Self

King Saud University, Department of Mathematics

Second Midterm Exam, S2, 2023/24

M-205

Marks: 25, Time: 90 minutes

Question 1 **[3+2+3 Marks]**

(a) Find the parametric equation of the line passing through point $A(0,1,1)$ and perpendicular to the plane containing points $A(0,1,1)$, $B(1,0,1)$ and $C(1,1,0)$.

(b) Determine the angle between vectors

$$\boldsymbol{a} = < 2,-2,1 > \text{ and } \boldsymbol{b} = < 3,0,4 >.$$

(c) Find the distance from point $D(-1,-1,-1)$ to the plane through points $A(0,1,1)$, $B(1,0,1)$ and $C(1,1,0)$.

Question 2 **[4+4 Marks]**

(a) Let $\boldsymbol{u} = < 2,-2,1 >$ and $\boldsymbol{v} = < 4,-1,2 >$. Find $Proj_v\, \boldsymbol{u}$ and show that $(\boldsymbol{u} - Proj_v\, \boldsymbol{u})$ is orthogonal to $\boldsymbol{v}$.

(b) Identify the surface $z = \frac{y^2}{2} + \frac{x^2}{4}$, find its traces on the coordinate planes and sketch the surface.

Question 3 **[5+4 Marks]**

(a) If the acceleration of a moving particle is given by

$$\boldsymbol{a}(t) = 6t\boldsymbol{i} + (2t+2)\boldsymbol{j} + (\cos t)\boldsymbol{k}.$$

Find its velocity and position given that the initial velocity is $\boldsymbol{v}(0) = \boldsymbol{i} - \boldsymbol{k}$, and the initial position is $\boldsymbol{r}(0) = \boldsymbol{2j} + \boldsymbol{3k}$.

(b) Let C be the curve determined by the vector valued function

$$\boldsymbol{r}(t) = (\tfrac{1}{2}t^2 + 1)\boldsymbol{i} + (t^2 - 2)\boldsymbol{j} + (t^3 + 3)\boldsymbol{k}.$$

Find the parametric equations of tangent line to the curve C at the point $t = 1$.

Q1° a/ $N = \vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = i + j + k$

parameter eq of the line is $x = t,\ y = 1+t,\ z = 1+t,\ t \in \mathbb{R}$.

b/ $a = \langle 2,-2,1\rangle,\quad b = \langle 3,0,4\rangle,\quad a\cdot b = 6+0+4 = 10$

$\|a\| = \sqrt{4+4+1} = \sqrt{9} = 3,\quad \|b\| = \sqrt{9+16} = \sqrt{25} = 5.$

$\cos(\theta) = \frac{a\cdot b}{\|a\|\|b\|} = \frac{10}{3\cdot 5} = 2/3 \Rightarrow \theta = \cos^{-1}(2/3)$

c/ eq of plan $N = \vec{AB} \times \vec{AC} = i+j+k$

$1(x-0) + 1(y-1) + 1(z-1) = 0$

$x+y+z = 2$, distance from $D(-1,-1,-1)$ to the plane $x+y+z=2$. $\Rightarrow d = \frac{|-1-1-1-2|}{\sqrt{1+1+1}} = \sqrt{5}/3$.

Q2 a/ $u = \langle 2,-2,1\rangle,\ v = \langle 4,-1,2\rangle$

$\text{comp}_v u = \frac{u\cdot v}{\|v\|} = \frac{8+2+6}{\sqrt{16+1+4}} = \frac{16}{\sqrt{21}}$, $\text{proj}_v u = \frac{u\cdot v}{\|v\|}\cdot\frac{v}{\|v\|}$

$= \frac{16}{\sqrt{21}}\ \frac{\langle 4,-1,2\rangle}{\sqrt{21}}$

$= \frac{16}{21}\langle 4,-1,2\rangle$

$= \langle \frac{64}{21}, \frac{-16}{21}, \frac{32}{21}\rangle$

$u - \text{proj}_v u = \langle 2,-2,3\rangle - \langle \frac{64}{21}, \frac{-16}{21}, \frac{32}{21}\rangle$

$= \langle \frac{-22}{21}, \frac{-26}{21}, \frac{31}{21}\rangle$

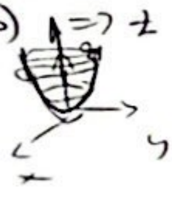
$(u - \text{proj}_v u)\cdot v = \frac{-88+26+62}{21} = 0 \Rightarrow$ $u - \text{proj}_v u \perp v$.

b/ paraboloid in the direction of z-axis, in xy-trace $(z=0)$

$\Rightarrow x = y = 0$ $(0,0)$ origin

in xy trace $(x=0) \Rightarrow z = \frac{y^2}{9}$ parabola, xz-plane $(y=0)$

$z = \frac{x^2}{4}$ parabola



Q3 a/ $a(t) = 6t\, i + (2t+2)j + \cos t\, k \Rightarrow v(t) = \int a(t)dt = 3t^2 i + (t^2+2t)j + \sin t\, k + c_1 i + c_2 j + c_3 k$

$v(0) = c_1 i + c_2 j + c_3 k = i - k \Rightarrow c_1 = 1,\ c_2 = 0,\ c_3 = -1$

$v(t) = (3t^2+1)i + (t^2+2t)j + (\sin t - 1)k$

$r(t) = \int [(3t^2+1)i + (t^2+2t)j + (\sin t - 1)k]dt$

$= (t^3+t)i + (\frac{t^3}{3} + t^2)j + (-\cos t - t)k + c_4 i + c_5 j + c_6 k$

at $t=0 \Rightarrow r(0) = -k + c_4 i + c_5 j + c_6 k = 2j + 3k \Rightarrow c_4 = 0,\ c_5 = 2,\ c_6 = 4$

$r(t) = (t^3+t)i + (t^3/3 + t^2 + 2)j + (\cos t - t + 4)k$

b/ at $t = 1 \Rightarrow x = 3/2 + t,\ y = -1+2t,\ z = 4+3t,\ t \in \mathbb{R}$