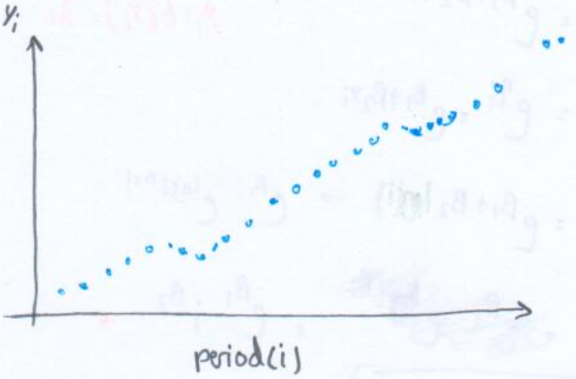
" Chapter 4 : Exercises Stat 333 page 69 "

Exercise 4.1:

Period i	Y_i = # Cases
1	1
2	6
3	16
4	23
5	27
⋮	⋮
20	159

(a)



by R:

* import Data:

```
install.packages("readxl")
  library(readxl)
  df <- read_excel("            ")
```

*a) plot y vs i :

```
plot(df$i, df$y)
```

(b)

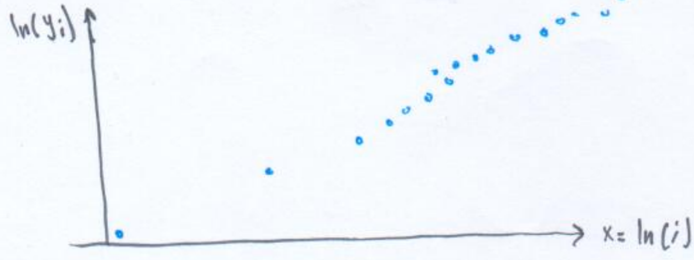
$$Y_i \sim \text{Poisson}(\lambda_i) \Rightarrow \begin{cases} \mu_i = E[Y_i] = \lambda_i \\ \sigma_i^2 = Var(Y_i) = \lambda_i \end{cases} \Rightarrow E[Y_i] = Var[Y_i]$$

$$\lambda_i = i^{\theta} \iff \ln(\lambda_i) = \theta \ln(i)$$

$$\begin{aligned} \ln(\lambda_i) &= \theta \ln(i) \\ &= \theta x_i \qquad (x_i = \ln(i)) \\ &= \eta_i \qquad (\eta_i = \eta_i(x) = \theta x_i) \end{aligned}$$

∴ link function:

$$g(\lambda_i) = \ln(\lambda_i)$$



by R:

*Add colum log(y) and log(i) :

```
df$logy <- log(df$y)
df$logi <- log(df$i)
Plot(df$logi, df$logy)
```

c) fit GzLM:

$$g(\lambda_i) = \log \lambda_i = \beta_1 + \beta_2 x_i$$

$$\Rightarrow \quad \lambda_i = e^{\beta_1 + \beta_2 x_i}$$

$\mu_i = E[y_i] = \lambda_i$, $\eta_i = \beta_1 + \beta_2 x_i$ where $x_i = \ln(i$

$$\mu_i = e^{\eta_i} = e^{\beta_1 + \beta_2 x_i}$$

$$= e^{\beta_1 + \beta_2 \ln(i)} = e^{\beta_1} e^{\ln(i^{\beta_2})}$$

$$= e^{\beta_1} i^{\beta_2} \quad *$$

1- $$\boxed{\mu_i = e^{\eta_i} \Rightarrow \frac{\partial \mu_i}{\partial \eta_i} = e^{\eta_i}}$$

$\boxed{y \sim \text{poisson} \quad var(y_i) = E[y_i]}$

2- The matrix of working weights is:

$$W = diag\left[\frac{1}{var(y_i)} \cdot \left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2 \right] = diag\left(\frac{1}{e^{\eta_i}} (e^{\eta_i})^2 \right)$$

$$= diag\left[e^{\eta_i} \right]$$

$\boxed{e^{\eta_i} = e^{\beta_1} \cdot i^{\beta_2} \; (*)}$

$$= diag\left[e^{\beta_1} i^{\beta_2} \right]$$

3- The information matrix is:

$$= \beta_1 + \beta_2 x_i \quad (x_i = \ln i)$$
$$= \beta_1 + \beta_2 \ln(i)$$

$$\mathcal{J} = X^t W X$$

$$X = \begin{bmatrix} 1 & \ln(1) \\ \vdots & \vdots \\ \vdots & \vdots \\ 1 & \ln(N) \end{bmatrix}_{N \times 2} \quad ; \quad X^t = \begin{bmatrix} 1 & \cdots & 1 \\ \ln(1) & \cdots & \ln(N) \end{bmatrix}_{2 \times N}$$

$$W = \begin{bmatrix} e^{\beta_1} 1^{\beta_2} & & 0 \\ & \ddots & \\ 0 & & e^{\beta_1} N^{\beta_2} \end{bmatrix}_{N \times N}$$

$$\therefore \quad \mathcal{J} = \begin{bmatrix} \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} & \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} \ln(i) \\ \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} \ln(i) & \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} [\ln(i)]^2 \end{bmatrix}$$

4- find U = Score statistics :

* log-likelihood function is :

$$\ell(\beta) = \sum_{i=1}^{N} \left[y_i \ln(\lambda_i) - \lambda_i - \ln(y_i!) \right]$$

$$\boxed{\lambda_i = \mu_i = e^{\beta_1 + \beta_2 \ln i}}$$

$$= \sum_{i=1}^{N} \left[y_i \ln(e^{\beta_1+\beta_2 \ln i}) - e^{\beta_1+\beta_2 \ln i} - \ln(y_i!) \right]$$

$$= \sum_{i=1}^{N} \left[y_i (\beta_1+\beta_2 \ln i) - e^{\beta_1+\beta_2 \ln i} - \ln(y_i!) \right]$$

* Score statistics are :

$$U_1 = \frac{\partial \ell}{\partial \beta_1} = \sum_{i=1}^{N} \left[y_i - e^{\beta_1+\beta_2 \ln(i)} \right]$$

$$U_2 = \frac{\partial \ell}{\partial \beta_2} = \sum_{i=1}^{N} \left[y_i \ln(i) - \ln(i) e^{\beta_1+\beta_2 \ln(i)} \right]$$

The vector of score statistics U is :

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{N} \left[y_i - e^{\beta_1+\beta_2 \ln(i)} \right] \\ \sum_{i=1}^{N} \left[y_i \ln(i) - \ln(i) e^{\beta_1+\beta_2 \ln(i)} \right] \end{pmatrix}$$

Another way for find information matrix $\mathfrak{I}$:

1- $$Var(U_1) = Var\left[\sum_{i=1}^{N} (y_i - e^{\beta_1+\beta_2 \ln(i)}) \right]$$

$$= \sum_{i=1}^{N} Var(y_i) = \sum_{i=1}^{N} e^{\beta_1+\beta_2 \ln(i)} = \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2}$$

2- $$Var(U_2) = Var\left[\sum_{i=1}^{N} (y_i \ln(i) - \ln(i) e^{\beta_1+\beta_2 \ln(i)}) \right]$$

$$= \sum_{i=1}^{N} var(y_i \ln(i)) = \sum_{i=1}^{N} [\ln(i)]^2 \, Var(y_i) = \sum_{i=1}^{N} [\ln(i)]^2 e^{\beta_1+\beta_2 \ln(i)}$$

$$= \sum_{i=1}^{N} [\ln(i)]^2 e^{\beta_1} i^{\beta_2}$$

3- $$Cov(U_1, U_2) = Cov\left(\sum_{i=1}^{N} (y_i - e^{\beta_1+\beta_2 \ln(i)}), \sum_{i=1}^{N} (y_i \ln(i) - \ln(i) e^{\beta_1+\beta_2 \ln(i)}) \right)$$

$$= \sum_{i=1}^{N} Cov(y_i , y_i \ln(i))$$

$$= \sum_{i=1}^{N} \ln(i) \; Cov(y_i , y_i)$$

$$= \sum_{i=1}^{N} \ln(i) \; var(y_i)$$

$$= \sum_{i=1}^{N} \ln(i) \; e^{\beta_1+\beta_2 \ln(i)} = \sum_{i=1}^{N} \ln(i) \; e^{\beta_1} \; i^{\beta_2}$$

4- Information matrix J is :

$$J = \begin{bmatrix} Var(U_1) & Cov(U_1 , U_2) \\ Cov(U_1, U_2) & Var(U_2) \end{bmatrix}$$

$$= \begin{bmatrix} \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} & \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} \ln(i) \\ \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} \ln(i) & \sum_{i=1}^{N} e^{\beta_1} i^{\beta_2} [\ln(i)]^2 \end{bmatrix}_{2\times 2}$$

5- we use the following iterative equation to find an approximate estimate of $\beta = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$b^{(m+1)} = b^{(m)} + \left[J^{(m)}\right]^{-1} U^{(m)} \qquad m = 0, 1, \ldots$$

• Step (0) : $m=0$

let initial value of b be $b^{(0)} = \begin{pmatrix} b_1^{(0)} \\ b_2^{(0)} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad b_1^{(0)} = 1$, $b_2^{(0)} = 1$

for this initial value, we have:

$$J^{(0)} = \begin{bmatrix} \sum_{i=1}^{N} e^{1} i^{(1)} & \sum_{i=1}^{N} e^{1} i^{(1)} \ln(i) \\ \sum_{i=1}^{N} e^{1} i^{(1)} \ln(i) & \sum_{i=1}^{N} e^{1} i^{(1)} [\ln(i)]^2 \end{bmatrix}_{2\times 2} = \begin{bmatrix} 570.839 & 1439.61 \\ 1439.61 & 3768.84 \end{bmatrix}_{2\times 2}$$

$$\left[J^{(0)}\right]^{-1} = \begin{bmatrix} 0.04775 & -0.01824 \\ -0.01824 & 0.00723 \end{bmatrix}$$

$$U^{(0)} = \begin{pmatrix} \sum_{i=1}^{N} \left(y_i - e^{1} \, i^{(1)} \right) \\ \\ \sum_{i=1}^{N} \left(y_i \ln(i) - \ln(i) \, e^{1} \, i^{(1)} \right) \end{pmatrix} = \begin{pmatrix} 740.1608 \\ \\ 1956.771 \end{pmatrix}$$

$$\therefore b^{(1)} = b^{(0)} + \left[J^{(0)} \right]^{-1} U^{(0)}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0.04775 & -0.01824 \\ -0.01824 & 0.00723 \end{bmatrix} \begin{bmatrix} 740.1608 \\ 1956.771 \end{bmatrix}$$

$$= \begin{bmatrix} 0.652354 \\ 1.651991 \end{bmatrix}$$

- Step $(m=1)$

we repet Step(0) using $\begin{cases} b_1^{(1)} = 0.652354 \\ b_2^{(1)} = 1.651991 \end{cases}$

we will obtain : $\begin{cases} b_1^{(2)} = 0.841856 \\ b_2^{(2)} = 1.429548 \end{cases}$

The following table Summarizes the results:

m	0	1	2	3	4	5	6
$b_1^{(m)}$	1	0.652354	0.841856	0.98454	0.995952	0.995998	0.995998
$b_2^{(m)}$	1	1.651991	1.429548	1.33373	1.326639	1.32661	1.32661

Same

we Stop the iteration process at step $m = 6$

$$\therefore b = \begin{bmatrix} 0.995998 \\ 1.32661 \end{bmatrix}$$

by R :

```
model <- glm ( y~ log(i) , family = poisson ( link = "log" ), data = df )
Summary(model)
```

output :

Cofficients :

	Estimate		s.t Error	z value	Pr(>|z|)
(intercept) :	0.996	b_1			
log(i) :	1.32661	b_2			

by minitab :

	c_1	c_2	c_3
	i	y	log i
1	1	1	0
2	2	6	0.69315
3	3	16	1.09861
⋮	⋮	⋮	⋮
20	20	159	2.99573

session

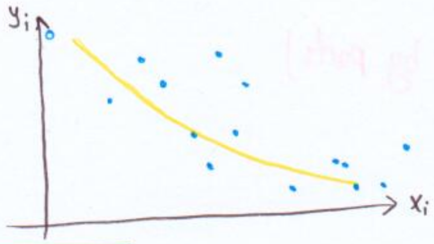
let $c_3 = \log(c_1)$ ↵

Stat → Reg → Poisson Reg → Fit possion model

→ Response [y] continuous predictor: [log i] → options → link function [log] → ok → ok

Exercise 4.2:

ⓐ



$\Rightarrow$ This graph shows that (y_i) exponentially decreases with (x_i).

by R:
plot(x, y)

ⓑ

$E[Y_i] = \mu_i$

$$\boxed{E[Y_i] = e^{\beta_1+\beta_2 x_i}}$$

$$\mu_i = e^{\beta_1+\beta_2 x_i} = e^{\eta_i} \quad ; \ \eta_i = \eta(x_i) = \beta_0 + \beta_1 x_i$$

$$\ln(\mu_i) = \eta_i$$
$$= \beta_1 + \beta_2 x_i$$

$\therefore$ The link function is:

$$g(\mu_i) = \ln(\mu_i)$$

ⓒ $f(y;\theta) = \theta e^{-y\theta} \quad ; \ y>0 \ , \ \theta>0$

we need to show that $E[Y] = \frac{1}{\theta}$ and $var[Y] = \frac{1}{\theta^2}$.

• First method:

$$E[y] = \int_0^\infty y f(y)\, dy = \int_0^\infty y\, \theta e^{-y\theta}\, dy \quad \text{(using integration by part)}$$

$u = y \qquad dv = \theta e^{-y\theta}$

$du = dy \qquad v = -e^{-y\theta}$

$$= -ye^{-y\theta}\Big|_0^\infty - \int_0^\infty -e^{-y\theta}\, dy$$

$$= 0 - \left[\frac{1}{\theta} e^{-y\theta}\right]_0^\infty$$

$$= 0 - \left[0 - \frac{1}{\theta}\right] = \frac{1}{\theta} \ \divideontimes$$

$e^{\infty} = \infty$
$e^{-\infty} = 0$
$e^{0} = 1$

2

$Var(y) = E[y^2] - (E[y])^2$

$E[y^2] = \int_0^\infty y^2 f(y)\, dy = \int_0^\infty y^2 \theta e^{-y\theta}\, dy$ (using integration by parts)

$u = y^2 \qquad dv = \theta e^{-y\theta}$

$du = 2y\, dy \qquad v = -e^{-y\theta}$

$= -y^2 e^{-y\theta}\Big|_0^\infty - \int_0^\infty -2y e^{-y\theta}\, dy$

$= 0 + \frac{2}{\theta} \int_0^\infty \theta y e^{-y\theta}\, dy \Rightarrow E[y] = \int_0^\infty y\theta e^{-y\theta}\, dy = \frac{1}{\theta}$

$= \frac{2}{\theta}\left(\frac{1}{\theta}\right) = \frac{2}{\theta^2}$

$Var(y) = E[y^2] - (E[y])^2 = \frac{2}{\theta^2} - \left(\frac{1}{\theta}\right)^2 = \frac{2}{\theta^2} - \frac{1}{\theta^2} = \frac{1}{\theta^2}$ ⋕

- Seconed method:

The exponential distribution belong to the "EF"

$f(y;\theta) = \theta e^{-\theta y}$

$= e^{\ln\theta - \theta y} = e^{a(y)b(\theta) + c(\theta) + d(y)}$

$a(y) = y \quad b(\theta) = -\theta \quad c(\theta) = \ln\theta \quad d(y) = 0$

$b' = -1 \qquad b'' = 0$

$c' = \frac{1}{\theta} \qquad c'' = -\frac{1}{\theta^2}$

$E[a(y)] = E[y] = -\frac{c'(\theta)}{b'(\theta)} = \frac{-\frac{1}{\theta}}{-1} = \frac{1}{\theta}$ ⋕

$Var(a(y)) = \frac{b''(\theta)\, c'(\theta) - c''(\theta)\, b'(\theta)}{[b'(\theta)]^3} \Rightarrow Var(y) = \frac{0 - \left[-\frac{1}{\theta^2}(-1)\right]}{[-1]^3} = \frac{1}{\theta^3}$ ⋕

d) $f(y_i;\theta_i) = \theta_i\, e^{-\theta_i y_i} \quad , \; y_i > 0 \quad , \; \theta_i > 0$

$$\left.\begin{array}{l} E[Y_i] = \mu_i = \dfrac{1}{\theta_i} \\ Var(Y_i) = \sigma^2 = \dfrac{1}{\theta_i} \end{array}\right\} \quad Var(Y_i) = \left[E[Y_i]\right]^2$$

$$\mu_i = e^{\beta_1 + \beta_2 x_i} = e^{\eta_i} \quad ; \quad \eta_i = \beta_1 + \beta_2 x_i$$

$\Leftrightarrow \ln(\mu_i) = \beta_1 + \beta_2 x_i = \eta_i$

- link function:

$g(\mu_i) = \ln(\mu_i)$

1- $\dfrac{\partial \mu_i}{\partial \eta_i} = e^{\eta_i}$

$Y \sim exp.$
$Var(Y) = (E[Y])^2$

2- The matrix of working weights is:

$$W = diag\left[\frac{1}{Var(Y_i)} \cdot \left(\frac{\partial \mu_i}{\partial \eta_i}\right)^2 \right] = diag\left[\frac{1}{(e^{\eta_i})^2} \cdot (e^{\eta_i})^2 \right]$$

$$= diag[1]$$

$$= I = \begin{bmatrix} 1 & & 0 \\ & 1 \ddots & \\ 0 & & 1 \end{bmatrix}_{N \times N}$$

3- The information matrix is:

$$\mathcal{J} = X^t W X$$

$$X = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix}_{N \times 2} , \quad X^t = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_N \end{bmatrix}_{2 \times N} , \quad W = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}_{N \times N}$$

$$\therefore \mathcal{J} = X^t W X = \begin{bmatrix} 1 & \cdots & 1 \\ x_1 & \cdots & x_N \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \end{bmatrix}_{2 \times 2}$$

4- find Score Statistics (U):

* log-likelihood function is:

$$l(\beta) = \sum_{i=1}^{N} \left[\ln(\theta_i) - y_i \theta_i \right]$$

$$= \sum_{i=1}^{N} \left[\ln\left(\frac{1}{\mu_i}\right) - \frac{y_i}{\mu_i} \right]$$

$$= \sum_{i=1}^{N} \left[\ln(\mu_i)^{-1} - \frac{y_i}{\mu_i} \right]$$

$$= \sum_{i=1}^{N} \left[-\beta_1 - \beta_2 x_i - \frac{y_i}{e^{\beta_1+\beta_2 x_i}} \right]$$

$$\boxed{\begin{array}{l} \bullet\ \mu_i = \frac{1}{\theta} \Rightarrow \theta = \frac{1}{\mu_i} \\ \bullet\ \mu_i = e^{\beta_1+\beta_2 x_i} \end{array}}$$

* The Score statistics are:

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix}$$

$$U_1 = \frac{\partial l}{\partial \beta_1} = \sum_{i=1}^{N} \left[-1 + \frac{y_i}{e^{\beta_1+\beta_2 x_i}} \right]$$

$$U_2 = \frac{\partial l}{\partial \beta_2} = \sum_{i=1}^{N} \left[-x_i + \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}} \right]$$

The vector of U:

$$U = \begin{bmatrix} \sum_{i=1}^{N} \left[-1 + \frac{y_i}{e^{\beta_1+\beta_2 x_i}} \right] \\ \sum_{i=1}^{N} \left[-x_i + \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}} \right] \end{bmatrix}$$

Another way for find information matrix $\Im$:

1- $$Var(U_1) = Var\left[\sum_{i=1}^{N} \left[-1 + \frac{y_i}{e^{\beta_1+\beta_2 x_i}} \right] \right] = \sum_{i=1}^{N} \frac{var(y_i)}{(e^{\beta_1+\beta_2 x_i})^2} = \sum_{i=1}^{N} \frac{(e^{\beta_1+\beta_2 x_i})^2}{(e^{\beta_1+\beta_2 x_i})^2}$$

$$= \sum_{i=1}^{N} 1 = N$$

2- $$Var(U_2) = Var\left[\sum_{i=1}^{N} \left(-x_i + \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}} \right) \right] = \sum_{i=1}^{N} \frac{x_i^2\ var(y_i)}{(e^{\beta_1+\beta_2 x_i})^2} = \sum_{i=1}^{N} \frac{x_i^2 (e^{\beta_1+\beta_2 x_i})^2}{(e^{\beta_1+\beta_2 x_i})^2}$$

$$= \sum_{i=1}^{N} x_i^2$$

3- $$Cov(U_1, U_2) = Cov\left(\sum_{i=1}^{N} \left(-1 + \frac{y_i}{e^{\beta_1+\beta_2 x_i}} \right), \sum_{i=1}^{N} \left(-x_i + \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}} \right) \right)$$

$$= \sum_{i=1}^{N} \operatorname{Cov}\left(\left(-1 + \frac{y_i}{e^{\beta_1+\beta_2 x_i}}\right), \left(-x_i + \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}}\right) \right)$$

$$= \sum_{i=1}^{N} \operatorname{Cov}\left(\frac{y_i}{e^{\beta_1+\beta_2 x_i}}, \frac{x_i y_i}{e^{\beta_1+\beta_2 x_i}} \right)$$

$$= \sum_{i=1}^{N} \frac{x_i}{(e^{\beta_1+\beta_2 x_i})^2} \operatorname{Cov}(y_i, y_i)$$

$$= \sum_{i=1}^{N} \frac{x_i}{(e^{\beta_1+\beta_2 x_i})^2} \operatorname{Var}(y_i, y_2)$$

$$= \sum_{i=1}^{N} \frac{x_i}{(e^{\beta_1+\beta_2 x_i})^2} (e^{\beta_1+\beta_2 x_i})^2 = \sum_{i=1}^{N} x_i$$

4- Information matrix J is:

$$J = \begin{bmatrix} \operatorname{Var}(U_1) & \operatorname{Cov}(U_1, U_2) \\ \operatorname{Cov}(U_1, U_2) & \operatorname{Var}(U_2) \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^{N} x_i \\ \sum_{i=1}^{N} x_i & \sum_{i=1}^{N} x_i^2 \end{bmatrix}_{2\times 2}$$

$N = 17 \qquad \sum_{i=1}^{17} x_i = 69.63 \qquad \sum_{i=1}^{N} x_i^2 = 291.4571$

$$\therefore J = \begin{bmatrix} 17 & 69.63 \\ 69.63 & 291.4571 \end{bmatrix} \Rightarrow$$

$$\Rightarrow J^{-1} = \begin{bmatrix} 2.73838856 & -0.65420947 \\ -0.65420947 & 0.159723697 \end{bmatrix}$$

5- we use the following iterative equation to find an approximate estimate of $\beta = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$b^{(m+1)} = b^{(m)} + \left[J^{(m)}\right]^{-1} U^{(m)} \qquad m = 0, 1, \ldots$$

• step(0): $m = 0$

Let initial value of b be $b^{(0)} = \begin{pmatrix} b_1^{(0)} \\ b_2^{(0)} \end{pmatrix} = \begin{pmatrix} 11 \\ -2 \end{pmatrix}$ $\qquad b_1^{(0)} = 11$, $b_2^{(0)} = -2$

$$U^{(0)} = \begin{pmatrix} \sum_{i=1}^{N} \left(-1 + \frac{y_i}{e^{11-2x_i}}\right) \\ \\ \sum_{i=1}^{N} \left(-x_i + \frac{x_i y_i}{e^{11-2x_i}}\right) \end{pmatrix} = \begin{pmatrix} 43.14648 \\ \\ 195.2387 \end{pmatrix}$$

$$\therefore\ b^{(1)} = b^{(0)} + \left[J^{(0)}\right]^{-1} U^{(0)}$$

$$= \begin{bmatrix} 11 \\ -2 \end{bmatrix} + \begin{bmatrix} 2.738388 56 & -0.65420947 \\ -0.65420947 & 6.159723697 \end{bmatrix} \begin{bmatrix} 43.14648 \\ \\ 195.2387 \end{bmatrix}$$

$$= \begin{bmatrix} 1.424823 \\ 0.95741 \end{bmatrix}$$

• step (m=1)

we repet step (m=0) Using $\begin{cases} b_1^{(1)} = 1.424823 \\ b_2^{(1)} = 0.95741 \end{cases}$

we will obtain : $\begin{cases} b_1^{(2)} = \\ b_2^{(2)} = \end{cases}$

The following table summarizes the results :

m	0	1	2	3	4	5	6			9	10
$b_1^{(m)}$	11	1.4248	4.0584	6.0272	7.5509	8.2975	—		—	8.4775	8.4775
$b_2^{(m)}$	-2	0.9574	0.1854	-0.4072	-0.8501	-1.0608	—		—	-1.1093	-1.1093

Same

we stop the iteration process at step m=10

(e) The fitted value:

$$\hat{y}_i = \hat{\mu}_i = e^{b_1 + b_2 x_i}$$

$$= e^{8.4775 - 1.1093 x_i}$$

y_i	65	156	100	— — —	5	65
$\hat{y}_i$	115.61	196.90	85.69	— — —	25.57	18.75

Standardized Residuals:

$$r_i = \frac{y_i - \hat{y}_i}{\hat{y}_i}$$

احولها للستندر (اطرح المتوسط واقسم على الانحراف

بما ان المسئلة توزيع اسي

المتوسط يساوي الانحراف ا

i	1	2	3		17
r_i	-0.4378	-0.2077	0.1669		2.4673

$$\sum_{i=1}^{17} r_i^2 = (-0.4378)^2 + (-0.2077)^2 + \cdots + (2.4673)^2 = 14.0832$$

① H_0: Model fits data well. H_A: Model does not fit data well.

② $\sum r_i^2 = 14.0832$

③

$\sum r_i^2 = 14.0832$

$$\chi^2_{\alpha, N-p} = \chi^2_{0.05, 17-2} = \chi^2_{0.05, 15} = 24.9958$$

$\alpha = 0.05$

N = # of observation

P = # of parameters

④ Since $\sum r_i^2 < \chi^2_{0.05, 15}$ $\Leftrightarrow$ ($\sum r_i^2 \in$ Accept Area), we conclude that the model is adequate for describing the data.