

Department of Mathematics, College of Sciences
King Saud University, Riyadh.

M-203 (Differential and Integral Calculus)

2nd MidTerm Examination (2nd semester 1445) (2023/2024),

Time: 90 Minutes

Max. Marks: 25.

Note: All questions carry equal marks.

Q 1. Evaluate the double integral:

$$\int_0^1 \int_{y^2}^1 y \sin(x^2) dx dy.$$

Q 2. Find the volume of the solid in the first octant bounded by $z = y$; $x^2 + y^2 = 4$; $x = 0$; $y = 0$.

Q 3. Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 1$ in the first octant.

Q 4. Using triple integral, find the volume of the solid bounded by the graphs of the equations: $z = 2 - x^2 - y^2$ and $z = x^2 + y^2$.

Q 5. Using cylindrical coordinates evaluate the integral:

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^{x^2+y^2} \sqrt{x^2 + y^2} dz dy dx.$$

Department of Mathematics

(1)

College of Science

King Saud University, Riyadh

M-2013 II Mid-Term Examination

All questions carry equal marks (II semester 1445)

Solutions to the questions ^{2023/2024}

Time: 90 Minutes

Max. Marks: 25

Q#1) Evaluate the double integral $\int_0^1 \int_{y^2}^1 y \sin(x^2) dx dy$ [Marks: 5]

Soln. Given: $y^2 \leq x \leq 1$ and $0 \leq y \leq 1$

changing to $0 \leq y \leq \sqrt{x}$ and $0 \leq x \leq 1$

Hence, we have $\int_0^1 \int_0^{\sqrt{x}} y \sin(x^2) dy dx$ (3)

$$= \int_0^1 \sin(x^2) \left[\frac{y^2}{2} \right]_0^{\sqrt{x}} dx$$

$$= \frac{1}{2} \int_0^1 x \sin x^2 dx = \frac{1}{4} [-\cos^2 x]_0^1 = \frac{1 - \cos^2 1}{4}$$

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Q#2) Find the volume of the solid in the first octant bounded by $z = y$, $x^2 + y^2 = 4$, $x = 0$, $y = 0$ [Marks: 5]

Soln. Volume $V = \iint_R y dA = \int_0^2 \int_0^{\sqrt{4-x^2}} y dy dx$ (3)

$$= \frac{1}{2} \int_0^2 (\sqrt{4-x^2})^2 dx = \frac{1}{2} \int_0^2 (4-x^2) dx$$

$$= \frac{1}{2} \left[4x - \frac{x^3}{3} \right]_0^2 = \frac{1}{2} \left(8 - \frac{8}{3} \right) = \frac{16}{3} = \frac{8}{3}$$

(2) 3

②

Q #3) Find the surface area of the portion of the cone $z = \sqrt{x^2 + y^2}$ that lies inside the cylinder $x^2 + y^2 = 1$ in the first octant. [Marks: 5]

Soln. We have $z = \sqrt{x^2 + y^2} = g(x, y)$

$$g_x = \frac{x}{\sqrt{x^2 + y^2}} ; g_y = \frac{y}{\sqrt{x^2 + y^2}} \quad (1)$$

Hence,

$$S.A. = \iint_R \sqrt{\frac{x^2}{x^2 + y^2} + \frac{y^2}{x^2 + y^2} + 1} dA$$
$$= \int_0^{\pi/2} \int_0^1 \sqrt{2} r dr d\theta = \sqrt{2} \times \frac{\pi}{2} \left[\frac{r^2}{2} \right]_0^1$$

$$= \sqrt{2}$$

$$= \frac{\sqrt{2} \pi}{4} \quad (1)$$

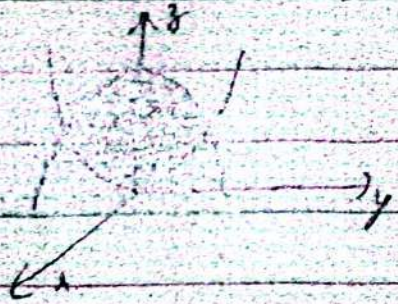
$$\sqrt{2} \int_0^1 \int_0^{\sqrt{1-x^2}} dy dx = \sqrt{2} \int_0^1 \sqrt{1-x^2} dx$$

Q #4) Using triple integral, find the Volume of the solid bounded by the graphs of the equations:

$$z = 2 - x^2 - y^2 \text{ and } z = x^2 + y^2 \quad \text{[Marks: 5]}$$

Soln. we have $2 - x^2 - y^2 = x^2 + y^2$

$$\text{or } 2(x^2 + y^2) = 2 \therefore x^2 + y^2 = 1$$



$$\text{Volume } V = \int_0^{2\pi} \int_0^1 \int_{r^2}^{2-r^2} r dz dr d\theta \quad (3)$$

$$= \int_0^{2\pi} \int_0^1 r \cdot (2 - \sqrt{r^2} - \sqrt{r^2}) dr d\theta$$

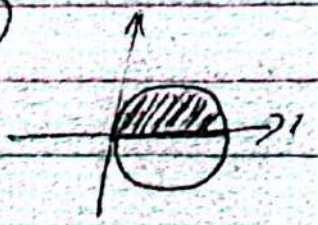
$$= \int_0^{2\pi} \int_0^1 2r \sqrt{1-2r^2} dr d\theta = 2\pi \left[\frac{1-2r^2}{2} \right]_0^1$$

$$= 2\pi \left[\frac{1-2}{2} - \frac{1-4}{2} \right] = 2\pi \left[1 - \frac{1}{2} \right]$$

Q #5) Using cylindrical coordinates evaluate the integral: $\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^{\sqrt{x^2+y^2}} \sqrt{x^2+y^2} dz dy dx$ [Markus 5]

Sol. $\int_0^{\pi/2} \int_0^{2\cos\theta} \int_0^r r \cdot r dz dr d\theta$

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$$= \int_0^{\pi/2} \int_0^{2\cos\theta} r^2 [z]_0^r dr d\theta$$

$$= \int_0^{\pi/2} \int_0^{2\cos\theta} r^4 dr d\theta = \int_0^{\pi/2} \left[\frac{r^5}{5} \right]_0^{2\cos\theta} d\theta$$

$$= \int_0^{\pi/2} \frac{2^5}{5} \cos^5 \theta d\theta$$

$$= \frac{2^5}{5} \int_0^{\pi/2} (-\sin^2 \theta)' \cos \theta d\theta$$

$$= \frac{2^5}{5} \times \frac{2^3}{15}$$

$$= \frac{2^8}{75} \quad \text{②}$$

Put $\sin \theta = t$

$$\cos \theta d\theta = dt$$

$$\int (1-t^2)^2 dt$$

$$= \int (-2t^2 + t^4) dt$$

$$= t - \frac{2}{3}t^3 + \frac{t^5}{5}$$

$$= \sin \theta - \frac{2}{3} \sin^3 \theta + \frac{1}{5}$$

$$= 1 - \frac{2}{3} + \frac{1}{5}$$

$$= \frac{15-10+3}{15} = \frac{8}{15}$$