …,7 (d) 3,4,…7 (e) None of these

27. $\mathbf{P(X \leq 3)}$

(a) 0.34 (b) 0.97 (c) 0.12 (d) 0.3 (e) None of these

College of Sciences

Department of Statistics and Operation Research

كلية العلوم

قسم الإحصاء وبحوث العمليات

First Midterm Exam

Tuesday, February 27, 2018	STAT 215	Academic year 1438-39H
12:00 – 1:00 pm	Probability 1	Second Semester

Student's Name		اسم الطالبة
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Classroom No.		رقم قاعة الاختبار
Teacher's Name		اسم أستاذة المقرر
Roll Number		رقم التحضير

Model B

Question	1	2	3	4	5	6	7	8	9	10
Answer	C	C	B	A	D	D	B	A	C	B

Question	11	12	13	14	15	16	17	18	19	20
Answer	A	B	A	C	D	C	B	B	C	A

Question	21	22
Answer	D	B

25

<u>Answer All Following Questions:</u>

<u>Question</u>:

1. Let X and Y be two independent random variables. If we define the mgf of X as $M_X(t) = (1-2t)^{-3}$ and the mgf of $Z = X + Y$ as $M_Z(t) = (1-2t)^{-7}$. Then, the mgf of Y is
   (A) $M_Y(t) = (1-2t)^4$ (C) $M_Y(t) = (1-2t)^{-4}$
   (B) $M_Y(t) = (1-2t)^{-2}$ (D) can't be known.
2. Let $f(x) = c(1-x),\ 0 \le x \le 1$ be a pdf of X. Then, the value of c is equal to
   (A) $\frac{1}{2}$ (B) -2 (C) 2 (D) $-\frac{1}{2}$

<u>Question:</u> If A and B are independent events, where $P(A) = 0.2$ and $P(B) = 0.3$. Then,

3. $P(A \cap B) =$
   (A) 0.2 (B) 0.06 (C) 0.5 (D) 0.44
4. $P(A|B) =$
   (A) 0.2 (B) 0.06 (C) 0.5 (D) 0.44
5. $P(A \cup B) =$
   (A) 0.2 (B) 0.06 (C) 0.5 (D) 0.44

<u>Question:</u> If X is a random variable with $M_X(t) = (1-4\,t)^{-1}$.Then,

6. $E(X) =$
   (A) 16 (B) $\frac{1}{4}$ (C) 32 (D) 4
7. $E(X^2) =$
   (A) 4 (B) 32 (C) 16 (D) 2
8. $Var(X) =$
   (A) 16 (B) 0 (C) 4 (D) 32
9. Let $Y = 2X$. Then, the mgf of Y is

   (A) $M_Y(t) = (1-2\,t)^{-1}$ (C) $M_Y(t) = (1-8\,t)^{-1}$
   (B) $M_Y(t) = (1-4\,t)^{-1}$ (D) none of these

<u>Question:</u>

Let X be a continuous random variable has $f(x) = 6x(1-x)\,,\ 0 \le\ x < 1$. Find:

10. $P(0.4 < X < 0.7) =$
    (A) 0.25 (B) 0.432 (C) 0 (D) 1
11. $F(x) =$

    (A) $3x^2 - 2x^3,\ 0 \le x < 1$ (C) $2x^3 - 3x^2,\ 0 \le x < 1$
    (B) $6x^2 - 6x^3,\ 0 \le x < 1$ (D) none of these
12. $E(X) =$
    (A) 0.5 (B) 0.25 (C) 0.75 (D) 1

$$M_Z(t) = M_X(t)\, M_Y(t)$$

$$(1-2t)^3 \frac{1}{(1-2t)^{7}} = \frac{1}{(1-\cancel{2t})^3}\, M_Y(t) \cdot \cancel{(1-2t)^3}$$

$$M_Y(t) = \frac{1}{(1-2t)^4}$$

3) A, B ind. $\Rightarrow P(A \cap B) = P(A)\,P(B)$

$= (0.2)(0.3) = 0.06$

4) $P(A|B) = \dfrac{P(A\cap B)}{P(B)} = \dfrac{0.06}{0.3} = 0.2$

5) $P(A\cap B) = P(A) + P(B) - P(A\cup B)$

$0.06 = 0.2 + 0.3 - x$

$x = 0.44$

$M_X(t) = (1-4t)^{-1}$.

6) $E(X) = M'_X(t) = -(1-4t)^{-2} \cdot -4 \big|_{t=0} = 4$

7) $E(X^2) = M''_X(t) = -8(1-4t)^{-3} \cdot -4 \big|_{t=0} = 32$

8) $\text{Var} = E(X^2) - (E(X))^2$

$= 32 - 16 = 16$

9) $Y = 2X \Rightarrow M_X(2t)$

$$= (1-8t)^{-1}$$

10) $\int_{0.4}^{0.7} (6x - 6x^2)\,dx = 3x^2 - 2x^3 \Big|_{0.4}^{0.7}$

$$= 3(0.7)^2 - 2(0.7)^3 - \left[3(0.4)^2 - 2(0.4)^3\right]$$

11) $\int_0^x (6x - 6x^2) = 3x^2 - 2x^3 \Big|_0^x$

$$= 3x^2 - 2x^3 \qquad 0 \le x < 1$$

12) $\int_0^1 x(6x - 6x^2)\,dx = 2x^3 - \frac{3}{2}x^4 \Big|_0^1$

$$= 2 - \frac{3}{2} = \frac{1}{2}$$

Question:

Suppose that X is a random variable that represents the number of people waiting at the line at a fast-food restaurant with pmf

$$f(x) = \frac{x}{10}, \quad x = 2, 3, 5.$$

Find:

13. $P(X = 1) =$

bc $x = 2, 3, 5$ ⇐ (A) 0 (B) 1 (C) $\frac{1}{5}$ (D) $\frac{4}{5}$

14. $P(1 < X < 5) =$

(A) $\frac{2}{10}$ (B) 0 (C) $\frac{5}{10}$ (D) 1

Question:

The blood groups of 200 people is distributed as follows: 50 have type **A** blood, 65 have **B** blood type, 70 have **O** blood type and 15 have type **AB** blood. If a person from this group is selected at random, find

15. The probability that this person has **O** blood type

(A) $\frac{50}{200}$ (B) 0 (C) $\frac{115}{200}$ (D) $\frac{70}{200}$

16. The probability that this person has **A** blood type or **B** blood type

(A) $\frac{50}{200}$ (B) 0 (C) $\frac{115}{200}$ (D) $\frac{70}{200}$

17. The probability that this person has **O** blood type and **AB** blood type

(A) $\frac{50}{200}$ (B) 0 (C) $\frac{115}{200}$ (D) $\frac{70}{200}$

* one person cannot take 2 type of blood

Question:

Consider the following distribution of X:

x	-2	-1	0	1	3
$f(x)$	0.1	0.09	0.4	0.21	0.2

Then,

18. $P(X = -1) =$

(A) 0.4 (B) 0.09 (C) 0.1 (D) 0.2

19. $P(X \leq 0) =$

(A) 0.1 (B) 0.19 (C) 0.59 (D) 0.4

20. $P(X = 2) =$

(A) 0 (B) 0.2 (C) 0.21 (D) 1

21. $E(X) =$

(A) 0.92 (B) 2.5 (C) 0.5 (D) 0.52

22. $E(3X - 1) =$

(A) 1.56 (B) 0.56 (C) 0.52 (D) 1

---

14) $f(2) + f(3) = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$

16) $P(A \cup B) = \frac{50}{200} + \frac{65}{200} = \frac{115}{200}$

19) $P(x \leq 0) = f(0) + f(-1) + f(-2)$

$= 0.4 + 0.09 + 0.1 = 0.59$

21) $E(x) =$

x	-2	-1	0	1	3	
$f(x)$	0.1	0.09	0.4	0.21	0.2	
$xf(x)$	-0.2	-0.09	0	0.21	0.6	$\Sigma\, 0.52$

22) $E(3x-1) = 3(0.52) - 1 = 0.56$

Question: If the mgf of a continuous random variable X is given by

$$M_X(t) = 1 + \frac{4}{5}t + \frac{4}{10}t^2 + \frac{4}{30}t^3 + \cdots.$$

Find the expected value and variance of *X*.

$E(X) = \frac{4}{5} \times 1! = \frac{4}{5}$

$E(x^2) = \frac{4}{10} \cdot 2! = \frac{4}{5}$

$Var = E(x^2) - (E(x))^2$

$= \frac{4}{5} - \left(\frac{4}{5}\right)^2 = \frac{4}{25}$

End of Questions.

King Saud University

College of Science

Statistics and Operation Research Department

STAT 215 Probability 1

First Semester 1438 – 1439

Second Midterm Exam Monday: 13 – 11 – 2017 1:00 – 3:00 PM	
	اسم الطالبة
	الرقم الجامعي
	الرقم التسلسلي
	رقم الشعبة

Question	1	2	3	4	5	6	7	8	9	10
Answer										

Question	11	12	13	14	15	16	17	18	19
Answer									

Question: In each of 5 races, the Democrats have a 60% chance of winning. Assuming that the races are independent of each other

$$f(x) = \binom{5}{x}(0.6)^x(0.4)^{5-x}, \qquad x = 0,1,2,3,4,5.$$

Find the following:

1. The probability that the Democrats will win 1 race or less is
   (A) 0.9139 (B) 0.9898 (C) 0.08704 (D) 0.0102
2. The probability that the Democrats will win exactly 2 races
   (A) 0.03686 (B) 0.2304 (C) 0.9678 (D) 0.7696
3. The standard deviation of X is
   (A) 1.2 (B) 1.0954 (C) 3 (D) 2

Question: A crate contains 50 light bulbs of which 5 are defectives and 45 are not. A Quality Control Inspector randomly samples 4 bulbs without replacement. Let X=the number of defective bulbs selected

dependent $\Rightarrow$ Hyper geometric (M, n, k)

$$f(x) = \frac{\binom{5}{x}\binom{45}{4-x}}{\binom{50}{4}}, \quad x = 0, 1, 2, 3, 4.$$

Then,

4. The distribution of X is M, n, k
   (A) $Bin(50,4)$ (B) $H(50,4,5)$ (C) $NBin(50,5)$ (D) $H(50,5,4)$
5. ؟ تعويض بالدالة ← $P(X = 1) =$
   (A) 0.5483 (B) 0.6469 (C) 0.69190 (D) 0.3081
6. $E(X) =$
   (A) 0.4 (B) 0.36 (C) 0.1 (D) 0.6
7. Find the variance of X when the bulbs are replaced independend $\Rightarrow Bin(n, P = \frac{k}{M})$
   (A) 0.4 (B) 0.36 (C) 0.1 (D) 0.6

Possion

Question: A life insurance sales man sells on the average 3 life insurance policies per week. If X=the number of life insurance sales man sells per week. Use Poisson distribution to calculate the following:

8. The probability that in a week he will sell 3 policies
   (A) 0.1493 (B) 0.224 (C) 0.6396 (D) 0.3426
9. $E(X) =$
   (A) 1.7321 (B) 6 (C) 3 (D) 2.4495
10. The standard deviation of X is
    (A) 1.7321 (B) 6 (C) 3 (D) 2.4495
11. The probability that he will sell one policy in 2 weeks
    (A) 6 (B) 0.0149 (C) 0.0446 (D) 0.9851

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1) $p(x \le 1) = \binom{5}{1}(0.6)^1 (0.4)^4 + \binom{5}{0}(0.6)^0 (0.4)^5$

2) $p(x=2) = \binom{5}{2}(0.6)^2 (0.4)^3$

3) $S.d = \sqrt{Var} = \sqrt{npq} = \sqrt{1.2} = 1.09$

6) $\frac{nk}{m} = \frac{4(5)}{50} = 0.4$

7) $p = \frac{k}{m} = 0.1 \Rightarrow q = 0.9$

$npq = 4 \times 0.1 \times 0.9 = 0.36$

uniform

Question: Suppose a random number k is taken from 13 to 20 in uniform distribution. Find:

$$f(x) = \frac{1}{8}, x = 13,..,20.$$

12. $P(X < 16) =$
    (A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{3}{20}$ (D) $\frac{1}{8}$
13. $P(10 < X < 14) =$
    (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{1}{20}$ (D) $\frac{3}{8}$
14. $E(X) =$
    (A) 12 (B) 3.5 (C) 6.67 (D) 16.5

Question: If unfair coin is tossed until first tail is obtained, then the pmf of X is given by:

$$f(x) = \frac{2}{3}\left(\frac{1}{3}\right)^{x-1}, \quad x = 1,2,...$$

Find:

15. The variance of X is equal to
    (A) $\frac{1}{3}$ (B) $\frac{2}{3}$ (C) $\frac{3}{2}$ (D) $\frac{3}{4}$
16. The mean of X is equal to
    (A) $\frac{1}{3}$ (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$
17. $P(X = 4) =$ تعويض
    (A) 0.0247 (B) 0.0082 (C) 0.074 (D) 0.9753
18. The moment generating function of X is
    (A) $(0.7\, e^t + 0.3)^3$ (B) $\frac{0.7\, e^t}{1-0.3\, e^t}$ (C) $\left(\frac{0.7\, e^t}{1-0.3\, e^t}\right)^3$ (D) $\frac{0.3\, e^t}{1-0.7\, e^t}$

?? 19. The mean if we toss the coin until <u>fourth</u> tail is obtained
    (A) $\frac{8}{3}$ (B) $\frac{3}{8}$ (C) 6 (D) $\frac{2}{3}$

Question:

20. If the pmf is defined as $f(x) = \frac{e^{-\lambda}\lambda^x}{x!}$; $x = 0,1,2,....$

    Derive the <u>mean</u> of the random variable X. Hint: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$.

$E(x) = \lambda$ sum start = 1

$$\sum_{x=0}^{\infty} x \cdot \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=1}^{\infty} \frac{\cancel{x}\,\lambda^{x-1}\lambda}{\cancel{x}(x-1)!}$$

$y = x-1$, $x=1$, $y=0$

$$= \lambda e^{-\lambda}\sum_{y=0}^{\infty}\frac{\lambda^y}{y!} = \lambda \cancel{e^{-\lambda}e^{\lambda}} = \lambda$$

15) geomtric $\Rightarrow$ $\sigma^2 = \frac{q}{p^2}$

$$= \frac{1/3}{(2/3)^2} = \frac{3}{4}$$

16) $E(x) = \frac{1}{p} = \frac{1}{2/3} = \frac{3}{2}$

18) $M_X(t) = \frac{pe^t}{1 - qe^t}$

$$= \frac{\frac{2}{3} e^t}{1 - \frac{1}{3} \cdot e^t} = \frac{0.7e^t}{1 - 0.3e^t}$$

21. If $X \sim Bin(n, p)$. Prove that the moment generating function of X is

$$M_X(t) = (pe^t + q)^n$$

Hint: $\sum_{x=0}^{n} \binom{n}{x} u^x v^{n-x} = (u+v)^n$.

$$\Rightarrow M_X(t) = \sum_{x=0}^{n} e^{tx} \binom{n}{x} p^x q^{x-1} = \underbrace{\sum_{x=0}^{n} (e^t p)^x q^{x-1} \binom{n}{x}}_{\text{from hint}}$$

$$M_X(t) = (pe^t + q)^n$$

22. Let X be a random variable has a geometric distribution with parameter p

$$f(x) = pq^{x-1}, \qquad x = 1, 2, \ldots$$

Prove that $E(X) = \frac{1}{p}$.

Hint: $\sum_{n=1}^{\infty} n\, i^{n-1} = \frac{1}{(1-i)^2}$, $i \in R: |i| < 1$

$$\sum_{x=1}^{\infty} x \cdot pq^{x-1} = p \sum_{x=1}^{\infty} x q^{x-1} \quad \text{from hint}$$

$$p \cdot \frac{1}{(1-q)^2} = \frac{p}{p^2} = \frac{1}{p}$$

End of Questions

College of Sciences

كلية العلوم

Department of Statistics and Operation Research

قسم الإحصاء وبحوث العمليات

جامعة الملك سعود
King Saud University

Final Exam

Tuesday, January 9, 2018	STAT 215	Academic year 1438-39H
8:00 – 11:00 am	Probability 1	First Semester

Student's Name		اسم الطالبة
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Classroom No.		رقم قاعة الاختبار
Teacher's Name		اسم أستاذة المقرر
Roll Number		رقم التحضير

Model A

Question	1	2	3	4	5	6	7	8	9	10
Answer	A	D	C	A	B	C	A	D	C	A

Question	11	12	13	14	15	16	17	18	19	20
Answer	B	B	D	D	C	A	C	A	D	B

Question	21	22	23	24	25	26	27	28	29
Answer	D	A	A	D	C	C	A	C	D

40

Question:

1. If $E(X|Y) = 3.46$, then $E(2X - 3|Y) =$ II. $E[(aX + b)|Y] = aE(X|Y) + b.$ $\rightarrow 2(3.46)-3$

(A) 3.92 (B) 6.92 (C) 3.46 (D) 0.46

2. If $X \sim Uniform[0, b]$ and $Var(X) = 3$, then the value of b is equal to

(A) 12 (B) 36 (C) 0 (D) 6

3. Let $\rho_{X,Y} = 0.7$, then $\rho_{2X,Y-1} =$ V. $\rho_{(aX\pm b),(cY\pm d)} = \rho_{X,Y}.$

(A) 0.29 (B) 0.4 (C) 0.7 (D) 1

4. If the MGF of X is given as $M_X(t) = (1 - 2t)^{-6}$, then the distribution of X is

(A) χ^2_{12} (B) $Gamma(3,2)$ (C) $Uniform(0,6)$ (D) $Exponential(6)$

5. T-distribution is symmetric about

(A) 1 (B) 0 (C) μ (D) σ

6. If $X \sim Gamma(7,2)$. Find $P(X < 6) =$

(A) 0.699 (B) 0.394 (C) 0.954 (D) 0.323

7. Consider the random variable X has a beta distribution which is given as

$$f(x) = 60\, x^2(1 - x)^3,$$

Then, $E(x) =$

(A) $\frac{3}{7}$ (B) $\frac{2}{6}$ (C) $\frac{2}{5}$ (D) 12

8. Suppose the MGF of X is $M_X(t) = e^{2t+5t^2}$, then the distribution of X is normal with

(A) $\mu = 10, \sigma^2 = 2$ (B) $\mu = 2, \sigma = 10$ (C) $\mu = 2, \sigma^2 = 5$ (D) $\mu = 2, \sigma^2 = 10$

9. Find the value of c in the probability joint distribution $f(x, y) = c\, xy, 0 < x < 1,$ $0 < y < 1$:

(A) 2 (B) 1 (C) 4 (D) $\frac{1}{4}$

Question:

If $E(X) = 2.6, E(Y) = 1.4$ and $E(XY) = 4.8$, then

10. $Cov(X, Y) =$ $E(xy) - E(x)E(y) = 4.8 - (2.6 \times 1.4) = 1.16$

(A) 1.16 (B) 3.4 (C) 2.2 (D) 0

11. X and Y are

If X and Y are two independent r.v.'s then

I. $Cov(X,Y) = 0.$

(A) independent (B) dependent (C) disjoint (D) none of these

Question:

Consider the probability joint distribution of X and Y is given by

$$f(x,y) = x + y, \qquad 0 < x < 1, \qquad 0 < y < 1$$

Calculate the following:

12. $f(x) =$ $\int_0^1 x+y\,dy = xy + \frac{y^2}{2}\Big|_0^1 = x + \frac{1}{2}, \quad 0<x<1$

(A) $x + y,\ 0 < x < 1$ (C) $x + y,\ 0 < y < 1$

(B) $x + \frac{1}{2},\ 0 < x < 1$ (D) $y + \frac{1}{2},\ 0 < y < 1$

13. $f(y) =$ $\int_0^1 x+y\,dx = x^2 + yx\Big|_0^1 = \frac{1}{2} + y, \quad 0<y<1$

(A) $x + y,\ 0 < x < 1$ (C) $x + y,\ 0 < y < 1$

(B) $x + \frac{1}{2},\ 0 < x < 1$ (D) $y + \frac{1}{2},\ 0 < y < 1$

14. $f(x|y) =$ $\frac{f(x,y)}{f(y)} = \frac{x+y}{\frac{1}{2}+y}$

(A) $x + \frac{1}{2},\ 0 < x < 1$ (C) $y + \frac{1}{2},\ 0 < y < 1$

(B) $\frac{x+y}{x+\frac{1}{2}}, 0 < x < 1,\ 0 < y < 1$ (D) $\frac{x+y}{y+\frac{1}{2}}, 0 < x < 1,\ 0 < y < 1$

15. $f\left(x|\frac{1}{2}\right) =$ $\frac{f(x,\frac{1}{2})}{f(\frac{1}{2})} = \frac{x+\frac{1}{2}}{1} = x + \frac{1}{2}$

(A) $\frac{x+2}{x+\frac{1}{2}}, 0 < x < 1$ (C) $x + \frac{1}{2}, 0 < x < 1$

(B) $y + \frac{1}{2}, 0 < y < 1$ (D) $\frac{x+y}{y+\frac{1}{2}}, 0 < x < 1,\ 0 < y < 1$

III. $E[E(X|Y)] = E(X)$, similarly $E[E(Y|X)] = E(Y)$.

16. $E\big(E(X|Y)\big) =$ $E(x) = \int_0^1 x^2 + \frac{1}{2}x\,dx = \frac{x^3}{3} + \frac{1}{4}x^2\Big|_0^1 = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

(A) $\frac{7}{12}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{3}{12}$

17. $E(XY) =$ $\int_0^1\int_0^1 xy(x+y)\,dx\,dy = \int_0^1\int_0^1 x^2y + xy^2\,dx\,dy = \int_0^1 \frac{x^3y}{3} + \frac{x^2y^2}{2}\Big|_0^1 dy = \int_0^1 \frac{1}{3}y + \frac{1}{2}y^2\,dy = \frac{1}{6}y^2 + \frac{1}{6}y^3\Big|_0^1 = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

(A) $\frac{1}{6}$ (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

Question:

Suppose the pdf of X is $f(x) = 2e^{-2x}, x > 0$. Then,

18. The MGF of X is

(A) $\frac{2}{2-t}$ (B) $(1-t)^{-2}$ (C) $\frac{1}{1-2t}$ (D) none of these

19. $Var(X) =$

(A) 4 (B) 2 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

Question:

The joint distribution of X and Y is defined as bellow

		X		
		-1	0	2
Y	0	0.01	0.05	0.2
	1	20) 0.05	0.2	0.05
	3	0.3	0.1	0.04

$f_x(-1) = 0.36$ $f_x(0) = 0.35$ $f_x(2) = .29$

$E(x) = -0.36 + 0 + 0.58 = 0.22$

$E(x^2) = 0.36 + 0 + 1.16 = 1.52$

Calculate:

20. $f(-1, 1) =$

(A) 0.01 (B) 0.05 (C) 0.3 (D) 0.26

21. $f_X(0) =$

(A) 0.17 (B) 0.29 (C) 0.26 (D) 0.35

22. $P(X > 0, Y < 3) =$ $f(2,1) + f(2,0) = 0.05 + 0.2$

(A) 0.25 (B) 0.29 (C) 0.45 (D) 0.32

23. $E(X) =$

(A) 0.22 (B) 0.34 (C) 0.72 (D) 0.14

24. $E(X^2) =$

(A) 1.32 (B) 0.74 (C) 2.52 (D) 1.52

25. $Var(X) =$ $1.52 - (0.22)^2 = 1.47$

(A) 2.41 (B) 2.72 (C) 1.47 (D) 0.34

Question:

Let $Var(X) = 1.4, Var(Y) = 1.8, Cov(X, Y) = 1.57$. Calculate:

26. $Var(X - Y) =$ $Var(x) + Var y - 2 Cov(x,y) = 1.4 + 1.8 - 2(1.57) = 0.06$

(A) 6.34 (B) 3.2 (C) 0.06 (D) 0.84

27. $Cov\left(6X, \frac{Y}{3}\right) =$ (V. $Cov(aX, bY) = abCov(X, Y)$. $= 6\left(\frac{1}{3}\right)(1.57) = 3.14$

(A) 3.14 (B) 9.42 (C) 0.52 (D) 2.49

28. $\rho_{X,Y} == \frac{Cov(X,Y)}{\sqrt{V(X)V(Y)}} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}$, $= \frac{1.57}{\sqrt{1.4 \times 1.8}} = 0.99$

(A) 0 (B) 0.62 (C) 0.99 (D) 0.64

29. The relationship between X and Y is

(A) week positive (B) strong negative (C) week negative (D) strong positive

---

Question:

30. If $X \sim Gamma(\alpha, \beta)$.Prove that $Var(X) = \frac{\alpha}{\beta^2}$.

31. Consider the pdf of X is defined as following

$$f(x) = \theta x^{-(\theta+1)}, \qquad 1 < x < \infty$$

Calculate the distribution of the random variable $Y = Ln\, X$

32. Let the two random variables X_1, X_2 have a joint probability distribution as follow:

		X_1	
		-1	0
X_2	0	0.2	0.3
	1	0.2	0.3

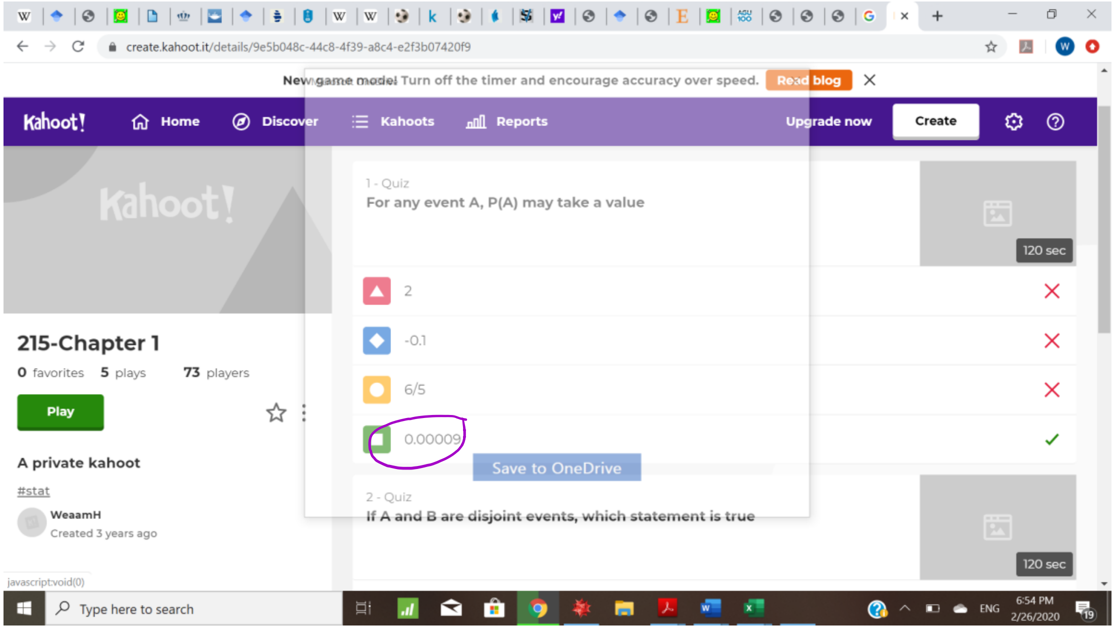
Find:

a) If the random variables X_1, X_2 are independent or not.
b) The pdf of $Y = X_1 + X_2$.

33. Find the CDF of X when $X \sim Uniform(0,1)$. Then, use the CDF transformation method to derive the distribution of $Y = \sqrt{X}$.

Table A.23 The Incomplete Gamma Function: $F(x;\alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y}\, dy$

	α									
x	1	2	3	4	5	6	7	8	9	10
1	0.6320	0.2640	0.0800	0.0190	0.0040	0.0010	0.0000	0.0000	0.0000	0.0000
2	0.8650	0.5940	0.3230	0.1430	0.0530	0.0170	0.0050	0.0010	0.0000	0.0000
3	0.9500	0.8010	0.5770	0.3530	0.1850	0.0840	0.0340	0.0120	0.0040	0.0010
4	0.9820	0.9080	0.7620	0.5670	0.3710	0.2150	0.1110	0.0510	0.0210	0.0080
5	0.9930	0.9600	0.8750	0.7350	0.5600	0.3840	0.2380	0.1330	0.0680	0.0320
6	0.9980	0.9830	0.9380	0.8490	0.7150	0.5540	0.3940	0.2560	0.1530	0.0840
7	0.9990	0.9930	0.9700	0.9180	0.8270	0.6990	0.5500	0.4010	0.2710	0.1700
8	1.0000	0.9970	0.9860	0.9580	0.9000	0.8090	0.6870	0.5470	0.4070	0.2830
9		0.9990	0.9940	0.9790	0.9450	0.8840	0.7930	0.6760	0.5440	0.4130
10		1.0000	0.9970	0.9900	0.9710	0.9330	0.8700	0.7800	0.6670	0.5420
11			0.9990	0.9950	0.9850	0.9620	0.9210	0.8570	0.7680	0.6590
12			1.0000	0.9980	0.9920	0.9800	0.9540	0.9110	0.8450	0.7580
13				0.9990	0.9960	0.9890	0.9740	0.9460	0.9000	0.8340
14				1.0000	0.9980	0.9940	0.9860	0.9680	0.9380	0.8910
15					0.9990	0.9970	0.9920	0.9820	0.9630	0.9300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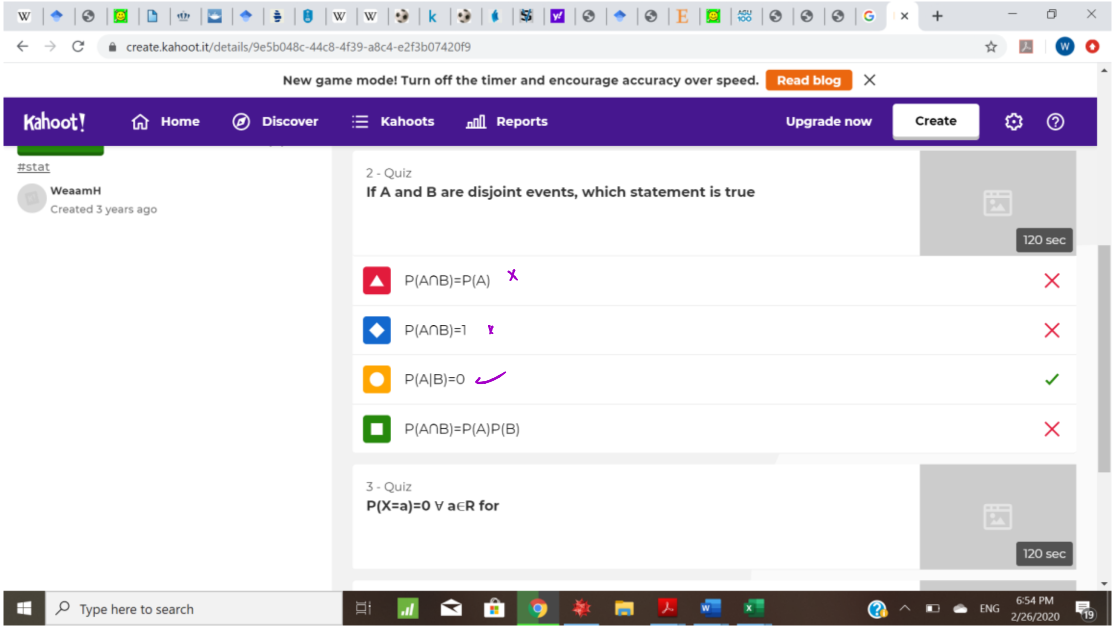
1 - Quiz
For any event A, P(A) may take a value
120 sec
2
-0.1
6/5
0.00009
Save to OneDrive
2 - Quiz
If A and B are disjoint events, which statement is true
120 sec
Type here to search
6:54 PM
2/26/2020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2 - Quiz
If A and B are disjoint events, which statement is true
120 sec
P(A∩B)=P(A)
P(A∩B)=1
P(A|B)=0
P(A∩B)=P(A)P(B)
3 - Quiz
P(X=a)=0 ∀ a∈R for
120 sec
Type here to search
6:54 PM
2/26/2020

P(A∩B)=P(A)P(B) ✗

3 - Quiz

P(X=a)=0 ∀ a∈R for

120 sec

- Discrete Variables ✗
- Continuous Variables ✓
- Discrete and Continuous Variables ✗
- Non of these ✗

4 - Quiz

Let x be a random variable takes 0,1,2. If the mass function is define as

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Non of these ✗

4 - Quiz

Let x be a random variable takes 0,1,2. If the mass function is define as f(0)=0.5 & f(1)=0.25 then f(2)=

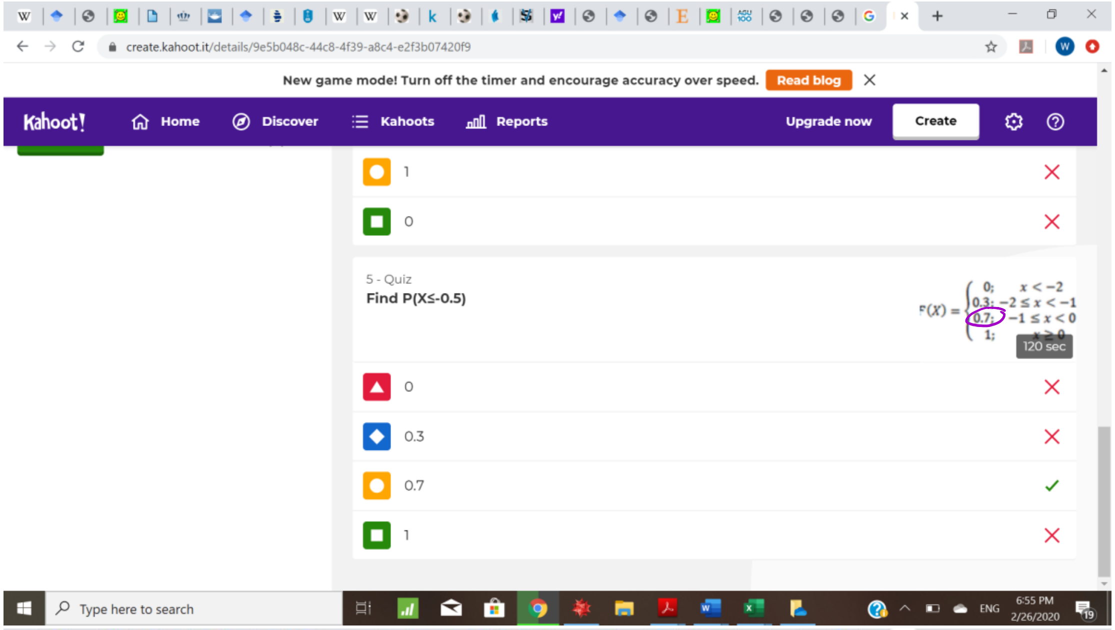
120 sec

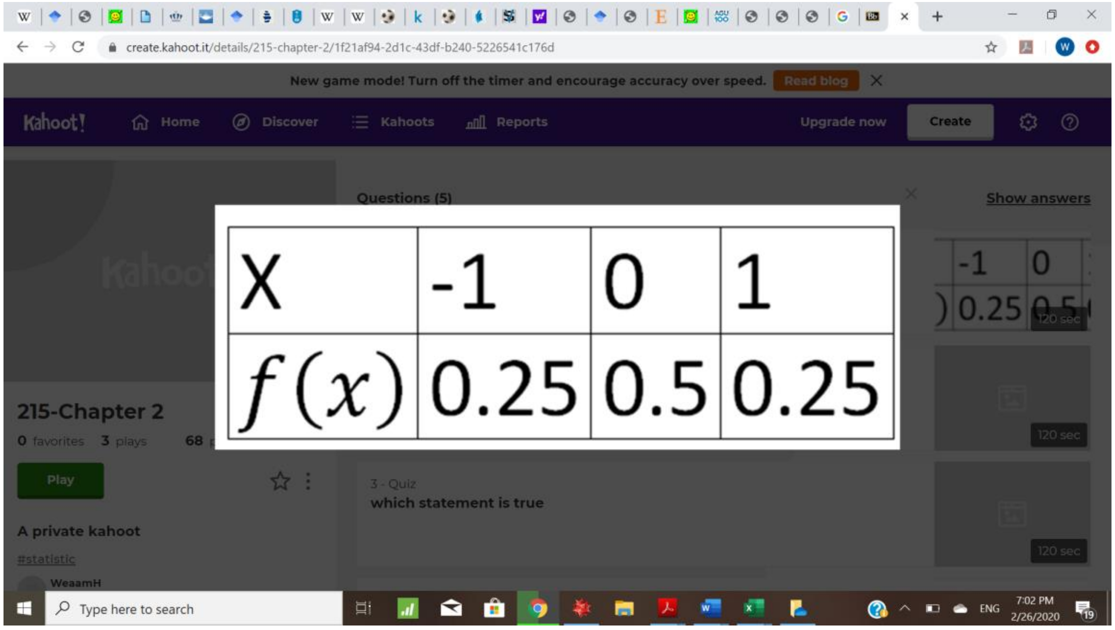
- 0.25 ✓
- 0.5 ✗
- 1 ✗
- 0 ✗

5 - Quiz

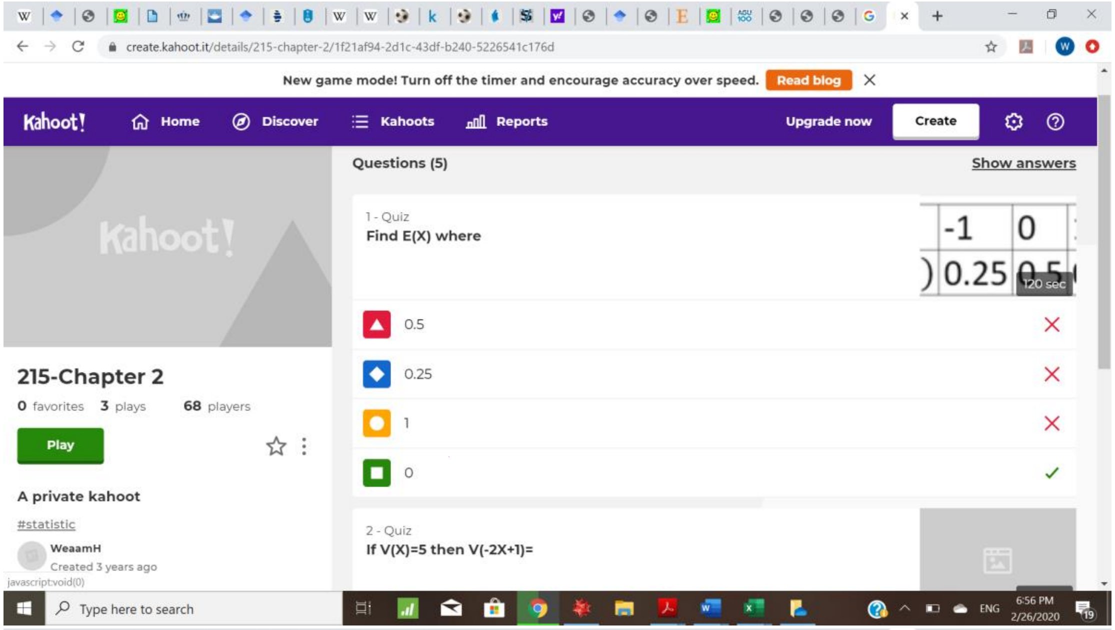
Find P(X≤-0.5)

$$F(X)=\begin{cases}0; & x<-2\\ 0.3; & -2\le x<-1\\ 0.7; & -1\le x<0\end{cases}$$





X	-1	0	1
$f(x)$	0.25	0.5	0.25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215-Chapter 2
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Play
A private kahoot
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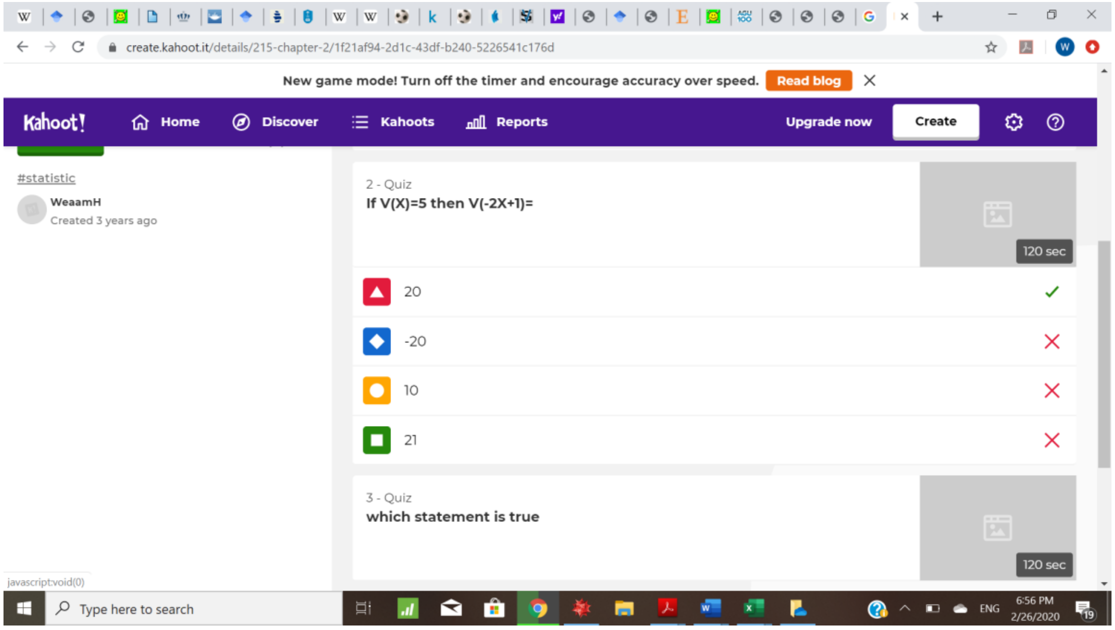
1 - Quiz
Find E(X) where
-1
0
0.25
120 sec
0.5
0.25
1
0
2 - Quiz
If V(X)=5 then V(-2X+1)=
Type here to search
ENG
6:56 PM
2/26/2020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2 - Quiz
If V(X)=5 then V(-2X+1)=
120 sec
20
-20
10
21
3 - Quiz
which statement is true
120 sec
Type here to search
ENG
6:56 PM
2/26/2020

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21

3 - Quiz

which statement is true

120 sec

- $\mu'_0 = \mu_0 = 1$ ✓
- $\mu_1 = 1$ ✗
- $\mu'_2 = V(X)$ ✗
- $\mu'_0 = 0$ ✗

4 - Quiz

Find the mean

215-Chapter 2

0 favorites 3 plays 68 players

Play

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WeaamH

$$M_X(t) = 7(7-t)^{-1}$$

5 - Quiz

Find $E(X^2)$=

120 sec

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$\mu'_0=0$

4 - Quiz
Find the mean

$= 7(7 \cdot$

120 sec

- 7
- 14
- 1/7 ✓
- 49

5 - Quiz
Find E(X^2)=

$+ 2t + 3t^2 \cdot$

Type here to search

6:57 PM 2/26/2020

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215-Chapter 2

0 favorites 3 plays 68 players

Play

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#statistic

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$$M_X(t) = 1 + 2t + 3t^2 + 5t^3 + \cdots$$

5 - Quiz
Find E(X^2)=

$+ 2t + 3t^2 \cdot$

120 sec

Type here to search

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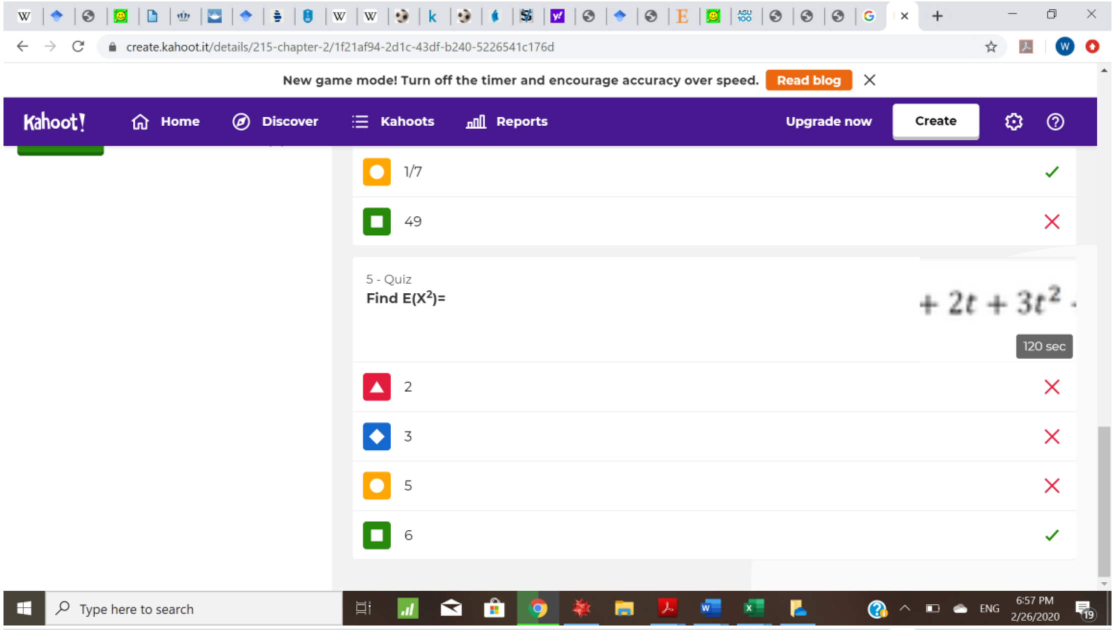

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1/7
49
5 - Quiz
Find E(X²)=
120 sec
2
3
5
6
Type here to search
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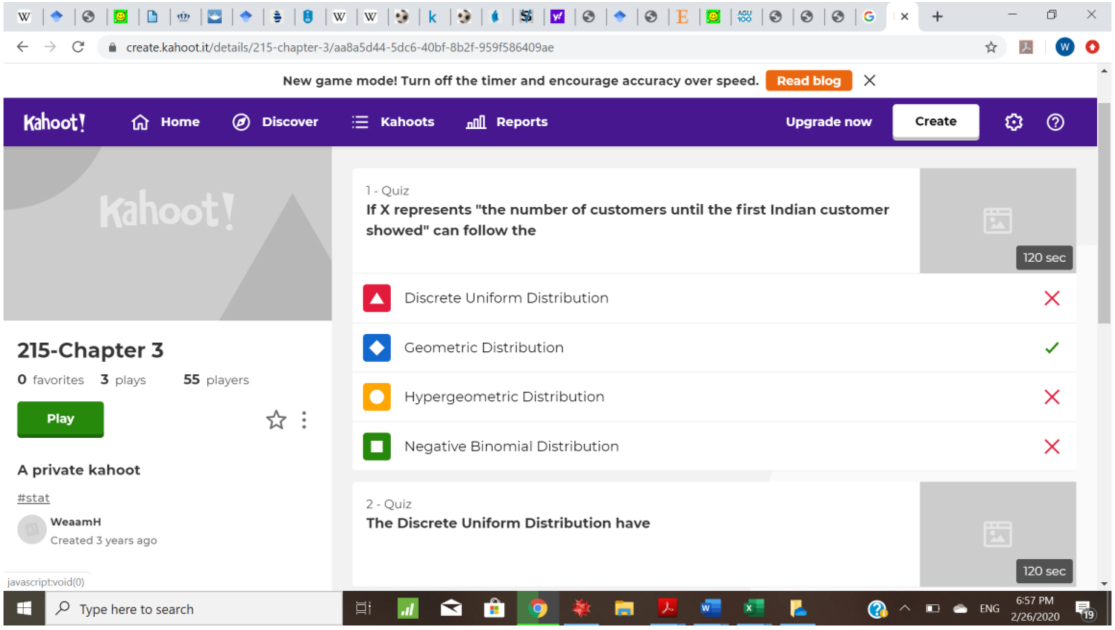

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215-Chapter 3
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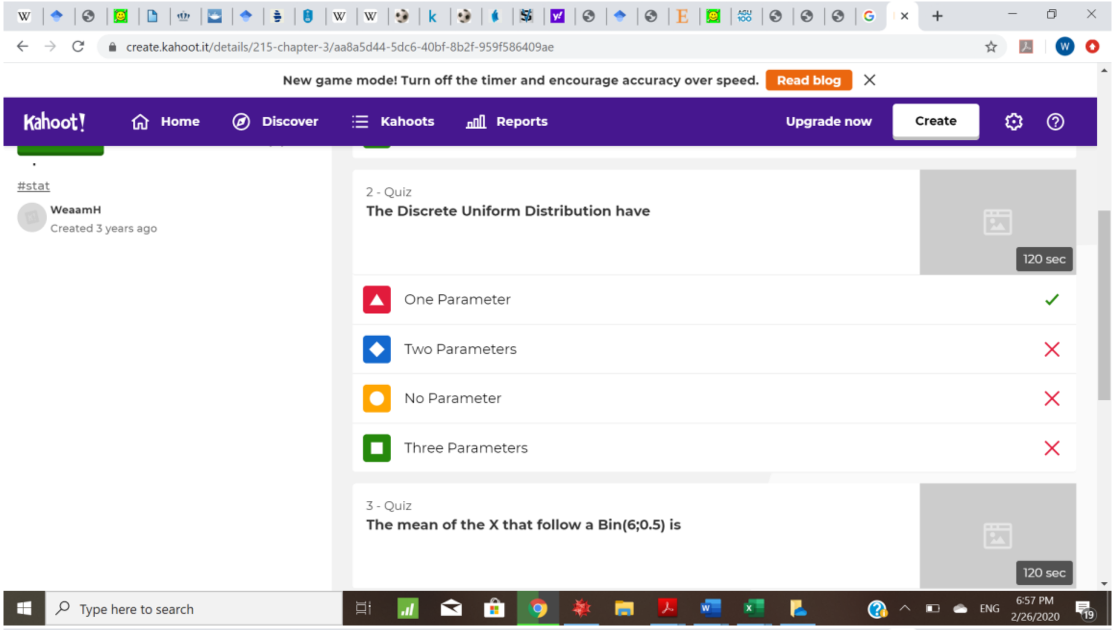
1 - Quiz
If X represents "the number of customers until the first Indian customer showed" can follow the
120 sec
Discrete Uniform Distribution
Geometric Distribution
Hypergeometric Distribution
Negative Binomial Distribution
2 - Quiz
The Discrete Uniform Distribution have
120 sec
javascript:void(0)
Type here to search
6:57 PM
2/26/2020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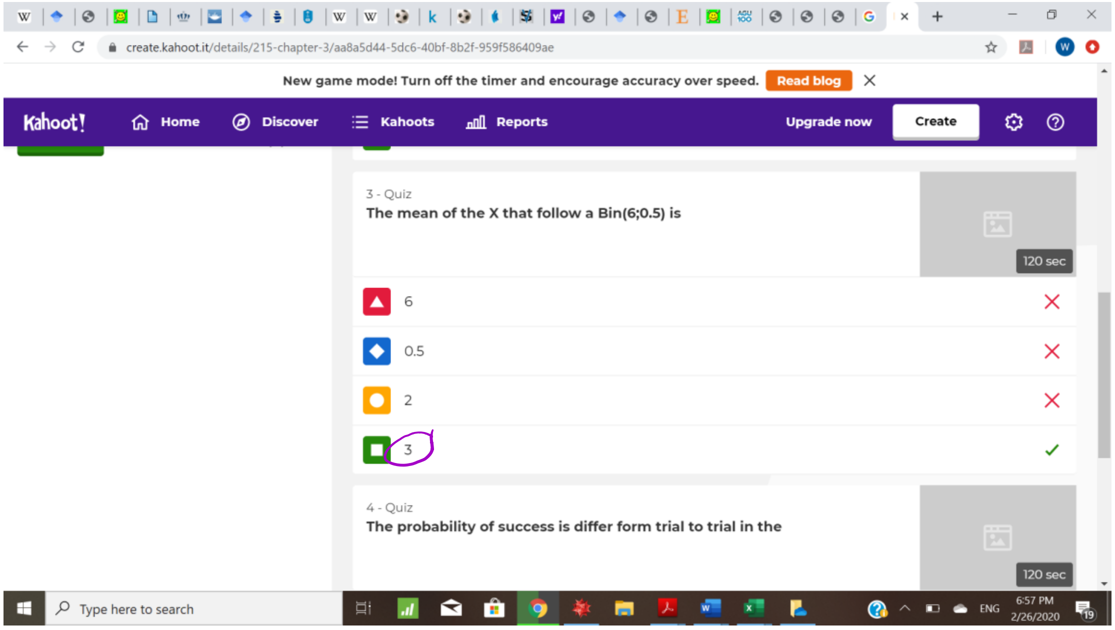
Created 3 years ago
2 - Quiz
The Discrete Uniform Distribution have
120 sec
One Parameter
Two Parameters
No Parameter
Three Parameters
3 - Quiz
The mean of the X that follow a Bin(6;0.5) is
120 sec
Type here to search
ENG
6:57 PM
2/26/2020

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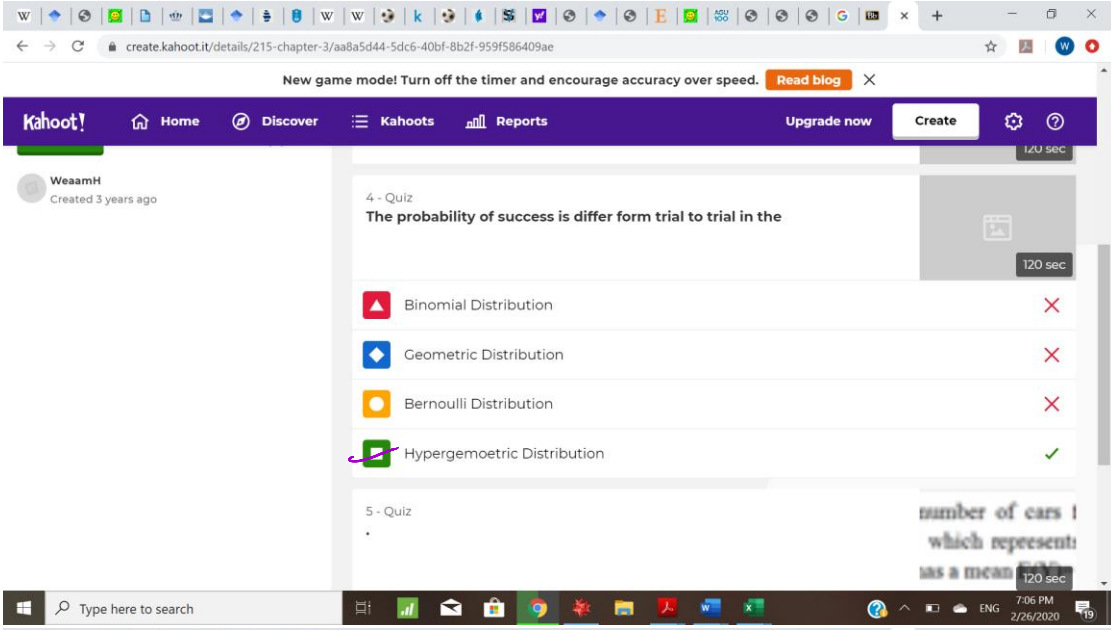
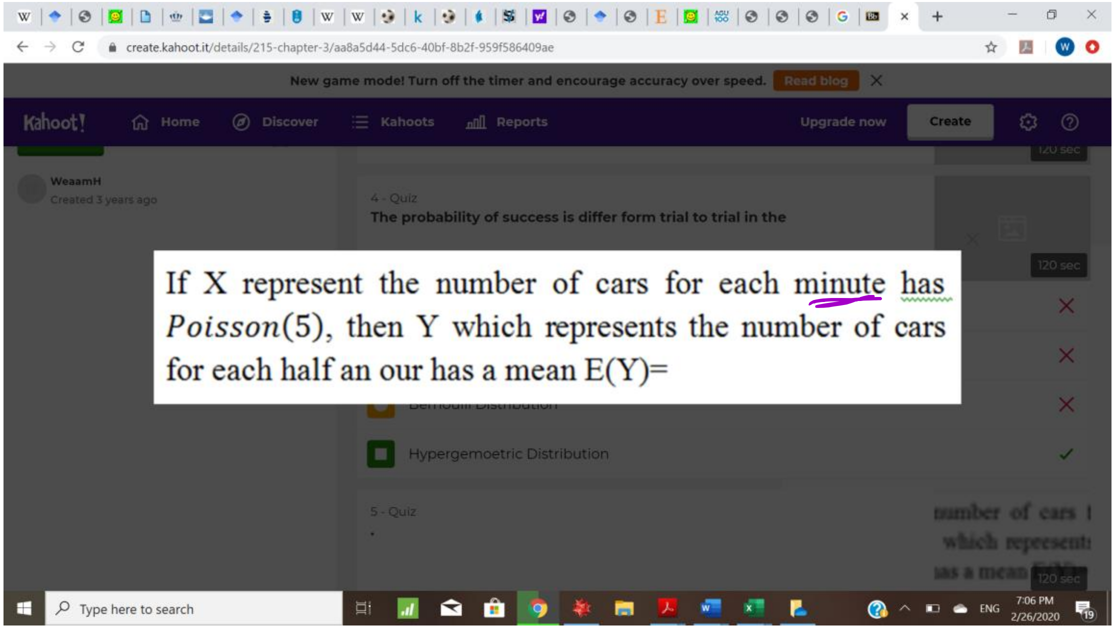
Create
3 - Quiz
The mean of the X that follow a Bin(6;0.5) is
120 sec
6
0.5
2
3
4 - Quiz
The probability of success is differ form trial to trial in the
120 sec
Type here to search
ENG
6:57 PM
2/26/2020





1 min → 5

30 min → ?

150

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- Bernoulli Distribution ✗
- Hypergemoetric Distribution ✓

5 - Quiz

.

number of cars
which represents
has a mean

120 sec

- 5 ✗
- 30 ✗
- 150 ✓
- 2.5 ✗

6 - Quiz

javascript:void(0)

Type here to search

6:58 PM 2/26/2020

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- 2.5 ✗

6 - Quiz

The variable X "represents the number of students before we have 7 students wears glasses" has

120 sec

- Binomial Distribution ✗
- Negative Binomial Distribtution ✓
- Hypergeometric Distrbution ✗
- Poisson Distribution ✗

7 - Quiz

Which Distribution that Does not depend on the values of X

Type here to search

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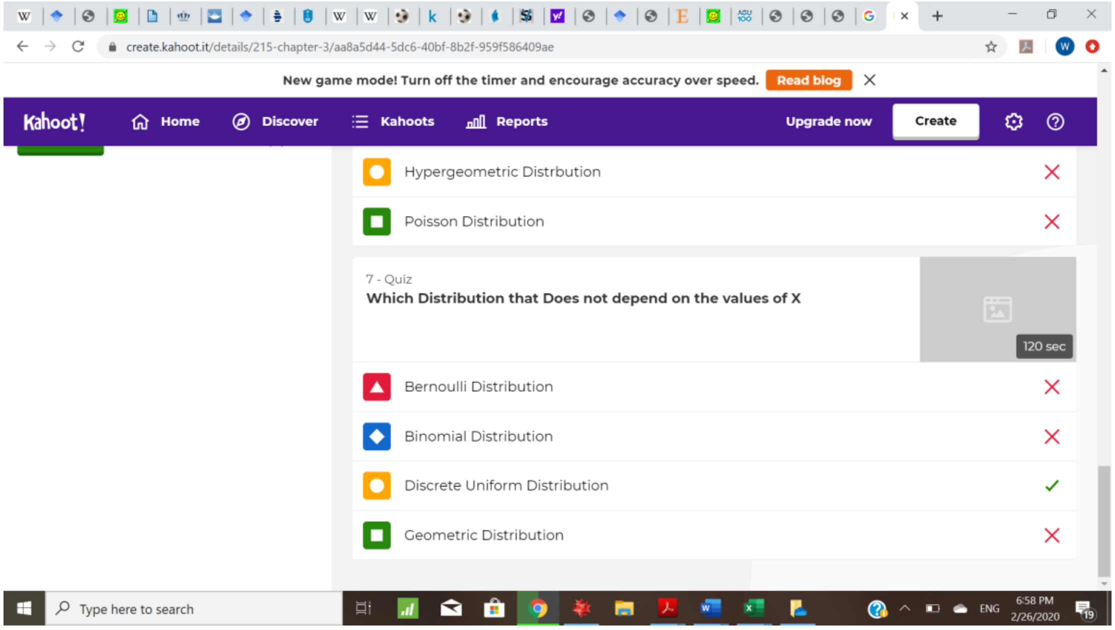
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College of Sciences

كلية العلوم

Department of Statistics and Operation Research

قسم الإحصاء وبحوث العمليات

Final Exam

Tuesday, May 15, 2018	STAT 215	Academic year 1438-39H
8:00 – 11:00 am	Probability 1	Second Semester

Student's Name		اسم الطالبة
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Classroom No.		رقم قاعة الاختبار
Teacher's Name		اسم أستاذة المقرر
Roll Number		رقم التسلسل

Model A

40

Answer All Following Questions:

Question:

1. If $X \sim \mathrm{Gamma}(7, 2)$. Find $P(X < 6) =$
   (A) 0.699 (B) 0.394 (C) 0.954 (D) 0.323
2. T-distribution is symmetric about
   (A) 1 (B) 0 (C) μ (D) σ
3. Let X be a continuous random variable has a $\mathrm{Uniform}(a, 7)$ and the mean of X is $\mu = 3$. Then, the value of a is equal to
   (A) 1 (B) -1 (C) 0 (D) 6
4. $\mathrm{Cov}(2X - 3Y,\ X + Y) =$
   (A) $2Var(X) - 3Var(Y) - 6Cov(X,Y)$
   (B) $2Var(X) - 3Var(Y) - Cov(X,Y)$
   (C) $4Var(X) + 9Var(Y) - 6Cov(X,Y)$
   (D) $4Var(X) - 9Var(Y) - 6Cov(X,Y)$
5. If X is a random variable with pdf $f(x) = 9\,x\,e^{-3x}, x > 0$. Then $E(X) =$
   (A) 3 (B) $\frac{1}{6}$ (C) $\frac{2}{3}$ (D) 6
6. The distribution of the random variable X which has the MGF as $M_X(t) = (1 - 2t)^{-\alpha}$ is
   (A) $Gamma\left(\alpha, \frac{1}{2}\right)$ (B) $\chi^2_{2\alpha}$ (C) Both A and B (D) None of these
7. $E(X - \mu_X) =$
   (A) E(X) (B) μ_X (C) $E(X) + \mu_X$ (D) 0
8. The value of c when $f(x) = c\,x^3(1 - x)^5, 0 < x < 1$ is equal to
   (A) $c = B(4,6)$ (B) $c = \frac{1}{B(4,6)}$ (C) $c = \frac{1}{B(3,5)}$ (D) $c = B(3,5)$
9. The distribution of the sample mean $\bar{X} = 2 + Z$ is
   (A) $\bar{X} \sim N(-2,1)$ (B) $\bar{X} \sim N(1,2)$ (C) $\bar{X} \sim N(2,1)$ (D) $\bar{X} \sim N(4,1)$
10. Given the normally distributed variable X with mean 16 and standard deviation 2, then $P(18 < X < 20) =$
    (A) 0.0735 (B) 0.1359 (C) 0.9901 (D) 0.47
11. Suppose the MGF of X is $M_X(t) = e^{2t + 5t^2}$, then the distribution of X is normal with
    (A) $\mu = 10, \sigma^2 = 2$ (B) $\mu = 2, \sigma = 10$ (C) $\mu = 2, \sigma^2 = 5$ (D) $\mu = 2, \sigma^2 = 10$

Question:

Let X and Y be random variables having the following joint density function

$$f(x,y) = \begin{cases} \frac{1}{8}(x^2 - y^2)e^{-x} & x > 0, \quad -x < y < x \\ 0 & otherwise \end{cases}. \quad Then$$

$\frac{1}{8}\int_{-x}^{x} (x^2 - y^2)\, e^{-x}\, dy$

12. The marginal pdf of X, $f_X(x)$ is given by
    (A) $\frac{1}{6}\ x^3\ e^{-x}, x > 0$ (B) $2e^{-2x}, x > 0$ (C) $e^{-x}, x > 0$ (D) $\frac{1}{24}\ x^4\ e^{-x}, x > 0$
13. $f_{Y|X=1}(y)$ is given by
    (A) $\frac{1}{8}\ (1 - y^2), -1 < y < 1$
    (B) $\frac{3}{4}\ (1 - y^2), -1 < y < 1$
    (C) $\frac{3}{4}\ (1 - y^2), -x < y < x$
    (D) $\frac{3}{4}\ (1 - e^{-2y}), -1 < y < 1$

Question:

The joint pmf of X and Y is given by

X	Y			
	0	**15**	**20**	**Total**
10	0.20	0.10	0.20	0.5
25	0.05	0.15	0.30	0.5
Total	0.25	0.25	0.5	1

14. $P(X - Y = X) =$
    (A) 0. 75 (B) 0.25 (C) 0.50 (D) 0.52

15. $P(Y > X) =$

(A) 0.30 (B) 0.25 (C) 0.50 (D) 0.75

16. $F_{XY}(25, 15)$ is

(A) 0.2 (B) 0.25 (C) 0.75 (D) 0.5

17. $E(X) =$ $(10 \times 0.5) + (25 \times 0.5)$

(A) 34.8 (B) 17.5 (C) 30.9 (D) 13.75

18. $E(X^2) =$ $(100 \times 0.5) + (625 \times 0.5)$

(A) 256.25 (B) 8.75 (C) 362.5 (D) 253.84

19. $Var(2X - 2) =$ $4(E(x^2) - (E(x))^2)$

(A) 227 (B) 112.5 (C) 114.5 (D) 225

20. $E(Y) =$

(A) 14 15 40 (B) 12.63 56.25 (C) 256.25 (D) 13.75

21. $E(XY) =$ $10 \times 15 \times 0.1 + 10 \times 20 \times 0.2 + 25 \times 15 \times 0.15 + 25 \times 20 \times 0.3$ 150

(A) 34.8 (B) 261.25 (C) 30.9 (D) 13.45

22. $Cov(X, Y) =$ $E(xy) - E(x)E(y)$

(A) 20.63 (B) 61.25 (C) 53.37 (D) 44.5

Question:

A random variables X and Y with the following results

$E(X)=2$, $E(Y)=3$, $E(X^2)=8$, $E(Y^2)=12$, $E(XY)=8$. Find :

23. $Cov(X, Y) =$ $8 - (2 \times 3) = 2$

(A) 2 (B) 3 (C) 4 (D) 11

24. $Var(Y) =$ $E(y^2) - [E(y)]^2 = 12 - 9 = 3$

(A) 2 (B) 3 1.73 (C) 4 (D) 11

25. $Var(X) =$ $E(x^2) - (E(x))^2 = 8 - 4$

(A) 2 (B) 3 (C) 4 2 (D) 11

26. $Var(X+Y) =$ VI. $V(X \pm Y) = V(X) + V(Y) \pm 2Cov(X, Y).$ $= 4 + 3 + 2(2) = 11$

(A) 2 (B) 3 (C) 4 (D) 11

27. Correlation between X ,Y = $\frac{2}{\sqrt{12}}$

(A) 0.816 (B) 0.167 (C) 0.577 (D) 0.204

28. $E[E(X|Y)] =$

(A) 2 (B) 3 (C) 4 (D) 11

Question:

If $f(x) = \frac{3}{8}x^2, \; 0 < x < 2$. Then find:

29. The cdf of X "F(X)" is:

(A) $\frac{3}{4}x$ (B) x^3 (C) $\frac{1}{8}x^3$ (D) $\frac{1}{24}x^3$

30. If $Y = \sqrt{X}$. The range of Y is:

(A) $0 < y < 4$ (B) $0 < y < \sqrt{2}$ (C) $0 < y < 2$ (D) $0 < y^2 < \sqrt{2}$

31. The cdf of Y is:

(A) $\frac{1}{8}y^6$ (B) $\frac{1}{8}y^5$ (C) $\frac{3}{4}y^5$ (D) $\frac{1}{8}y^2$

32. The pdf of Y (Use the cdf method) is:

(A) $\frac{3}{4}y^6$ (B) $\frac{1}{8}y^6$ (C) $\frac{5}{8}y^4$ (D) $\frac{3}{4}y^5$

Question:

33. If $X_1 \sim \chi^2_2$ and $X_2 \sim \chi^2_6$ are independent random variables. Find the distribution of $Y = X_1 + X_2$ (Use the moment generating function method). (3 marks).

34. Prove that the moment generating function (MGF) of the standard normal distribution is given as: $M_Z(t) = e^{\frac{t^2}{2}}$. (2 marks).

35. Let X_1 and X_2 are two random variables have the joint probability distribution as follows:

		X	
		0	1
Y	-1	0.21	0.41
	0	0.24	0.14

i. Prove that *X* and *Y* are dependent random variables. (1 mark).

ii. Find the pmf of the random variable $Z = X + Y$. (2 mark).

End of Questions

Table 4.3: Area to the Left of the Z score for Standard Normal Distribution.

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890

Table A.23 The Incomplete Gamma Function: $F(x;\alpha) = \int_0^x \frac{1}{\Gamma(\alpha)} y^{\alpha-1} e^{-y}\, dy$

	α									
x	1	2	3	4	5	6	7	8	9	10
1	0.6320	0.2640	0.0800	0.0190	0.0040	0.0010	0.0000	0.0000	0.0000	0.0000
2	0.8650	0.5940	0.3230	0.1430	0.0530	0.0170	0.0050	0.0010	0.0000	0.0000
3	0.9500	0.8010	0.5770	0.3530	0.1850	0.0840	0.0340	0.0120	0.0040	0.0010
4	0.9820	0.9080	0.7620	0.5670	0.3710	0.2150	0.1110	0.0510	0.0210	0.0080
5	0.9930	0.9600	0.8750	0.7350	0.5600	0.3840	0.2380	0.1330	0.0680	0.0320
6	0.9980	0.9830	0.9380	0.8490	0.7150	0.5540	0.3940	0.2560	0.1530	0.0840
7	0.9990	0.9930	0.9700	0.9180	0.8270	0.6990	0.5500	0.4010	0.2710	0.1700
8	1.0000	0.9970	0.9860	0.9580	0.9000	0.8090	0.6870	0.5470	0.4070	0.2830
9		0.9990	0.9940	0.9790	0.9450	0.8840	0.7930	0.6760	0.5440	0.4130
10		1.0000	0.9970	0.9900	0.9710	0.9330	0.8700	0.7800	0.6670	0.5420
11			0.9990	0.9950	0.9850	0.9620	0.9210	0.8570	0.7680	0.6590
12			1.0000	0.9980	0.9920	0.9800	0.9540	0.9110	0.8450	0.7580
13				0.9990	0.9960	0.9890	0.9740	0.9460	0.9000	0.8340
14				1.0000	0.9980	0.9940	0.9860	0.9680	0.9380	0.8910
15					0.9990	0.9970	0.9920	0.9820	0.9630	0.9300

كلية العلوم
جامعة الملك سعود

215 احص: احتمال (1)
الاختبار النهائي
الفصل الثاني: 1437 – 1438 هـ

اليوم: الاربعاء 1438/8/28 هـ	الوقت: 8:00 – 11:00

Version (A)	
اسم الطالبة:	
الرقم الجامعي:	
الرقم التسلسلي:	
رقم الشعبة:	
اسم الأستاذة:	

ملاحظات:

1- الاختبار يحتوي على 34 سؤال و 7 صفحات.

2- سيتم تصحيح الأسئلة الاختيارية من ورقة التصحيح الآلي **فقط**، وبذلك أي خطأ في نقل الإجابة تتحمل الطالبة مسؤوليته.

3- من الضروري كتابة **الاسم والرقم الجامعي والرقم التسلسلي واسم الأستاذة** على **الورقتين** (ورقة الأسئلة وورقة الإجابة).

Question:

If $E(X) = 0.4,\ E(Y) = 0.7,\ E(XY) = 0.5,\ V(X) = 0.24,\ V(Y) = 0.37$. Find:

1. $E(X^2) =$ $V(x) + (E(x))^2$

   (A) 0.86 (B) 0.53 (C) 0.4 (D) 0.08

2. $E(X^2 - 2X + 3) =$ $E(x^2) - 2E(x) + 3$

   (A) 2.73 (B) 2.6 (C) 2.28 (D) 3.06

3. $Cov(X, Y) =$ $0.5 - (0.4 \times 0.7)$

   (A) 0.22 (B) 0.04 (C) 0.41 (D) 0.36

4. $V(X - Y) =$ $V(x) + V(y) - 2\,cov(x,y) = 0.24 + 0.37 - 2(0.22)$

   (A) 1.05 (B) 0.17 (C) 0.83 (D) 1.43

5. $\rho_{X,Y} =$ $\dfrac{0.22}{\sqrt{0.24 \times 0.37}}$

   (A) 2.48 (B) 0.74 (C) 2.89 (D) 3.67

Question:

Let $f(x, y) = cy^2 + x,\ \ 0 \le x \le 1, 0 \le y \le 1.$ Then,

$\int_0^1\int_0^1 cy^2 + x \, dx\,dy = \int_0^1 cy^2x + \frac{x^2}{2}\Big|_0^1 dy$

$= \int_0^1 cy^2 + \frac{1}{2}\, dy = \frac{cy^3}{3} + \frac{1}{2}y\Big|_0^1$

$= \frac{c}{3} + \frac{1}{2} = 1$

$\frac{c}{3} = \frac{1}{2}$

6. $c =$

   (A) $\frac{1}{6}$ (B) $\frac{3}{2}$ (C) $\frac{1}{2}$ (D) $\frac{1}{3}$

7. $f(x) =$ $\int_0^1 \frac{3}{2}y^2 + x\, dy = \frac{1}{2}y^3 + xy\Big|_0^1 = \frac{1}{2} + x$

   (A) $x + \frac{1}{2}$ (B) $x^2 + \frac{1}{2}$ (C) $\frac{3}{2}y^2 + \frac{1}{2}$ (D) $\frac{1}{3}y^2 + \frac{1}{2}$

8. $f(y) =$ $\int_0^1 \frac{3}{2}y^2 + x\, dx = \frac{3}{2}y^2x + \frac{x^2}{2}\Big|_0^1 = \frac{3}{2}y^2 + \frac{1}{2}$

   (A) $x + \frac{1}{2}$ (B) $x^2 + \frac{1}{2}$ (C) $\frac{3}{2}y^2 + \frac{1}{2}$ (D) $\frac{1}{3}y^2 + \frac{1}{2}$

$V(X) = E_Y[V(X|Y)] + V_Y[E(X|Y)]$.

$V(Y) = E_X[V(Y|X)] + V_X[E(Y|X)]$.

Question:

V_x

If $E(E(Y|X)) = 2, V(E(Y|X)) = 3, E(V(Y|X)) = 3$ and $Cov(X, Y) = 2$. Then

$E(y)$

9. $E(Y) =$
(A) 2 (B) 0 (C) 6 (D) 3

10. $V(Y) =$ $E_x(V(y|x)) + V_x(E(y|x))$
(A) 0 (B) 6 (C) 3 (D) 2

11. $Cov(X + Y, Y) =$ V. $Cov(X+Y,Z) = Cov(X,Z) + Cov(Y,Z)$ $= Cov(x,y) + Cov(y,y) = 2 + Var(y) = 2+6$
(A) 2 (B) 4 (C) 5 (D) 8

12. $Cov(3, Y) =$
(A) 2 (B) 3 (C) 0 (D) 4

Question:

Let X and Y be random variables that take on values from the set {-1, 0, 1}:

Y \ X	-1	0	1
-1	3/15	2/15	1/15
0	1/15	1/15	2/15
1	1/15	1/15	3/15
	5/15	4/15	6/15

Find:

13. $P(X = 0, Y = 1) =$
(A) 0 (B) $\frac{2}{15}$ (C) $\frac{1}{15}$ (D) $\frac{3}{15}$

14. $P(X < 0, Y > -1) =$ $f(-1,0) + f(-1,1) = \frac{1}{15} + \frac{1}{15}$
(A) $\frac{2}{15}$ (B) $\frac{5}{15}$ (C) $\frac{4}{15}$ (D) $\frac{3}{15}$

15. $P(X + Y < 0) =$ $f(-1,-1) + f(-1,0) + f(0,-1) = \frac{3}{15} + \frac{1}{15} + \frac{2}{15} = \frac{6}{15}$
(A) $\frac{3}{15}$ (B) $\frac{5}{15}$ (C) $\frac{4}{15}$ (D) $\frac{6}{15}$

16. $f_X(x) =$

(A)

x	-1	0	1
$f(x)$	$\frac{5}{15}$	$\frac{4}{15}$	$\frac{6}{15}$

(B)

x	-1	0	1
$f(x)$	$\frac{6}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

(C)

x	-1	0	1
$f(x)$	$\frac{4}{15}$	$\frac{5}{15}$	$\frac{6}{15}$

(D)

x	-1	0	1
$f(x)$	$\frac{6}{15}$	$\frac{5}{15}$	$\frac{4}{15}$

$E(x^2) = (1)(\frac{5}{15}) + (\frac{6}{15})(1)$

$= \frac{11}{15}$

17. $E(X) =$ $(-1)(\frac{5}{15}) + (0)(\frac{4}{15}) + (1)(\frac{6}{15}) =$

(A) $\frac{3}{15}$ (B) $\frac{10}{15}$ (C) $\frac{11}{15}$ (D) $\frac{1}{15}$

18. $V(X) =$ $\frac{11}{15} - (\frac{1}{15})^2 =$

(A) $\frac{10}{15^2}$ (B) $\frac{1}{15}$ (C) $\frac{11}{15}$ (D) $\frac{164}{15^2}$

19. $f(x = 0|y = -1) =$ $\frac{f(x,y)}{f(y)} = \frac{f(0,-1)}{f(-1)} = \frac{\frac{2}{15}}{\frac{6}{15}} = \frac{2}{6} = \frac{1}{3}$

(A) $\frac{3}{15}$ (B) $\frac{1}{3}$ (C) $\frac{2}{15}$ (D) $\frac{6}{15}$

Question:

$\frac{3y^2}{x^3} = \frac{S}{3x^2}$

Let $f(x) = 3x^2,\ 0 < x < 1$ and $f(y|x) = \frac{3y^2}{x^3}$, $0 < y < x$. Determine:

$9x^2y^2 = x^3 S$

$\frac{9x^2y^2}{x^3} = S$

$S = \frac{9y^2}{x}$

20. $f(x, y) =$

(A) $\frac{9y^2}{x}$, $0 < x < 1;\ 0 < y < 1$
(B) $\frac{9y^2}{x}$, $0 < y < x < 1$
(C) $\frac{3y^2}{x^3}$, $0 < y < x < 1$
(D) $3x^2$, $0 < y < 1, 0 < x < 1$

21. $f(y) =$ $9\int_y^1 \frac{y^2}{x}\,dx = 9y^2 \ln x\Big|_y^1 = \underbrace{9y^2\ln(1)}_{\text{zero}} - 9y^2\ln y$

(A) $-9y^2Lny$, $0 < y < x$
(B) $-9y^2Lny$, $0 < y < 1$
(C) $9y^2Lny$, $0 < y < x$
(D) $9y^2Lny$, $0 < y < 1$

22. $f(x|y) =$ $\frac{\cancel{9y^2}}{x} \cdot \frac{1}{-\cancel{9y^2}\ln y}$

(A)) $Ln(x)$, $0 < y < x < 1$
(B) $\frac{x^2}{12}$, $0 < y < x < 1$
(C) $\frac{-1}{xLny}$, $0 < y < x < 1$
(D) $\frac{y^2}{2}$, $0 < y < x < 1$

23. $E(Y|X) =$ $\int_0^x y\frac{3y^2}{x^3}\,dy = \frac{3}{x^3}\int_0^x y^3\,dy = \frac{3}{x^3}\left(\frac{y^4}{4}\right) = \frac{3}{x^3}\cdot\frac{x^4}{4}$

(A) $\frac{3}{4}x$ (B) $\frac{x^2}{12}$ (C) $\frac{x}{2}$ (D) $\frac{y^2}{2}$

24. $F(y|x) =$ $(\int_{-\infty}^{x} f(u|y)\,du;$ $= \frac{3}{x^3}\int_0^y y^2\,dy = \frac{3}{x^3}\left(\frac{y^3}{3}\right)\Big|_0^y = \frac{3}{x^3}\cdot\frac{y^3}{3}$

(A) $2x$, $0 < y < x < 1$
(B) 1, $0 < y < x < 1$
(C) $\frac{y}{x}$, $0 < y < x < 1$
(D) $\left(\frac{y}{x}\right)^3$, $0 < y < x < 1$

25. X and Y are $f(x,y) \stackrel{?}{=} f(x)f(y) \Rightarrow \frac{9y^2}{x} \neq 3x^2\cdot(-9y^2\ln y)$

(A) independent (B) dependent

$M_{X,Y}(\mathrm{t}_1, 0) = M_X(t_1)$.

Question: $M_{X,Y}(0, \mathrm{t}_2) = M_Y(t_2)$.

26. $M_{X,Y}(0, t_2) =$
    (A) $M_X(t_1)$ (B) 0 (C) $M_Y(t_2)$ (D) 1

27. If $E(X|Y) = \frac{2}{7}$ and $Z = X + \frac{4}{7}$, then: $E(Z|Y) =$ $E(x+\frac{4}{7}|y) = E(x|y) + \frac{4}{7} = \frac{2}{7} + \frac{4}{7}$
    (A) $\frac{4}{7}$ (B) $\frac{8}{7}$ (C) $\frac{2}{7}$ (D) $\frac{6}{7}$

28. The value of $\rho_{2x,y+8}$ when $\rho_{X,Y} = 0.27$ is equal to
    (A) -0.27 (B) 0.27 (C) 0.54 (D) 0.3

Question:

29. Prove that $\rho_{X,X} = 1$. (1 mark).

30. Prove that $Cov(X+Y, Z) = Cov(X, Z) + Cov(Y, Z)$. (2 marks).

31. If $f(x) = \theta x^{\theta-1}$ $0 < x < 1$, find the distribution of $Y = -lnX$ **(Use the one-to-one transformation**). (2 marks).

32. If $f(x) = 1,\ 0 < x < 1$. Find the pdf of $Y = \sqrt{X}$**. (Use the CDF method)**. (2 marks).

33. Let X_1 and X_2 are two random variables have the joint probability distribution as follows:

		X_1	
		0	1
X_2	0	0.05	0.58
	1	0.3	0.07

(a) Prove that X_1 and X_2 are dependent random variables. (1 mark).
(b) Find the pmf of the random variable $Y = 2X_1 + X_2$. (2 marks).

34. If $X_1 \sim \chi_n^2$ and $X_2 \sim \chi_m^2$ are independent random variables. Find the distribution of $Y = X_1 + X_2$ **(Use the moment generating function method)**.

Table of some MGF:

Distribution Name	pdf	MGF
Uniform Distribution	$f(x) = \frac{1}{b-a}, \quad a \le x \le b$	$M_X(t) = \frac{e^{bt} - e^{at}}{t(b-a)}$
Exponential Distribution	$f(x) = \theta e^{-\theta x}, \quad x \ge 0$	$M_X(t) = \frac{\theta}{\theta - t}$
Gamma Distribution	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, x \ge 0$	$M_X(t) = \left(\frac{\beta}{\beta - t}\right)^\alpha$
Chi-Squared Distribution	$f(x) = \frac{1}{2^{\left(\frac{v}{2}\right)} \Gamma\left(\frac{v}{2}\right)} x^{\frac{v}{2}-1} e^{-\frac{x}{2}}, x \ge 0$	$M_X(t) = (1-2t)^{-\frac{v}{2}}$
Normal Distribution	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}, -\infty < x < \infty$	$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$

End of Questions

Question (1): (7 Marks)

Suppose X and Y have the following joint probability distribution:

$f(x,y)$		x = 2	x = 4
y	1	0.18	0.39
	5	0.29	0.14

Find the following:

1. Prove that $f(x,y)$ is a pmf.
2. Find the pmf of X.
3. Find $E(XY^2)$.
4. Find $E(X)$ using the moment generating function of X and Y.
5. Find the distribution of U when, $U = X + Y$.

Question (2): (5 Marks)

Suppose X and Y be two random variables where, $E(X) = 2.6$, $E(Y) = 3.1$, $E(XY) = 8.2$, $V(Y) = 2.4$, $V(X) = 0.54$. Find:

1. $Cov(X, 2)$. $= 0$
2. $Cov(Y, X)$. $= E(xy) - E(x)E(y) = 8.2 - 3.1 \times 2.6 = 8.2 - 8.06 = 0.14$
3. $V(X + Y)$. $= V(x) + V(y) + 2\,cov = 2.4 + 0.54 + 2(0.14)$
4. $Corr(X, Y)$. $= \frac{0.14}{\sqrt{2.4 \times 0.54}}$
5. Study the independence between X and Y.

Question (3): (5 Marks)

Let X denote the reaction time, in seconds, to a certain stimulus and Y denote the temperature (◦F) at which a certain reaction starts to take place. Suppose that two random variables X and Y have the joint density:

$$f(x,y) = 4xy\,, \quad 0 < x < 1,\ 0 < y < 1.$$

1. $f(x|y)$.
2. $F(x|y)$.
3. $E(X|Y = 0.2)$.
4. $V(X|Y = 0.2)$.

Question (4): (3 Marks)

Let X be a random variable with probability distribution

$$f(x) = 2x\,,\quad 0 < x < 1$$

Find the probability distribution of the random variable $Y = X^2$.