SOLUTION MIDTERM I EXAM., SEM II, 2025

DEPT. MATH., COLLEGE OF SCIENCE
KING SAUD UNIVERSITY
MATH: 107 FULL MARK: 25

Q1. [3+2=5]

$$\begin{pmatrix} 3 & 8 & 2 & -5 \\ 2 & 5 & -3 & 0 \\ 1 & 2 & -2 & -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 2 & -2 & -1 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$\Rightarrow x = -1 - 2y + 2z,\ y = 2 - z,\ z = -1$. Hence by back substitution, we get $z = -1,\ y = 3$ and $x = -9$.

Q2. [3+2=5]

Considering the form $(A|I)$, we have the following

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -1/8 & 1/8 & 3/8 \\ 0 & 1 & 0 & 0 & 4/8 & 0 \\ 0 & 0 & 1 & 3/8 & -3/8 & -1/8 \end{pmatrix}$$

which is in the form $(I|A^{-1})$, where

$$A^{-1} = \frac{1}{8}\begin{pmatrix} -1 & 1 & 3 \\ 0 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$$

and

$$(A^T)^{-1} = (A^{-1})^T = \frac{1}{8}\begin{pmatrix} -1 & 0 & 3 \\ 1 & 4 & -3 \\ 3 & 0 & -1 \end{pmatrix}$$

Q3. [3+2=5]

(a) Due to symmetry, we must have $x + 2y = -1$ and $-2x - 3y = 4$. Solving for x and y one obtains $x = -5$ and $y = 2$.

(b) We have $\left(BA^{-1}B^T\right)^T = (B^T)^T(A^{-1})^T B^T = BA^{-1}B^T$, as A is symmetric.

Q 4. [1+2+2+1=6]

Since $det(A) = -3$, A is inventible. The matrix of the cofactors is given by

$$C = \begin{pmatrix} -1 & -1 & 2 \\ -1 & -1 & -1 \\ 1 & -2 & 1 \end{pmatrix}$$

and hence

$$adj(A) = C^T = \begin{pmatrix} -1 & -1 & 1 \\ -1 & -1 & -2 \\ 2 & -1 & 1 \end{pmatrix}$$

$det(A^{-1}) = \frac{1}{det(A)} = -\frac{1}{3}$.

Q 5. [1+1+1+1=4]

We have $|A| = -3$, $|A_1| = -4$, $|A_2| = 1$, $|A_3| = 5$. Hence applying Cramer's rule, we get $x = \frac{|A_1|}{|A|} = \frac{4}{3}$, $y = \frac{|A_2|}{|A|} = -\frac{1}{3}$, $z = \frac{|A_3|}{|A|} = -\frac{5}{3}$