STAT415
probability(2)

part1 –problems from SOA

Addition rule: $\boldsymbol{P(A) + P(B) - P(A \cap B)}$

Disjoint Events:

$If\ A \cap B\ = \phi$; *we say that* A *and* B *are* ***disjoint*** *sets and* $P(A \cap B) = 0$

Complementary events: $P(A) = 1 - P(A^C)$

Conditional probability:

$P(A|B) = \frac{P(A \cap B)}{P(B)}$; $P(B|A) = \frac{P(A \cap B)}{P(A)}$

$P(A^c|B) = 1 - P(A|B)$

Independent Events: $\begin{cases} P(A \cap B) = P(A)P(B) \\ P(A|B) = P(A) \\ P(B|A) = P(B) \end{cases}$

De Morgan's laws

$P(A \cap B)^C = P(A^C \cup B^C)$

$P(A \cup B)^C = P(A^C \cap B^C)$

$P(A^C \cap B^C) = P(A \cup B)^C = 1 - P(A \cup B)$

Another rule:

$$P(A \cup B) = \begin{cases} P(A) + P(B) - P(A \cap B) \\ P(A) + P(A^C \cap B) \\ P(A^C \cap B^c)^c = 1 - P(A^C \cap B^C) \end{cases}$$

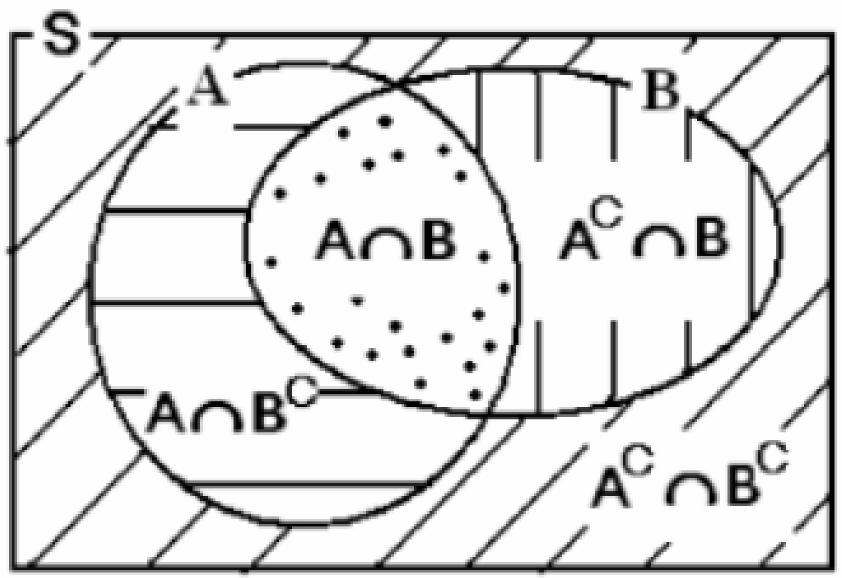
$$P(A \cap B) = P(A^C \cup B^C)^C = 1 - P(A^C \cup B^C) = 1 - [P(A^C) + P(B^C) - P(A^C \cap B^C)]$$

$$P(A) = P(A \cap B) + P(A \cap B^C)$$

$$P(B) = P(A \cap B) + P(A^C \cap B)$$

$$P(A \cap B^C) = P(A) - P(A \cap B)$$

$$P(A^c \cap B) = P(B) - P(A \cap B)$$



mutually exclusive (disjoint)

$$A \cap B = \emptyset$$

$$P(A \cap B) = 0$$

exhaustive

$$A \cup B = \Omega$$

$$P(A \cup B) = 1$$

Independent

$$P(A \cap B) = P(A)P(B)$$

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

1. A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics
(ii) 29% watched baseball
(iii) 19% watched soccer
(iv) 14% watched gymnastics and baseball
(v) 12% watched baseball and soccer
(vi) 10% watched gymnastics and soccer
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A) 24%
(B) 36%
(C) 41%
(D) 52%
(E) 60%

1. Solution: D

Let G = viewer watched gymnastics, B = viewer watched baseball, S = viewer watched soccer. Then we want to find

$$\Pr\left[(G \cup B \cup S)^c\right] = 1 - \Pr(G \cup B \cup S)$$

$$= 1 - \left[\Pr(G) + \Pr(B) + \Pr(S) - \Pr(G \cap B) - \Pr(G \cap S) - \Pr(B \cap S) + \Pr(G \cap B \cap S)\right]$$

$$= 1 - (0.28 + 0.29 + 0.19 - 0.14 - 0.10 - 0.12 + 0.08) = 1 - 0.48 = 0.52$$

2. The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work.

Calculate the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

(A) 0.05
(B) 0.12
(C) 0.18
(D) 0.25
(E) 0.35

2. Solution: A

Let R = referral to a specialist and L = lab work. Then

$$P[R\cap L]=P[R]+P[L]-P[R\cup L]=P[R]+P[L]-1+P[(R\cup L)^c]$$

$$=p[R]+P[L]-1+P[R^c\cap L']=0.30+0.40-1+0.35=0.05.$$

3. Solution: D

3. You are given $P[A \cup B] = 0.7$ and $P[A \cup B'] = 0.9$.

Calculate $P[A]$.

(A) 0.2
(B) 0.3
(C) 0.4
(D) 0.6
(E) 0.8

3. Solution: D

First note

$$P[A \cup B] = P[A] + P[B] - P[A \cap B]$$

$$P\left[A \cup B^c\right] = P[A] + P\left[B^c\right] - P\left[A \cap B^c\right]$$

Then add these two equations to get

$$P[A \cup B] + P\left[A \cup B^c\right] = 2P[A] + \left(P[B] + P\left[B^c\right]\right) - \left(P[A \cap B] + P\left[A \cap B^c\right]\right)$$

$$0.7 + 0.9 = 2P[A] + 1 - P\left[(A \cap B) \cup \left(A \cap B^c\right)\right]$$

$$1.6 = 2P[A] + 1 - P[A]$$

$$P[A] = 0.6$$

4. An urn contains 10 balls: 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are the same color is 0.44.

Calculate the number of blue balls in the second urn.

(A) 4
(B) 20
(C) 24
(D) 44
(E) 64

4. Solution: A

For $i = 1,2$, let R_i = event that a red ball is drawn from urn i and let B_i = event that a blue ball is drawn from urn i. Then, if x is the number of blue balls in urn 2,

$$\begin{aligned} 0.44 &= \Pr[(R_1 \cap R_2) \cup (B_1 \cap B_2)] = \Pr[R_1 \cap R_2] + \Pr[B_1 \cap B_2] \\ &= \Pr[R_1]\Pr[R_2] + \Pr[B_1]\Pr[B_2] \\ &= \frac{4}{10}\left(\frac{16}{x+16}\right) + \frac{6}{10}\left(\frac{x}{x+16}\right) \end{aligned}$$

Therefore,

$$2.2 = \frac{32}{x+16} + \frac{3x}{x+16} = \frac{3x+32}{x+16}$$

$$2.2x + 35.2 = 3x + 32$$

$$0.8x = 3.2$$

$$x = 4$$

5. An auto insurance company has 10,000 policyholders. Each policyholder is classified as

(i) young or old;
(ii) male or female; and
(iii) married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males.

Calculate the number of the company's policyholders who are young, female, and single.

(A) 280
(B) 423
(C) 486
(D) 880
(E) 896

5. Solution: D

Let $N(C)$ denote the number of policyholders in classification C. Then
N(Young and Female and Single)
= N(Young and Female) – N(Young and Female and Married)
= N(Young) – N(Young and Male) – [N(Young and Married) – N(Young and Married and Male)]
= 3000 – 1320 – (1400 – 600) = 880.

6. A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease.

Calculate the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

(A) 0.115
(B) 0.173
(C) 0.224
(D) 0.327
(E) 0.514

6. Solution: B

Let

H = event that a death is due to heart disease

F = event that at least one parent suffered from heart disease

Then based on the medical records,

$$P\left[H \cap F^c\right] = \frac{210-102}{937} = \frac{108}{937}$$

$$P\left[F^c\right] = \frac{937-312}{937} = \frac{625}{937}$$

and $P\left[H \mid F^c\right] = \dfrac{P\left[H \cap F^c\right]}{P\left[F^c\right]} = \dfrac{108}{937} \Big/ \dfrac{625}{937} = \dfrac{108}{625} = 0.173$

8. Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist.

Calculate the probability that a randomly chosen member of this group visits a physical therapist.

(A) 0.26
(B) 0.38
(C) 0.40
(D) 0.48
(E) 0.62

8. Solution: D

Let C = event that patient visits a chiropractor and T = event that patient visits a physical therapist. We are given that

$$\Pr[C] = \Pr[T] + 0.14$$

$$\Pr(C \cap T) = 0.22$$

$$\Pr(C^c \cap T^c) = 0.12$$

Therefore,

$$0.88 = 1 - \Pr[C^c \cap T^c] = \Pr[C \cup T] = \Pr[C] + \Pr[T] - \Pr[C \cap T]$$

$$= \Pr[T] + 0.14 + \Pr[T] - 0.22$$

$$= 2\Pr[T] - 0.08$$

or

$$P[T] = (0.88 + 0.08)/2 = 0.48$$

12. A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third have high blood pressure.
(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

Calculate the portion of the patients selected who have a regular heartbeat and low blood pressure.

(A) 2%
(B) 5%
(C) 8%
(D) 9%
(E) 20%

12. Solution: E

“Boxed” numbers in the table below were computed.

	High BP	Low BP	Norm BP	Total
Regular heartbeat	0.09	0.20	0.56	0.85
Irregular heartbeat	0.05	0.02	0.08	0.15
Total	0.14	0.22	0.64	1.00

From the table, 20% of patients have a regular heartbeat and low blood pressure.

17. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims.

Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A) 0.10
(B) 0.20
(C) 0.25
(D) 0.40
(E) 0.80

17. Solution: D

Let O = event of operating room charges and E = event of emergency room charges. Then

$$0.85 = P(O \cup E) = P(O) + P(E) - P(O \cap E)$$

$$= P(O) + P(E) - P(O)\mathrm{P}(E) \quad \text{(Independence)}$$

Because $P\left(E^c\right) = 0.25 = 1 - P(E)$, $P(E) = 0.75$,

$$0.85 = P(O) + 0.75 - P(O)(0.75)$$

$$P(O)(1 - 0.75) = 0.85 - 0.75 = 0.10$$

$$P(O) = 0.10 / 0.25 = 0.40.$$

19. An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of Driver	Probability of Accident	Portion of Company's Insured Drivers
16-20	0.06	0.08
21-30	0.03	0.15
31-65	0.02	0.49
66-99	0.04	0.28

A randomly selected driver that the company insures has an accident.

Calculate the probability that the driver was age 16-20.

(A) 0.13
(B) 0.16
(C) 0.19
(D) 0.23
(E) 0.40

19. Solution: B

Apply Bayes' Formula. Let

$A =$ Event of an accident

$B_1 =$ Event the driver's age is in the range 16-20

$B_2 =$ Event the driver's age is in the range 21-30

$B_3 =$ Event the driver's age is in the range 30-65

$B_4 =$ Event the driver's age is in the range 66-99

Then

$$P(B_1|A) = \frac{P(A|B_1)P(B_1)}{P(A|B_1)P(B_1)+P(A|B_2)P(B_2)+P(A|B_3)P(B_3)+P(A|B_4)P(B_4)}$$

$$= \frac{(0.06)(0.08)}{(0.06)(0.08)+(0.03)(0.15)+(0.02)(0.49)+(0.04)(0.28)} = 0.1584$$

20. An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year.

H.W

A policyholder dies in the next year.

Calculate the probability that the deceased policyholder was ultra-preferred.

(A) 0.0001
(B) 0.0010
(C) 0.0071
(D) 0.0141
(E) 0.2817

20. Solution: D

Let
S = Event of a standard policy
F = Event of a preferred policy
U = Event of an ultra-preferred policy
D = Event that a policyholder dies
Then

$$P[U \mid D] = \frac{P[D \mid U]P[U]}{P[D \mid S]P[S] + P[D \mid F]P[F] + P[D \mid U]P[U]}$$

$$= \frac{(0.001)(0.10)}{(0.01)(0.50) + (0.005)(0.40) + (0.001)(0.10)}$$

$$= 0.0141$$

21. Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

(i) 10% of the emergency room patients were critical;
(ii) 30% of the emergency room patients were serious;
(iii) the rest of the emergency room patients were stable;
(iv) 40% of the critical patients died;
(vi) 10% of the serious patients died; and
(vii) 1% of the stable patients died.

H.W

Given that a patient survived, calculate the probability that the patient was categorized as serious upon arrival.

(A) 0.06
(B) 0.29
(C) 0.30
(D) 0.39
(E) 0.64

20. Solution: D

Let
S = Event of a standard policy
F = Event of a preferred policy
U = Event of an ultra-preferred policy
D = Event that a policyholder dies
Then

$$P[U \mid D] = \frac{P[D \mid U]P[U]}{P[D \mid S]P[S] + P[D \mid F]P[F] + P[D \mid U]P[U]}$$

$$= \frac{(0.001)(0.10)}{(0.01)(0.50) + (0.005)(0.40) + (0.001)(0.10)}$$

$$= 0.0141$$

23. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are:

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05
Total	100%	

H.W

Given that a driver has been involved in at least one collision in the past year, calculate the probability that the driver is a young adult driver.

(A) 0.06
(B) 0.16
(C) 0.19
(D) 0.22
(E) 0.25

23. Solution: D

Let
C = Event of a collision
T = Event of a teen driver
Y = Event of a young adult driver
M = Event of a midlife driver
S = Event of a senior driver
Then,

$$P[Y \mid C] = \frac{P[C|Y]P[Y]}{P[C|T]P[T] + P[C|Y]P[Y] + P[C|M]P[M] + P[C|S]P[S]}$$

$$= \frac{(0.08)(0.16)}{(0.15)(0.08) + (0.08)(0.16) + (0.04)(0.45) + (0.05)(0.31)} = 0.22.$$