Lecture ② : Simplix Method

Ex (2) Solve the following Lp problem

$$\min \; 4x_1 - x_2 \qquad\qquad \text{min} \rightarrow \text{minimum}$$

$$\text{s.t} \quad 2x_1 + x_2 \leq 8$$
$$x_2 \leq 5$$
$$x_1 - x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Ans * Change the given Lp problem to max Lp problem as follows:

$$\max \; -4x_1 + x_2$$
$$\text{s.t} \quad 2x_1 + x_2 \leq 8$$
$$x_2 \leq 5$$
$$x_1 - x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

$\Rightarrow$ **Canonical form**

$$\max \; -4x_1 + x_2$$
$$2x_1 + x_2 + x_3 = 8$$
$$x_2 + x_4 = 5$$
$$x_1 - x_2 + x_5 = 4$$

where x_3, x_4 and x_5 are slack variables

Let $x_1 = x_2 = 0$

i.e x_1 and x_2 are non-basic variables NBVs

$\Rightarrow$ x_3, x_4 and x_5 are basic variables BVs

i.e $\{x_3, x_4, x_5\}$ is the basis

So, we can write the dictionary as follows:

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BVs

$x_3 = 8 - 2x_1 - x_2$

$x_4 = 5 - x_2$

$x_5 = 4 - x_1 + x_2$

$z = -4x_1 + x_2$

first dictionary

* We pick a variable x_2 which having +ve coefficient in the eqn for z

$x_2 \longrightarrow$ incoming variable

Ratio test

$x_3 : \frac{8}{1} = 8$, $x_4 : \frac{5}{1}$, x_5 : positive coefficient for x_2 (no constraint)

$\therefore x_4 \longrightarrow$ outgoing variable

$\Rightarrow \boxed{x_2 = 5 - x_4}$

$$\Rightarrow \begin{aligned} x_2 &= 5 - x_4 \\ x_3 &= 8 - 2x_1 - (5 - x_4) \\ x_5 &= 4 - x_1 + (5 - x_4) \\ z &= -4x_1 + (5 - x_4) \end{aligned} \Rightarrow$$

BVs

$x_2 = 5 - x_4$

$x_3 = 3 - 2x_1 + x_4$

$x_5 = 9 - x_1 - x_4$

$z = 5 - 4x_1 - x_4$

STOP

-ve coeffs

Second dictionary

$\therefore$ the optimal soln is

$x_1 = 0$, $x_2 = 5$, $x_3 = 3$, $x_4 = 0$, $x_5 = 9$

where $z = 5$

i.e max $z = 5$ and consequently the min of $4x_1 - x_2$ is -5.

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