جامعة الملك سعود
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STAT415
probability(2)

part5 –problems from SOA

100. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold.

$$P[X = 0, Y = 0] = 1/6$$
$$P[X = 1, Y = 0] = 1/12$$
$$P[X = 1, Y = 1] = 1/6$$
$$P[X = 2, Y = 0] = 1/12$$
$$P[X = 2, Y = 1] = 1/3$$
$$P[X = 2, Y = 2] = 1/6$$

Calculate the variance of X.

(A) 0.47
(B) 0.58
(C) 0.83
(D) 1.42
(E) 2.58

100. Solution: B

$P(X = 0) = 1/6$
$P(X = 1) = 1/12 + 1/6 = 3/12$
$P(X = 2) = 1/12 + 1/3 + 1/6 = 7/12$.
$E[X] = (0)(1/6) + (1)(3/12) + (2)(7/12) = 17/12$
$E[X^2] = (0)^2(1/6) + (1)^2(3/12) + (2)^2(7/12) = 31/12$
$\text{Var}[X] = 31/12 - (17/12)^2 = 0.58$.

116. An actuary determines that the annual number of tornadoes in counties P and Q are jointly distributed as follows:

		Annual number of tornadoes in county Q			
		0	1	2	3
Annual number of tornadoes in county P	0	0.12	0.06	0.05	0.02
	1	0.13	0.15	0.12	0.03
	2	0.05	0.15	0.10	0.02

Calculate the conditional variance of the annual number of tornadoes in county Q, given that there are no tornadoes in county P.

(A) 0.51
(B) 0.84
(C) 0.88
(D) 0.99
(E) 1.76

116. Solution: D

With no tornadoes in County P the probabilities of 0, 1, 2, and 3 tornadoes in County Q are 12/25, 6/25, 5/25, and 2/25 respectively.
The mean is (0 + 6 + 10 + 6)/25 = 22/25.
The second moment is (0 + 6 + 20 + 18)/25 = 44/25.
The variance is $44/25 - (22/25)^2 = 0.9856$.

156. The probability of x losses occurring in year 1 is $(0.5)^{x+1}$ for $x = 0, 1, 2, \ldots$.

The probability of y losses in year 2 given x losses in year 1 is given by the table:

Number of losses in year 1 (x)	Number of losses in year 2 (y) given x losses in year 1				
	0	1	2	3	4+
0	0.60	0.25	0.05	0.05	0.05
1	0.45	0.30	0.10	0.10	0.05
2	0.25	0.30	0.20	0.20	0.05
3	0.15	0.20	0.20	0.30	0.15
4+	0.05	0.15	0.25	0.35	0.20

Calculate the probability of exactly 2 losses in 2 years.

(A) 0.025
(B) 0.031
(C) 0.075
(D) 0.100
(E) 0.131

156. Solution: E

$$P(x=1, y=1) = P(y=1 \mid x=1)P(x=1) = 0.3(0.5)^2 = 0.075$$

$$P(x=2, y=0) = P(y=0 \mid x=2)P(x=2) = 0.25(0.5)^3 = 0.03125$$

$$P(x=0, y=2) = P(y=2 \mid x=0)P(x=0) = 0.05(0.5)^1 = 0.025$$

The total is 0.13125.

225. An insurance company will cover losses incurred from tornadoes in a single calendar year. However, the insurer will only cover losses for a maximum of three separate tornadoes during this timeframe. Let X be the number of tornadoes that result in at least 50 million in losses, and let Y be the total number of tornadoes. The joint probability function for X and Y is

$$p(x,y) = \begin{cases} c(x+2y), & \text{for } x = 0,1,2,3,\ y = 0,1,2,3,\ x \le y \\ 0, & \text{otherwise,} \end{cases}$$

where c is a constant.

Calculate the expected number of tornadoes that result in fewer than 50 million in losses.

(A) 0.19
(B) 0.28
(C) 0.76
(D) 1.00
(E) 1.10

225. Solution: E

The possible events are (0,0), (0,1), (0,2), (0,3), (1,1), (1,2), (1,3), (2,2), (2,3), and (3,3). The probabilities (without c) sum to $0 + 2 + 4 + 6 + 3 + 5 + 7 + 6 + 8 + 9 = 50$. Therefor $c = 1/50$. The number of tornadoes with fewer than 50 million in losses is $Y - X$. The expected value is $(1/50)[0(0) + 1(2) + 2(4) + 3(6) + 0(3) + 1(5) + 2(7) + 0(6) + 1(8) + 0(9)]=55/50 = 1.1$.

231. Let N denote the number of accidents occurring during one month on the northbound side of a highway and let S denote the number occurring on the southbound side.

Suppose that N and S are jointly distributed as indicated in the table.

$N \setminus S$	0	1	2	3 or more
0	0.04	0.06	0.10	0.04
1	0.10	0.18	0.08	0.03
2	0.12	0.06	0.05	0.02
3 or more	0.05	0.04	0.02	0.01

Calculate Var $(N \mid N + S = 2)$.

(A) 0.48
(B) 0.55
(C) 0.67
(D) 0.91
(E) 1.25

231. Solution: B

Given $N + S = 2$, there are 3 possibilities $(N,S) = (2,0), (1,1), (0,2)$ with probabilities 0.12, 0.18, and 0.10 respectively.
The associated conditional probabilities are

$P(N = 0 \mid N + S = 2) = 0.10/0.40 = 0.25$,
$P(N = 1 \mid N + S = 2) = 0.18/0.40 = 0.45$,
$P(N = 2 \mid N + S = 2) = 0.12/0.40 = 0.30$.

The mean is $0.25(0) + 0.45(1) + 0.30(2) = 1.05$.
The second moment is $0.25(0) + 0.45(1) + 0.30(4) = 1.65$.
The variance is $1.65 – (1.05)(1.05) = 0.5475$.