Note:

- **Organize your answers in the same order as the problems.**
- **Write legibly.**

1- Let $F_n = 2^{2^n} + 1$.

(a) Show that $F_0 F_1 \ldots F_{n-1} = F_n - 2,$ for all $n \geq 1$.

(b) Use (a) to show that $(F_m, F_n) = 1$ for all $m \neq n$.

2- Use Fermat factorization algorithm to completely factorize 836 (no calculators).

3- Find the least positive integer x that satisfies the system:

$$x \equiv 8 \bmod 9$$
$$x \equiv 5 \bmod 15$$
$$x \equiv 10 \bmod 20$$

4- If $(a, n) = 1$, then show that $a^{\varphi(n)} \equiv 1 \bmod n$, where $\varphi(n)$ is Euler function.

5- Let $p \neq q$ be primes. Prove that $p^{q-1} + q^{p-1} \equiv 1 \bmod pq$.