Definite Integral

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1 Last Lecture

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Thus, $f(x) = 5x^4 + 3x \leq 6x^5 + 3x^2 + 2 = g(x)$. Hence by property (7) we get

$$\int_1^9 f(x)dx \leq \int_1^9 g(x)dx.$$

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Therefore, $z = -1 \in (-2, 0)$.
(ii) $f_{av} = \frac{1}{b-a}\int_a^b f(x)dx = \frac{1}{0-(-2)}(0) = 0.$