SOLUTION MIDTERM EXAM. II, SEMESTER II, 2025

Dept. Math., College of Science

KING SAUD UNIVERSITY

Math: 107 Full Mark: 25 Time: 90 Min

Q1. [2+2+2=6] (a) The displacement vector $\mathbf{PQ} = \langle 8, 3, 6\rangle$. Hence, work done $= \mathbf{F}\cdot\mathbf{PQ} = \langle 5, -3, 1\rangle \cdot \langle 8, 3, 6\rangle = 37$ units.

(b) $\langle 1, 2, 3\rangle = \mathbf{a} + 2\mathbf{b} + 3\mathbf{c} = \langle x, 0, 0\rangle + 2\langle 0, y, 0\rangle + 3\langle 0, 0, z\rangle = \langle x, 2y, 3z\rangle \implies x = y = z = 1$.

(c) $\mathbf{a}\cdot\mathbf{b} = \langle 1, 0, 0\rangle \cdot \langle 0, 1, 0\rangle = 0$. Similarly, $\mathbf{b}\cdot\mathbf{c} = 0$ and $\mathbf{c}\cdot\mathbf{a} = 0$.

Q2. [3+2+2+2=9] (a) Let $\mathbf{a} = \mathbf{PQ} = \langle 1, 2, 3\rangle$ and $\mathbf{b} = \mathbf{PR} = \langle -1, 0, -3\rangle$. Then $\mathbf{a}\times\mathbf{b} = \langle -6, 0, 2\rangle$. So the plane contains $P(1, -2, 0)$ and has normal vector $\mathbf{a}\times\mathbf{b} = \langle -6, 0, 2\rangle$. This plane, then, has equation $-6(x-1) + 2(z-0) = 0$, that is, $-6x + 2z + 6 = 0$.

(b) (i) We have normal vectors $\mathbf{n}_1 = \langle 1, -2, 2\rangle$ and $\mathbf{n}_2 = \langle 2, 1, -1\rangle$ show that $\frac{1}{2} \neq \frac{-2}{1}$, then $\wp_1 \nparallel \wp_2$.

(ii) Considering the augmented matrix, we have

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & -1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & -1 \end{bmatrix}$$

where, we have $x = 3 + 2y - 2z$ and $y = -1 + z$. Using $z = t$, we get $x = 1$, $y = -1 + t$ and $z = t$, for all $t \in \Re$.

(iii) The distance from the point $A(1, -1, 3)$ to the plane is $d = \frac{|1-2(-1)+2(3)-3|}{\sqrt{1^2+(-2)^2+2^2}} = \frac{6}{\sqrt{9}} = 2 \Rightarrow d = 2$.

Q3. [3+4+3=10] (a) Domain of $\mathbf{r}$ is $(-\infty, 1)$. $\mathbf{r}'(t) = \frac{-1}{1-t}\mathbf{i} + \cos t\mathbf{j} + 2t\mathbf{k}$. $\mathbf{r}''(t) = \frac{-1}{(1-t)^2}\mathbf{i} - \sin t\mathbf{j} + 2\mathbf{k}$.

(b) $\mathbf{r}(t) = e^t\cos t\mathbf{i} + e^t \sin t + e^t\mathbf{k}$

$\mathbf{v}(t) = \mathbf{r}'(t) = (e^t\cos t - e^t \sin t)\mathbf{i} + (e^t\cos t + e^t\sin t)\mathbf{j} + e^t\mathbf{k}$

$\mathbf{a}(t) = \mathbf{r}''(t) = -2e^t\sin t\mathbf{i} + 2e^t\cos t\mathbf{j} + e^t\mathbf{k}$.

$\mathbf{v}(\frac{\pi}{2}) = -e^{\frac{\pi}{2}}\mathbf{i} + e^{\frac{\pi}{2}}\mathbf{j} + e^{\frac{\pi}{2}}\mathbf{k} = e^{\frac{\pi}{2}}(-\mathbf{i} + \mathbf{j} + \mathbf{k})$

$\mathbf{a}(\frac{\pi}{2}) = -2e^{\frac{\pi}{2}}\mathbf{i} + e^{\frac{\pi}{2}}\mathbf{k} = e^{\frac{\pi}{2}}(-2\mathbf{i} + \mathbf{k})$.

Speed$=\sqrt{e^\pi + e^\pi + e^\pi} = \sqrt{3e^\pi} = \sqrt{3}e^{\frac{\pi}{2}}$.

(c) The given equation of the surface can be written as $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{4} = 1$, which is an ellipsoid. xy-trace is $\frac{x^2}{4} + \frac{y^2}{9} = 1$, which is an ellipse; yz-trace is $\frac{y^2}{9} + \frac{z^2}{4} = 1$, which is an ellipse; xz-trace is $\frac{x^2}{4} + \frac{z^2}{4} = 1$, which is a circle. Sketch is given at the last page.