STAT415
probability(2)

part3 –problems from SOA

30. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims.

The number of claims filed has a Poisson distribution.

Calculate the variance of the number of claims filed.

(A) $\frac{1}{\sqrt{3}}$

(B) 1

(C) $\sqrt{2}$

(D) 2

(E) 4

30. Solution: D

Let N be the number of claims filed. We are given $P[N=2]=\frac{e^{-\lambda}\lambda^2}{2!}=3P[N=4]=3\frac{e^{-\lambda}\lambda^4}{4!}$.

Then,

$\frac{1}{2}\lambda^2=\frac{3}{24}\lambda^4$ or $\lambda^2=4$ or $\lambda=2$, which is the variance of N.

39. A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.
(ii) In any calendar year, the probability of a hurricane is 0.05.
(iii) The numbers of hurricanes in different calendar years are mutually independent.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A) 0.06
(B) 0.19
(C) 0.38
(D) 0.62
(E) 0.92

39. Solution: E

The number of hurricanes has a binomial distribution with $n = 20$ and $p = 0.05$. Then $P[X < 3] = 0.95^{20} + 20(0.95)^{19}(0.05) + 190(0.95)^{18}(0.05)^2 = 0.9245.$

41. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants).

Calculate the probability that at least nine participants complete the study in one of the two groups, but not in both groups?

(A) 0.096
(B) 0.192
(C) 0.235
(D) 0.376
(E) 0.469

41. Solution: E

The number completing the study in a single group is binomial (10,0.8). For a single group the probability that at least nine complete the study is $\binom{10}{9}(0.8)^9(0.2)+\binom{10}{10}(0.8)^{10}=0.376$

The probability that this happens for one group but not the other is 0.376(0.624) + 0.624(0.376) = 0.469.

48. An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4.

Calculate the expected benefit under this policy.

(A) 2234
(B) 2400
(C) 2500
(D) 2667
(E) 2694

48. Solution: E

$$E[Y] = 4000(0.4) + 3000(0.6)(0.4) + 2000(0.6)^2(0.4) + 1000(0.6)^3(0.4)$$
$$= 2694$$

67. A baseball team has scheduled its opening game for April 1. If it rains on April 1, the game is postponed and will be played on the next day that it does not rain. The team purchases insurance against rain. The policy will pay 1000 for each day, up to 2 days, that the opening game is postponed.

The insurance company determines that the number of consecutive days of rain beginning on April 1 is a Poisson random variable with mean 0.6.

Calculate the standard deviation of the amount the insurance company will have to pay.

(A) 668
(B) 699
(C) 775
(D) 817
(E) 904

67. Solution: B

The expected amount paid is (where N is the number of consecutive days of rain)

$$1000P[N=1]+2000P[N>1]=1000\frac{e^{-0.6}0.6}{1!}+2000\left(1-e^{-0.6}-e^{-0.6}0.6\right)=573.$$

The second moment is

$$1000^2 P[N=1]+2000^2 P[N>1]=1000^2\frac{e^{-0.6}0.6}{1!}+2000^2\left(1-e^{-0.6}-e^{-0.6}0.6\right)=816{,}893.$$

The variance is 816,893 – 573^2 = 488,564 and the standard deviation is 699.

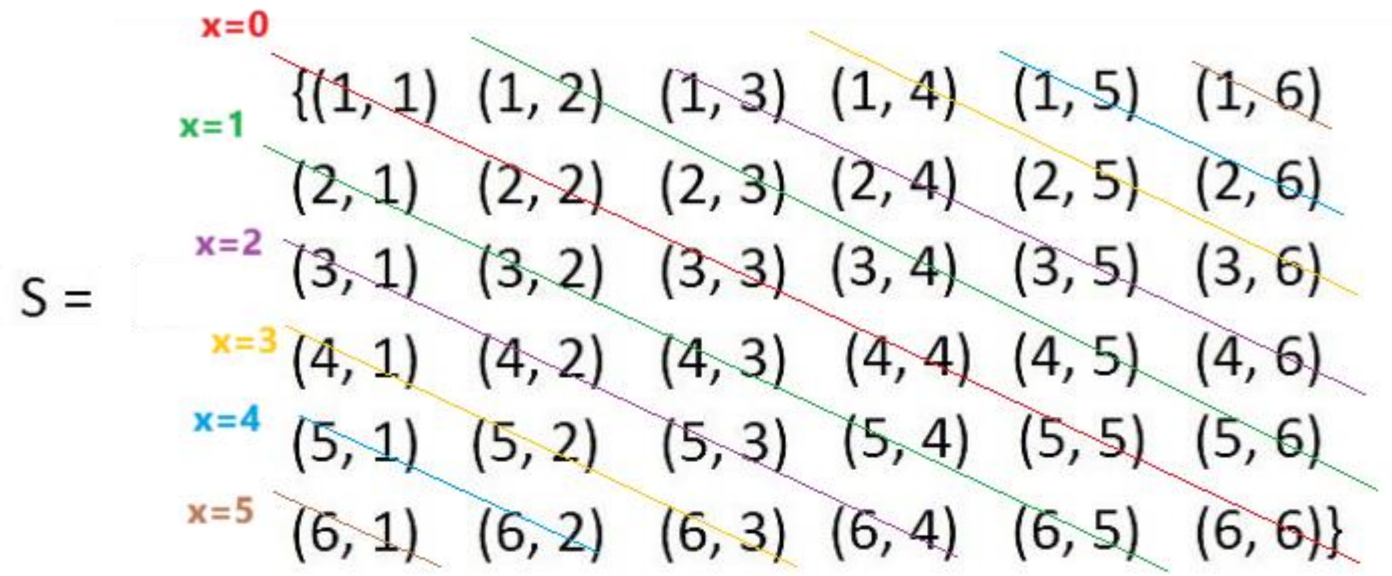
159. Two fair dice are rolled. Let X be the absolute value of the difference between the two numbers on the dice.

Calculate the probability that $X < 3$.

(A) 2/9
(B) 1/3
(C) 4/9
(D) 5/9
(E) 2/3

159. Solution: E

The dice rolls that satisfy this event are (1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (3,5), (4,2), (4,3), (4,4), (4,5), (4,6), (5,3), (5,4), (5,5), (5,6), (6,4), (6,5), and (6,6). They represent 24 of the 36 equally likely outcomes for a probability of 2/3.



160. An actuary analyzes a company's annual personal auto claims, M, and annual commercial auto claims, N. The analysis reveals that $\text{Var}(M) = 1600$, $\text{Var}(N) = 900$, and the correlation between M and N is 0.64.

Calculate $\text{Var}(M + N)$.

(A) 768
(B) 2500
(C) 3268
(D) 4036
(E) 4420

160. Solution: D

$$0.64 = \rho = \frac{Cov(M,N)}{\sqrt{Var(M)Var(N)}}$$

$$Cov(M,N) = 0.64\sqrt{1600(900)} = 768$$

$$Var(M+N) = Var(M) + Var(N) + 2Cov(M,N) = 1600 + 900 + 2(768) = 4036$$

164. In each of the months June, July, and August, the number of accidents occurring in that month is modeled by a Poisson random variable with mean 1. In each of the other 9 months of the year, the number of accidents occurring is modeled by a Poisson random variable with mean 0.5. Assume that these 12 random variables are mutually independent.

Calculate the probability that exactly two accidents occur in July through November.

(A) 0.084
(B) 0.185
(C) 0.251
(D) 0.257
(E) 0.271

164. Solution: B

The months in question have 1, 1, 0.5, 0.5, and 0.5 respectively for their means. The sum of independent Poisson random variables is also Poisson, with the parameters added. So the total number of accidents is Poisson with mean 3.5. The probability of two accidents is

$$\frac{e^{-3.5}3.5^2}{2!} = 0.185.$$

173. In a given region, the number of tornadoes in a one-week period is modeled by a Poisson distribution with mean 2. The numbers of tornadoes in different weeks are mutually independent.

Calculate the probability that fewer than four tornadoes occur in a three-week period.

(A) 0.13
(B) 0.15
(C) 0.29
(D) 0.43
(E) 0.86

173. Solution: B

The sum of independent Poisson variables is also Poisson, with the means added. Thus the number of tornadoes in a three week period is Poisson with a mean of 3x2 = 6. Then,

$$P(N < 4) = p(0) + p(1) + p(2) + p(3) = e^{-6}\left(\frac{6^0}{0!} + \frac{6^1}{1!} + \frac{6^2}{2!} + \frac{6^3}{3!}\right) = 0.1512.$$

174. An electronic system contains three cooling components that operate independently. The probability of each component's failure is 0.05. The system will overheat if and only if at least two components fail.

Calculate the probability that the system will overheat.

(A) 0.007
(B) 0.045
(C) 0.098
(D) 0.135
(E) 0.143

174. Solution: A

The number of components that fail has a binomial(2, 0.05) distribution. Then,

$$P(N \geq 2) = p(2) + p(3) = \binom{3}{2}(0.05)^2(0.95) + \binom{3}{3}(0.05)^3 = 0.00725.$$

197. On any given day, a certain machine has either no malfunctions or exactly one malfunction. The probability of malfunction on any given day is 0.40. Machine malfunctions on different days are mutually independent.

Calculate the probability that the machine has its third malfunction on the fifth day, given that the machine has not had three malfunctions in the first three days.

(A) 0.064
(B) 0.138
(C) 0.148
(D 0.230
(E) 0.246

197. Solution: C

The intersection of the two events (third malfunction on the fifth day and not three malfunctions on first three days) is the same as the first of those events. So the numerator of the conditional probability is the negative binomial probability of the third success (malfunction) on the fifth day, which is

$$\binom{4}{2}(0.4)^2(0.6)^2(0.4) = 0.13824 .$$

The denominator is the probability of not having three malfunctions in three days, which is

$1-(0.4)^3 = 0.936$.

The conditional probability is $0.13824/0.936 = 0.1477$.

212. The number of days an employee is sick each month is modeled by a Poisson distribution with mean 1. The numbers of sick days in different months are mutually independent.

Calculate the probability that an employee is sick more than two days in a three-month period.

(A) 0.199
(B) 0.224
(C) 0.423
(D) 0.577
(E) 0.801

212. Solution: D

Let N be the number of sick days for an employee in three months. The sum of independent Poisson variables is also Poisson and thus N is Poisson with a mean of 3.. Then,

$$P[N \leq 2] = e^{-3}\left(\frac{3^0}{0!} + \frac{3^1}{1!} + \frac{3^2}{2!}\right) = e^{-3}(1+3+4.5) = 0.423 .$$

The answer is the complement, 1 – 0.423 = 0.577.

213. The number of traffic accidents per week at intersection Q has a Poisson distribution with mean 3. The number of traffic accidents per week at intersection R has a Poisson distribution with mean 1.5.

Let A be the probability that the number of accidents at intersection Q exceeds its mean. Let B be the corresponding probability for intersection R.

Calculate $B - A$.

(A) 0.00
(B) 0.09
(C) 0.13
(D) 0.19
(E) 0.31

213. Solution: B

$$A = P(N > 3) = 1 - \left[P(N=0) + P(N=1) + P(N=2) + P(N=3)\right]$$

$$= 1 - e^{-3}\left(1 + \frac{3}{1} + \frac{9}{2} + \frac{27}{6}\right) = 1 - 13e^{-3} = 0.3528$$

$$B = P(N > 1.5) = 1 - \left[P(N=0) + P(N=1)\right]$$

$$= 1 - e^{-1.5}\left(1 + \frac{1.5}{1}\right) = 1 - 2.5e^{-1.5} = 0.4422$$

$B - A = 0.4422 - 0.3528 = 0.0894.$

237. In a group of 15 health insurance policyholders diagnosed with cancer, each policyholder has probability 0.90 of receiving radiation and probability 0.40 of receiving chemotherapy. Radiation and chemotherapy treatments are independent events for each policyholder, and the treatments of different policyholders are mutually independent.

The policyholders in this group all have the same health insurance that pays 2 for radiation treatment and 3 for chemotherapy treatment.

Calculate the variance of the total amount the insurance company pays for the radiation and chemotherapy treatments for these 15 policyholders.

(A) 13.5
(B) 37.8
(C) 108.0
(D) 202.5
(E) 567.0

237. Solution: B

Let X represent the number of policyholders who undergo radiation.
Let Y represent the number of policyholders who undergo chemotherapy.
X and Y are independent and binomially distributed with 15 trials each and with "success" probabilities 0.9 and 0.4, respectively.

The variances are $15(0.9)(0.1) = 1.35$ and $15(0.4)(0.6) = 3.6$.

The total paid is $2X + 3Y$ and so the variance is $4(1.35) + 9(3.6) = 37.8$.