Q1. 4+4+4=12 (a) Since the given matrix is symmetric, we have: $x - y + z = 0$; $x + y + z = 0$; $4x + 2y + z = 0$. Solving these homogeneous linear system, one obtains: $x = y = z = 0$.
(b) The $|A| = 6 \neq 0$, the matrix is invertible. Now,

$$B = A + 3I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

(c) By applying elementary row operations on the augmented matrix, we get

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & \lambda^2 - 4 & \lambda + 2 \end{bmatrix}$$

Hence, $\lambda = 2$ makes the given system inconsistent.
Q2. 4+4+4=12 (a) $\mathbf{AB} = \langle -2, 1, 1\rangle$, and $\mathbf{AC} = \langle -1, 1, -1\rangle$. So, $\mathbf{AB} \times \mathbf{AC} = \langle -2, -3, -1\rangle$. Hence, the equation of the plane is $-2(x-1) - 3(y-1) - 1(z-0) = 0$, i.e., $2x + 3y + z = 5$.
(b) The area of the triangle ABC is given by $\frac{1}{2}\|\mathbf{AB} \times \mathbf{AC}\| = \frac{1}{2}\sqrt{14}$.
(c) The volume of the box $ABCD$ is given by $|(\mathbf{AB}\times\mathbf{AC})\cdot\mathbf{AD}| = |\langle -2, -3, -1\rangle\cdot\langle 2, 0, 1\rangle| = |-5| = 5$.
Q3. 4+4+4=12 (a) Given C: $\mathbf{r}(t) = \langle \sin t + \cos t, (\sin t)e^t, \cos t\rangle$. Then $\mathbf{r}'(t) = \langle \cos t - \sin t, (\cos t)e^t + (\sin t)e^t, -\sin t\rangle$. The point $P(1, 0, 1)$ corresponds to $t = 0$, and $\mathbf{r}'(0) = \langle 1, 1, 0\rangle$. So, the equation of the tangent line to C at $P(1, 0, 1)$ has parametric equations: $x = 1 + t$, $y = t$, $z = 1$, $t \in \Re$.
(b) We get $\mathbf{r}'(t) = \langle 2(t-1), 2, \frac{1}{t}\rangle$ and $\mathbf{r}''(t) = \langle 2, 0, -\frac{1}{t^2}\rangle$.
Then $\mathbf{v}(1) = \mathbf{r}'(1) = \langle 0, 2, 1\rangle$, $v(1) = \sqrt{5}$ and $\mathbf{a}(1) = \mathbf{r}''(1) = \langle 2, 0, -1\rangle$.
(c) We have $\mathbf{r}'(t) = \langle -\sin t, \cos t, -e^{-t}\rangle$, and $\mathbf{r}''(t) = \langle -\cos t, -\sin t, e^{-t}\rangle$. Then $\mathbf{r}'(0) = \langle 0, 1, -1\rangle$, $\mathbf{r}''(0) = \langle -1, 0, 1\rangle$, and $\|\mathbf{r}'(0)\| = \sqrt{2}$. So, $a_T(0) = \frac{\mathbf{r}'(0)\cdot\mathbf{r}''(0)}{\|\mathbf{r}'(0)\|} = -\frac{1}{\sqrt{2}}$. Next $\mathbf{r}'(0) \times \mathbf{r}''(0) = \langle 1, 1, 1\rangle$, and so, $a_N(0) = \frac{\|\mathbf{r}'(0)\times\mathbf{r}''(0)\|}{\|\mathbf{r}'(0)\|} = \sqrt{\frac{3}{2}}$.
Q4. 2+2+3+4+3=14 (a) We have $\frac{\partial\omega}{\partial x} = -e^{-x}\cos y - e^{-y}\sin x$, and $\frac{\partial^2\omega}{\partial x^2} = e^{-x}\cos y - e^{-y}\cos x$. $\frac{\partial\omega}{\partial y} = -e^{-x}\sin y - e^{-y}\cos x$, and $\frac{\partial^2\omega}{\partial y^2} = -e^{-x}\cos y + e^{-y}\cos x$. Hence we obtain $\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2} = 0$.
(b) By applying chain rule, we get $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = \frac{-2e^{-2t}}{x+y} + \frac{3t^2-2t}{x+y} = \frac{-2e^{-2t}+3t^2-2t}{e^{-2t}+t^3-t^2+5}$.
(c) We get $\nabla f = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{i} = (2x - 5y)\mathbf{i} + (6y - 5x)\mathbf{j}$. $\nabla f|_P = 11\mathbf{i} - 21\mathbf{j}$, and $\mathbf{u} = \frac{1}{\sqrt{2}}(\mathbf{i} + \mathbf{j})$. Hence, $D_{\mathbf{u}}f|_P = \nabla f|_p \cdot \mathbf{u} = \frac{-10}{\sqrt{2}} \approx -7.07$
(d) We have $f_x(x, y) = 3x^2+3y$, and $f_y(x, y) = 3x-3y^2$. By solving $x^2+y = 0$ and $x-y^2 = 0$, we obtain the critical points $(0, 0)$ and $(1, -1)$. Now $D(x, y) = -36xy-9$. Thus, we get $D(0, 0) = -9 < 0$ implies $(0, 0, 0)$ is a saddle point. $D(1, -1) = 27 > 0$ and $f_{xx}(1, -1) = 6 > 0$ implying $f(1, -1) = -1$ is a local minimum.
(e) Applying Lagrange multiplier, we get $\nabla f = \lambda\nabla g \implies \langle 2x, 2y, 2z\rangle = \lambda\langle 1, -1, 1\rangle$. Thus, $x = \frac{\lambda}{2}, y = \frac{-\lambda}{2}$ and $z = \frac{\lambda}{2}$; using $x - y + z = 1$, we get $\lambda = \frac{2}{3}$, and hence, $x = \frac{1}{3}, y = \frac{-1}{3}$ and $z = \frac{1}{3}$. Therefore, the minimal value of f is $f(\frac{1}{3}, \frac{-1}{3}, \frac{1}{3}) = \frac{1}{3}$.