STAT 333

Nonparametric Statistics Methods

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Chapter 1

INTRODUCTION

- Some types of tests presented and their parametric counterparts.

Type of analysis	Nonparametric test	Parametric equivalent
Comparing two related samples	Wilcoxon signed ranks test and sign test	*t*-Test for dependent samples
Comparing two unrelated samples	Mann–Whitney *U*-test and Kolmogorov–Smirnov two-sample test	*t*-Test for independent samples
Comparing three or more related samples	Friedman test	Repeated measures, analysis of variance (ANOVA)
Comparing three or more unrelated samples	Kruskal–Wallis *H*-test	One-way ANOVA
Comparing categorical data	Chi square χ^2 tests and Fisher exact test	None
Comparing two rank-ordered variables	Spearman rank-order correlation	Pearson product–moment correlation
Comparing two variables when one variable is discrete dichotomous	Point-biserial correlation	Pearson product–moment correlation
Comparing two variables when one variable is continuous dichotomous	Biserial correlation	Pearson product–moment correlation
Examining a sample for randomness	Runs test	None

Parametric tests rely on six main assumptions. If these assumptions are not met, the results may be misleading, and nonparametric tests should be used instead.

1. The data are randomly drowned from a normally distributed population.
2. The populations are approximately equal variances.
3. The sample distribution is approximately normal.
4. Observations are independent of each other, expect for paired values.
5. The data are measured on an interval or ratio scale.
6. The sample size is sufficiently large.

Exercise 1:

1. Which of the following is not true of parametric statistics?

A	They are inferential tests.
B	They assume certain characteristics of population parameters.
C	They assume normality of the population.
D	They are distribution-free.

2. A collection of statistical methods that generally requires very few, if any assumptions about the population distribution is known as..............................

A	Parametric methods	B	Nonparametric methods
C	Semiparametric	D	None of these

3. A nonparametric method for determining the differences between two populations based on two matched samples where only preference data is required is the

A	Mann-Whitney-Wilcoxon test	B	Wilcoxon signed-rank test
C	Sign test	D	Kruskal-Wallis Test

4. Parametric tests are based on some restrictive assumptions about the ....

A	Random sample	B	Census	C	Sample	D	Population

5. Nonparametric tests for examining a sample for randomness ....

A	Fridman test	B	Runs test	C	T - test	D	U - test

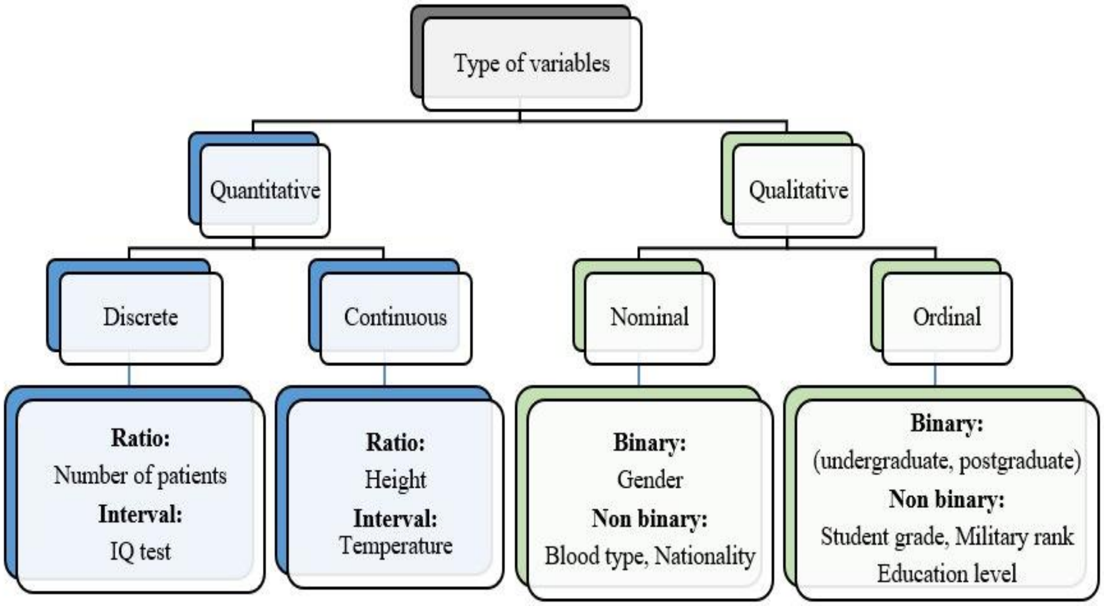
6. Point- biserial correlation is used for ....

A	Comparing two rank-ordered variables.
B	Comparing two variables when one variable is discrete dichotomous.
C	Comparing two variables when one variable is continuous dichotomous.
D	Comparing two related samples.

7. Parametric test equivalent to Kruskal–Wallis *H*-test is: ....

A	One-way ANOVA	B	Repeated measures
C	T-test for dependent samples	D	Fridman test

- **Measurement scales:**



Interval scale: Consider as pertinent information not only the relative order of the measurements as in the ordinal scale but also the size of the interval between measurements.

For example:

- Temperature.
- TQ-test.
- Time of the day (00:00 midnight, 14:00 afternoon)

Ratio scale: Not only the order and interval size are important, but also the ratio between two measurements is meaningful.

For example:

- Crop yields (إنتاج المحاصيل).
- Distances.
- Time to finish an exam.
- Weights.
- Heights.
- Income.

Exercise 2: Choose the correct measurement scale.

1. Gender (Male, Female) A) Nominal B) Ordinal C) Interval D) Ratio	8. Height in centimeters A) Ordinal B) Interval C) Ratio D) Nominal
2. Blood type (A, B, AB, O) A) Ordinal B) Nominal C) Ratio D) Interval	9. Saudi national ID number A) Interval B) Ratio C) Nominal D) Ordinal
3. Satisfaction level (Low, Medium, High) A) Nominal B) Ordinal C) Interval D) Ratio	10. Calendar year (2015, 2020, 2025) A) Ratio B) Ordinal C) Interval D) Nominal
4. Age in years A) Nominal B) Ordinal C) Interval D) Ratio	11. Marital status (Single, Married, Divorced) A) Ordinal B) Interval C) Ratio D) Nominal
5. Temperature in Celsius (°C) A) Nominal B) Ordinal C) Interval D) Ratio	12. Pain level (Mild, Moderate, Severe) A) Nominal B) Ordinal C) Interval D) Ratio
6. Exam grades (A, B, C, D) A) Ratio B) Interval C) Ordinal D) Nominal	13. Distance traveled (km) A) Ordinal B) Interval C) Nominal D) Ratio
7. Number of students in a class A) Nominal B) Ordinal C) Interval D) Ratio	14. Eye color A) Ratio B) Interval C) Ordinal D) Nominal

- **Ranking data:**

Example: Rank the following data:

Students who ate breakfast	Students who skipped breakfast
90	75
85	80
95	55
70	90

After ordering	Rank ignoring ties values		Rank accounting for ties values
55	1		1
70	2		2
75	3		3
80	4		4
85	5		5
90	6	$\frac{6+7}{2} = 6.5$	6.5
90	7		6.5
95	8		8

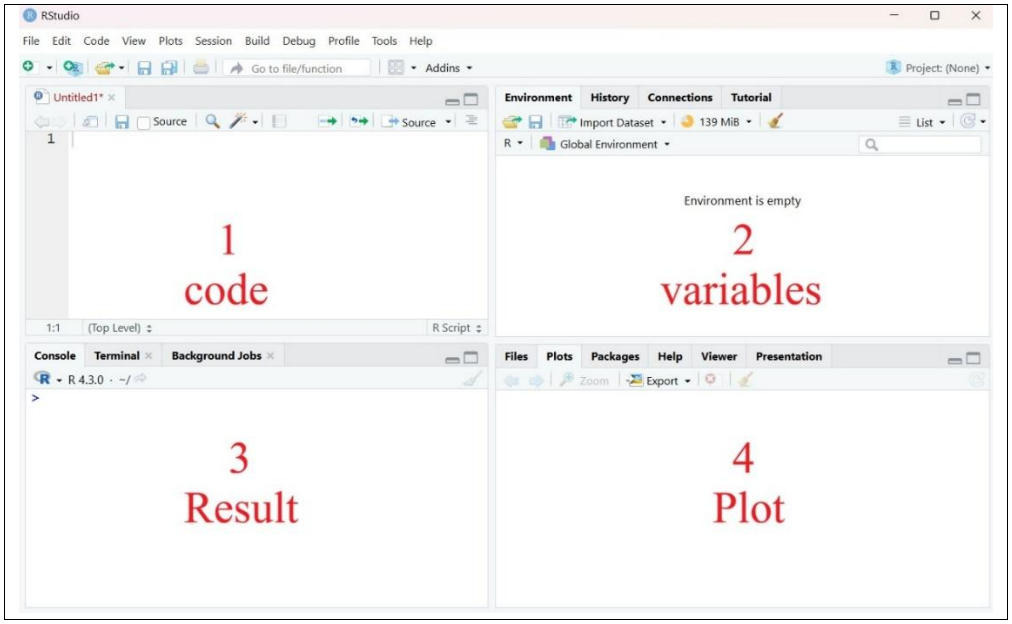
Example: the following data represent quiz score om math.

100	60	70	90	80	100	80	20	100	50

Rank the quiz score.

Quiz score	After ordering	Rank ignoring ties values		Rank accounting for ties values
100	20	1		1
60	50	2		2
70	60	3		3
90	70	4		4
80	80	5	$\frac{5+6}{2} = 5.5$	5.5
100	80	6		5.5
80	90	7		7
20	100	8	$\frac{8+9+10}{3} = 9$	9
100	100	9		9
50	100	10		9

Using R studio:



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R - code

```
> x=c(100,60,70,90,80,100,80,20,100,50)
> rank(x)
```

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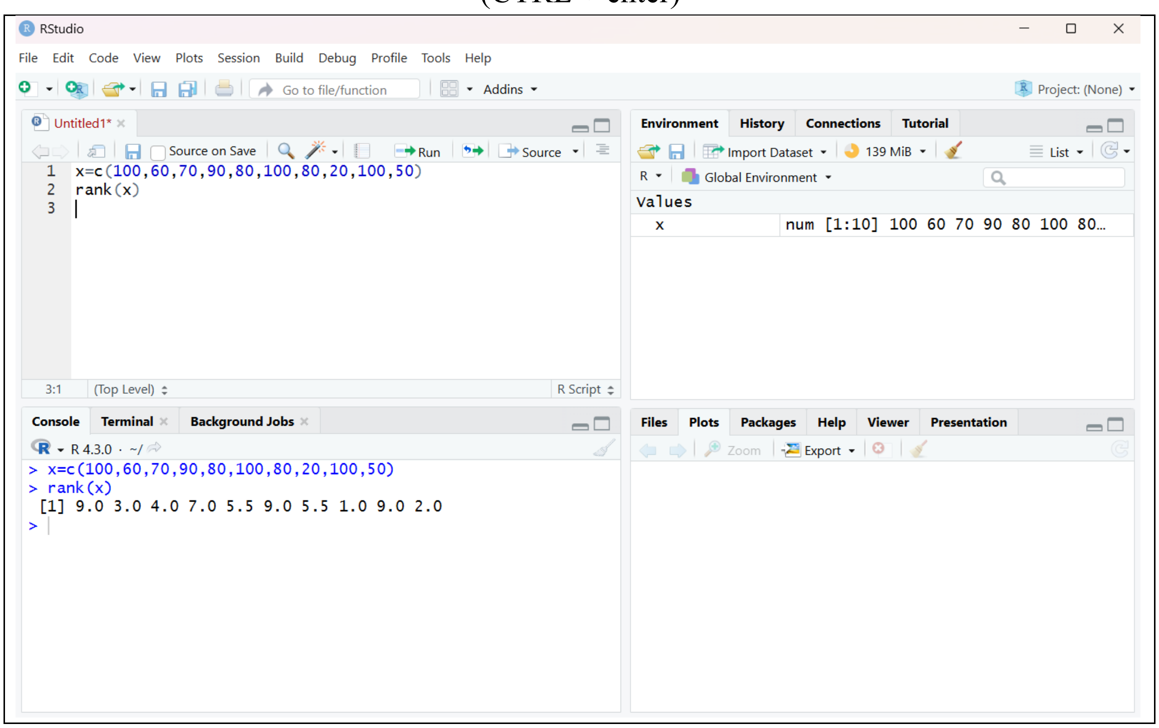


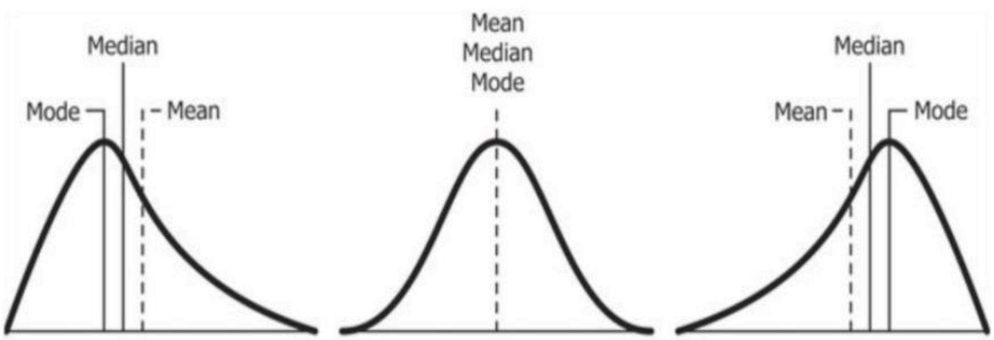
Chapter 2

TESTING DATA FOR NORMALITY

- **Skewness and Kurtosis:**

	Skewness:	Kurtosis:
Formula	$S_k = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_i - \bar{x}}{s} \right)^3$	$K = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_i - \bar{x}}{s} \right)^4 \right] - \frac{3(n-1)^2}{(n-2)(n-3)}$ $s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$
Standard error (SE)	$SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$	$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$
Z - score	$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$ If $Z_{S_k} \in (-1.96, 1.96)$ pass the normality assumption for $\alpha = 0.05$	$Z_k = \frac{K - 0}{SE_K}$ If $Z_k \in (-1.96, 1.96)$ pass the normality assumption for $\alpha = 0.05$

Skewness:

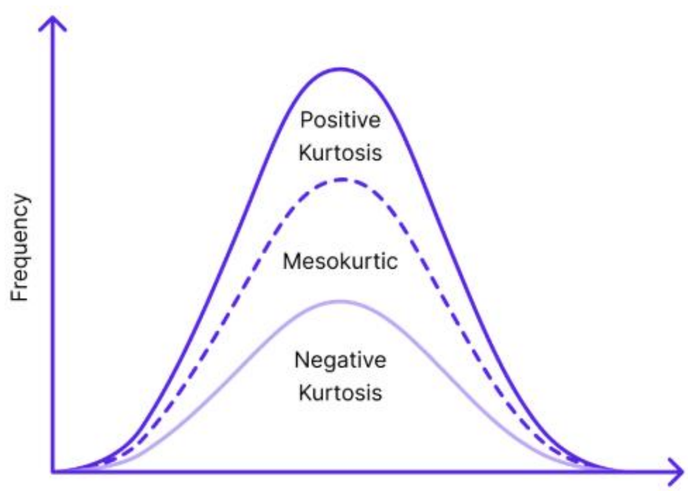


Positive skew
Right skew

No skewness

Negative skew
Left skew

Kurtosis:



Positive kurtosis

Kurtosis = 0

Negative kurtosis

Exercise 1:

The following data represent a samples of week 1 quiz score.
Calculate the skewness and kurtosis.

90	72	90
64	95	89
74	88	100
77	57	35
100	64	95
65	80	84
90	100	76

	data	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$\left(\frac{x_i-\bar{x}}{s}\right)^3$	$\left(\frac{x_i-\bar{x}}{s}\right)^4$
1	90	9.761905	95.29478	0.202561	0.118962
2	64	-16.2381	263.6757	-0.9323	0.91077
3	74	-6.2381	38.91383	-0.05286	0.019837
⋮	⋮	⋮	⋮	⋮	⋮
19	95	14.7619	217.9138	0.700452	0.622068
20	84	3.761905	14.15193	0.011592	0.002624
21	76	-4.2381	17.96145	-0.01658	0.004226
Total	1685		5525.81	-18.4149	69.01972

$$\bar{x} = \frac{\sum x}{n} = \frac{1685}{21} = 80.2381$$

$$S = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}} = \sqrt{\frac{5525.80}{20}} = 6.62199$$

$$\sum\left(\frac{x_i-\bar{x}}{s}\right)^3 = -18.4149$$

$$\sum\left(\frac{x_i-\bar{x}}{s}\right)^4 = 69.01972$$

$$S_k = \frac{n}{(n-1)(n-2)}\sum\left(\frac{x_i-\bar{x}}{s}\right)^3$$

$$= \frac{21}{(21-1)(21-2)} \times -18.4149 = -1.01766$$

$$SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}} = \sqrt{\frac{6\times 21(21-1)}{(21-2)(21+1)(21+3)}} = 0.50119$$

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}}$$

$$= \frac{-1.01766-0}{0.50119} = -2.032$$

$$Z_{S_k} \notin (-1.96, 1.96)$$

$$K = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left(\frac{x_i-\bar{x}}{s}\right)^4\right] - \frac{3(n-1)^2}{(n-2)(n-3)}$$

$$= \left[\frac{21(21+1)}{(21-1)(21-2)(21-3)} \times 69.02\right] - \frac{3(21-1)^2}{(21-2)(21-3)} = 1.153$$

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$$

$$= \sqrt{\frac{24\times 21(21-1)^2}{(21-2)(21-3)(21+5)(21+3)}} = 0.971941$$

$$Z_k = \frac{K - 0}{SE_K}$$

$$= \frac{1.153-0}{0.971941} = 1.186$$

$$Z_k \in (-1.96, 1.96)$$

R – code

```
x=c(90,72,90,64,95,89,74,88,100,77,57,35,100,64,95,65,80,84,90,100,76)
m=mean(x)
s=sd(x)
n=length(x)

i3=sum(((x-m)/s)^3)
i4=sum(((x-m)/s)^4)

sk=n/((n-1)*(n-2))*i3
SEs=sqrt(6*n*(n-1)/(n-2)/(n+1)/(n+3))

kur=(n*(n+1)/(n-1)/(n-2)/(n-3)*i4)-(3*(n-1)^2/(n-2)/(n-3))
SEk=sqrt((24*n*(n-1)^2)/(n-2)/(n-3)/(n+5)/(n+3))
```

`x=c(90,72,90,64,95,89,74,88,100,77,57,35,100,64,95,65,80,84,90,100,76)`

`m=mean(x)`

`s=sd(x)` ← $S = \sqrt{\frac{\sum(x_i-\bar{x})^2}{n-1}}$

`n=length(x)`

`i3=sum(((x-m)/s)^3)` ← $\sum\left(\frac{x_i-\bar{x}}{s}\right)^3$

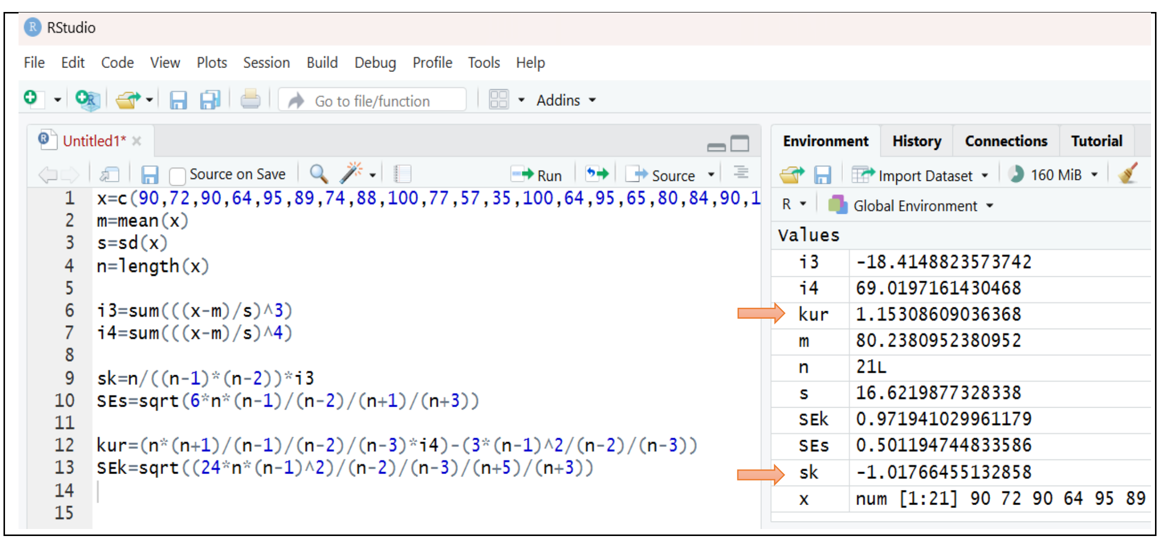
`i4=sum(((x-m)/s)^4)` ← $\sum\left(\frac{x_i-\bar{x}}{s}\right)^4$

`sk=n/((n-1)*(n-2))*i3` ← $S_k = \frac{n}{(n-1)(n-2)}\sum\left(\frac{x_i-\bar{x}}{s}\right)^3$

`SEs=sqrt(6*n*(n-1)/(n-2)/(n+1)/(n+3))` ← $SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}}$

`kur=(n*(n+1)/(n-1)/(n-2)/(n-3)*i4)-(3*(n-1)^2/(n-2)/(n-3))` ← $K = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)}\sum\left(\frac{x_i-\bar{x}}{s}\right)^4\right] - \frac{3(n-1)^2}{(n-2)(n-3)}$

`SEk=sqrt((24*n*(n-1)^2)/(n-2)/(n-3)/(n+5)/(n+3))` ← $SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}}$



Exercise 2:

A department store has decided to evaluate customer satisfaction. The store provides customers with a survey to rate employee friendliness. The survey uses a scale of 1–10. The survey results are:

7	3	3	6
4	4	4	5
5	5	8	9
5	5	5	7
6	8	6	2

Calculate the skewness and kurtosis.

	data	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$\left(\frac{x_i-\bar{x}}{s}\right)^3$	$\left(\frac{x_i-\bar{x}}{s}\right)^4$
1	7				
2	4				
3	5				
⋮	⋮	⋮	⋮	⋮	⋮
18	9				
19	7				
20	2				
Total	107		62.55	4.09576	45.0707

$\bar{x} =$ ....

$s =$ ....

$\sum \left(\frac{x_i-\bar{x}}{s}\right)^3 =$ ....

$\sum \left(\frac{x_i-\bar{x}}{s}\right)^4 =$ ....

$$S_k = \frac{n}{(n-1)(n-2)} \sum \left(\frac{x_i-\bar{x}}{s}\right)^3 = \text{....................................}$$

$$SE_{S_k} = \sqrt{\frac{6n(n-1)}{(n-2)(n+1)(n+3)}} = \text{....................................}$$

$$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}} =$$

$$K = \left[\frac{n(n+1)}{(n-1)(n-2)(n-3)} \sum \left(\frac{x_i-\bar{x}}{s}\right)^4\right] - \frac{3(n-1)^2}{(n-2)(n-3)} =$$

....

$$SE_K = \sqrt{\frac{24n(n-1)^2}{(n-2)(n-3)(n+5)(n+3)}} = \text{....................................}$$

$$Z_k = \frac{K - 0}{SE_K} =$$

R – code

```
x=c(7,4,5,5,6,3,4,5,5,8,3,4,8,5,6,6,5,9,7,2)
```

Exercise 3:

Calculate the skewness and kurtosis for the following data.

	data	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$\left(\frac{x_i-\bar{x}}{S}\right)^3$	$\left(\frac{x_i-\bar{x}}{S}\right)^4$
1	25				
2	30				
3	12				
4	18				
5	20				
Total					

$\bar{x} =$ ....

$S =$ ....

$\sum \left(\frac{x_i-\bar{x}}{S}\right)^3 =$ ....

$\sum \left(\frac{x_i-\bar{x}}{S}\right)^4 =$ ....

$S_k =$ ....
....
....

$Z_{S_k} = \frac{S_k - 0}{SE_{S_k}} =$

$SE_{S_k} =$ ....
....
....

$K =$ ....
....
....

$Z_k = \frac{K-0}{SE_K} =$

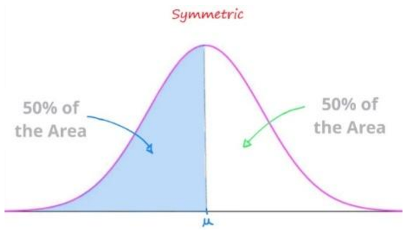
$SE_K =$ ....
....
....

R – code

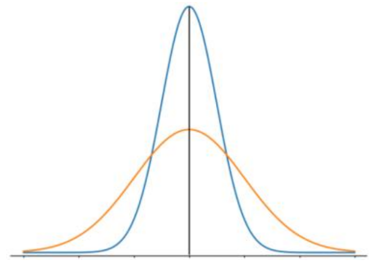
```
x=c(25,30,12,18,20)
```

The Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < X < \infty$$



Mean = Median = Mode



μ

$\sigma > \sigma$

Normal distribution	$X \sim N(\mu, \sigma^2)$	
Standard normal	$Z \sim N(0, 1)$	$Z = \frac{X-\mu}{\sigma} \sim N(0, 1)$

Testing Data for Normality: Kolmogorov–Smirnov

H_0: The data approximately follow normal distribution.
H_A: The data do not follow normal distribution.

Question: Suppose that we have a random sample of size n. For α = 0.05, if the Z-score of the skewness of the sample is (−2.032), then the sample has Therefore, either the sample must be modified and rechecked or you must use a nonparametric statistical test.

A	Pass the normality assumption for kurtosis
B	Pass the normality assumption for skewness.
C	Failed the normality assumption for kurtosis
D	Failed the normality assumption for skewness.

Exercise 4:

For the following data:

8.1	8.2	8.2	8.7	8.7	8.8	8.8	8.9	8.9	8.9
9.2	9.2	9.2	9.3	9.3	9.3	9.4	9.4	9.4	9.4
9.5	9.5	9.5	9.5	9.6	9.6	9.6	9.7	9.7	9.9

(a) Find: Skewness, Standard error of the skewness, Kurtosis, Standard error of the kurtosis

Using R

```
y=c(8.1,9.2,9.5,8.2,9.2,9.5,8.2,9.2,9.5,8.7,9.3,9.5 ,8.7,9.3,9.6,8.8,9.3,9.6,8.8,9.4,9.6,8.9,9.4,9.7
        ,8.9,9.4,9.7,8.9,9.4,9.9)
m=mean(y)
s=sd(y)
n=length(y)
i3=sum(((y-m)/s)^3)
i4=sum(((y-m)/s)^4)
sk=n/((n-1)*(n-2))*i3
SEs=sqrt(6*n*(n-1)/(n-2)/(n+1)/(n+3))
kur=(n*(n+1)/(n-1)/(n-2)/(n-3)*i4)-(3*(n-1)^2/(n-2)/(n-3))
SEk=sqrt((24*n*(n-1)^2)/(n-2)/(n-3)/(n+5)/(n+3))

> sk
[1] -0.9043788
> SEs
[1] 0.4268924
> kur
[1] 0.1877582
> SEk
[1] 0.8327456
```

(b) Using a Kolmogorov–Smirnov one-sample test, is the data follow normal distribution

Using R

```
ks.test(y, "pnorm", mean = mean(y), sd = sd(y))
            Asymptotic one-sample Kolmogorov-Smirnov test
data:  y
D = 0.18377, p-value = 0.263 ⇐
alternative hypothesis: two-sided
```

H_0: The data approximately follow normal distribution.
H_A: The data do not follow normal distribution.

$P - value = 0.263 > 0.05$,
We accept H_0, The data approximately follow normal distribution

Exercise 5:

A department store has decided to evaluate customer satisfaction. The store provides customers with a survey to rate employee friendliness. The survey uses a scale of 1–10. The survey results are:

7	3	3	6
4	4	4	5
5	5	8	9
5	5	5	7
6	8	6	2

Use the Kolmogorov–Smirnov one-sample test to decide if survey results approximately matching a normal distribution.

Using R

```
x=c(7,4,5,5,6,3,4,5,5,8,3,4,8,5,6,6,5,9,7,2)
ks.test(y, "pnorm", mean = mean(x), sd = sd(x))
        Asymptotic one-sample Kolmogorov-Smirnov test

data:  x
D = 0.17648, p-value = 0.5617
alternative hypothesis: two-sided
```

H_0: The data approximately follow normal distribution.
H_A: The data do not follow normal distribution.

$P - value = 0.5617 > 0.05$,
We accept H_0, The data approximately follow normal distribution

Chapter 3

THE WILCOXON SIGNED RANK AND THE SIGN TEST

The Wilcoxon signed test and sing test:
are for comparing two samples that are paired or related.

$$H_0: \mu_D = 0$$
$$H_A: \mu_D \neq 0$$

Parametric equivalent: *t*-Test for dependent samples (Paired test)

The sum of the ranks with positive differences	$\sum R_+$
The sum of the ranks with negative differences	$\sum R_-$
Test statistics	$T = \min(\sum R_+ , \sum R_-)$
The mean	$\bar{x}_T = \frac{n(n+1)}{4}$
Standard deviation	$S_T = \sqrt{\frac{n(n+1)(2n+1)}{24}}$
z-score	$Z = \frac{T-\bar{x}_T}{S_T}$
Effect size (ES): Determine the degree of association between the groups	$ES = \frac{\lvert Z \rvert}{\sqrt{n}}$; $0 < ES < 1$ ES closer to 0.10 small ES closer to 0.30 medium ES closer to 0.50 large

Exercise 1:

The counseling staff (فريق الإرشاد) in a school started a new program this year to reduce bullying (التنمر) in elementary schools. To see if the program worked, they compared the percentage of successful interventions (التدخلات) before the program (last year) with the percentage after the program (this year). The data were reported by 12 elementary school counselors.

(a) Is their difference in percentage of successful interventions in the two years.

Participants	Successful intervention	
	last year	this year
1	31	31
2	14	14
3	53	50
4	18	30
5	21	28
6	44	48
7	12	35
8	36	32
9	22	23
10	29	34
11	17	27
12	40	42

Participants	Successful intervention		Different	Rank	Sign
	last year	this year		without zero	
1	31	31	0	-	
2	14	14	0	-	
3	53	50	-3	3	-
4	18	30	12	9	+
5	21	28	7	7	+
6	44	48	4	4.5	+
7	12	35	23	10	+
8	36	32	-4	4.5	-
9	22	23	1	1	+
10	29	34	5	6	+
11	17	27	10	8	+
12	40	42	2	2	+

1. Using Wilcoxon test:

$H_0: \mu_D = 0$ (There is no difference in the percentages)

$H_A: \mu_D \neq 0$ (There is a difference in the percentages)

$\sum R_- = 3 + 4.5 = 7.5$

$\sum R_+ = 9 + 7 + 4.5 + 10 + 1 + 6 + 8 + 2 = 47.5$

$T = \min(\sum R_+, \sum R_-)$

$= \min(47.5, 7.5) = 7.5$

We use (Table B.3 page 244)

TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics *T*.

n	$\alpha_{\text{two-tailed}} \leq 0.10$, $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$, $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$, $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$, $\alpha_{\text{one-tailed}} \leq 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15

$T = 7.5 < 8$ then we reject H_0 (There is a difference in the percentages)

Using R

```
x=c(31,14,53,18,21,44,12,36,22,29,17,40)
y=c(31,14,50,30,28,48,35,32,23,34,27,42)
wilcox.test(x,y, paired = TRUE, alternative = "two.sided")
        Wilcoxon signed rank test with continuity correction

data:  x and y
V = 7.5, p-value = 0.04671  <----------  Reject H0
alternative hypothesis: true location shift is not equal to 0
```

2. Using the sign test:

$H_0: P = 0.5$ (There is no difference in the percentages)
$H_A: P \neq 0.5$ (There is a difference in the percentages)

If $n < 25$ we calculate p using $P(X) = \frac{n!}{(n-X)!X!} P^X (1-P)^{n-X}$

If $n \geq 25$ we calculate p using $Z_c = \frac{max(n_p, n_n) - 0.5(n_p + n_n) - 0.5}{0.5\sqrt{n_p + n_n}}$

$n = n_p + n_n$, n_p = number of the positive differences
n_n = number of the negative differences

Participants	Successful intervention		Sign of the difference
	last year	this year	
1	31	31	
2	14	14	
3	53	50	-
4	18	30	+
5	21	28	+
6	44	48	+
7	12	35	+
8	36	32	-
9	22	23	+
10	29	34	+
11	17	27	+
12	40	42	+

$$n = n_p + n_n = 10 < 25 \text{ , then: } P(X) = \frac{n!}{(n-X)!\,X!} P^X (1-P)^{n-X}$$

$P(X = 0) = \frac{10!}{(10-0)!0!} 0.5^0 (1-0.5)^{10-0} = 0.0010$

$P(X = 1) = \frac{10!}{(10-1)!1!} 0.5^1 (1-0.5)^{10-1} = 0.0098$

$P(X = 2) = \frac{10!}{(10-2)!2!} 0.5^2 (1-0.5)^{10-2} = 0.0439$

P-values for each tail,
we sum the probabilities for each tail until we find a probability $\geq \frac{\alpha}{2} = 0.025$

X	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.001	0.0098	0.0439	0.1173	0.2051	0.1172	0.2051	0.1172	0.0439	0.0098	0.001

0.0547 (X = 0 to 2) median (X = 5) 0.0547 (X = 8 to 10)

$\text{P} - \text{value} = 0.0547 + 0.0547$
$\text{P} - \text{value} = 0.1094 > 0.05$

We fail to reject H_0 (There is no difference in the percentages)

Using R

```
x=c(31,14,53,18,21,44,12,36,22,29,17,40)
y=c(31,14,50,30,28,48,35,32,23,34,27,42)
d=y-x
d_nz=d[d!=0]
np=sum(d_nz>0)
nn=sum(d_nz<0)
n=length(d_nz)
#n=np+nn
binom.test(np,n,p=0.5,alternative = "two.sided")

        Exact binomial test
data:  np and n
number of successes = 8, number of trials = 10, p-value = 0.1094  <------  Accept H0
alternative hypothesis: true probability of success is not equal to 0.5
95 percent confidence interval:
 0.4439045 0.9747893
sample estimates:
probability of success
                   0.8
#or
# you have to install BSDA package first using install.packages("BSDA")
library(BSDA)
SIGN.test(x,y,md=0,alternative = "two.sided")

Dependent-samples Sign-Test
data:  x and y
S = 2, p-value = 0.1094  <------  Accept H0
alternative hypothesis: true median difference is not equal to 0
95 percent confidence interval:
 -9.680909  0.000000
sample estimates:
median of x-y
           -3
Achieved and Interpolated Confidence Intervals:
                  Conf.Level   L.E.pt
Lower Achieved CI     0.8540  -7.0000
Interpolated CI       0.9500  -9.6809
Upper Achieved CI     0.9614 -10.0000
                  U.E.pt
Lower Achieved CI      0
Interpolated CI        0
Upper Achieved CI      0
```

(b) Construct a 95% median confidence interval based on the Wilcoxon signed rank test

	-3	12	7	4	23	-4	1	5	10	2
-3	-3	4.5	2	0.5	10	-3.5	-1	1	3.5	-0.5
12		12	9.5	8	17.5	4	6.5	8.5	11	7
7			7	5.5	15	1.5	4	6	8.5	4.5
4				4	13.5	0	2.5	4.5	7	3
23					23	9.5	12	14	16.5	12.5
-4						-4	-1.5	0.5	3	-1
1							1	14	5.5	1.5
5								5	7.5	3.5
10									10	6
2										2

$$U_{ij} = \frac{D_i + D_j}{2}$$

$$1 \le i \le j \le n$$

$$\mathrm{K} = \mathrm{T} + 1 = 8 + 1 = 9$$

1	2	3	4	5	6	7	8	9	…
-4	-3.5	-3	-1.5	-1	-1	-0.5	0	0.5	

⋮

…	47	48	49	50	51	52	53	54	55
	12	12	12.5	13.5	14	15	16.5	17.5	23

95% confidence that the difference number of activities lies between (0.5 , 12)

(c) Determine the degree of association.

$$\bar{x}_T = \frac{n(n+1)}{4} = \frac{10(10+1)}{4} = 27.5$$

$$S_T = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{10(10+1)(20+1)}{24}} = 9.81$$

$$Z = \frac{T - \bar{x}_T}{S_T} = \frac{7.5 - 27.5}{9.81} = -2.0387$$

Effect size:

$ES = \frac{|Z|}{\sqrt{n}} = \frac{|-2.0387|}{\sqrt{10}} = 0.64$ (Which indicates a strong measure of association.)

Exercise 2:

A school is trying to get more students to participate in activities that will make learning more desirable. Table below shows the number of activities that each of the 10 students in one class participated in last year before a new activity program was implemented and this year after it was implemented (تم تطبيقه). Construct a 95% median confidence interval based on the Wilcoxon signed rank test to determine whether the new activity program had a significant positive effect on the student participation.

Participants	Activities	
	last year	this year
1	18	20
2	22	28
3	10	18
4	25	23
5	16	20
6	14	21
7	21	17
8	13	18
9	28	22
10	12	21

Activities		Difference
last year X_j	this year X_i	$D_i = X_i - X_j$
18	20	2
22	28	6
10	18	8
25	23	-2
16	20	4
14	21	7
21	17	-4
13	18	5
28	22	-6
12	21	9

TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics *T*.

n	$\alpha_{\text{two-tailed}} \le 0.10$ $\alpha_{\text{one-tailed}} \le 0.05$	$\alpha_{\text{two-tailed}} \le 0.05$ $\alpha_{\text{one-tailed}} \le 0.025$	$\alpha_{\text{two-tailed}} \le 0.02$ $\alpha_{\text{one-tailed}} \le 0.01$	$\alpha_{\text{two-tailed}} \le 0.01$ $\alpha_{\text{one-tailed}} \le 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15

$n = 10$
$\alpha = 0.05$
$T = 8$

	2	6	8	-2	4	7	-4	5	-6	9
2	2	4	5	0	3	4.5	-1	3.5	-2	5.5
6		6	7	2	5	6.5	1	5.5	0	7.5
8			8	3	6	7.5	2	6.5	1	8.5
-2				-2	1	2.5	-3	1.5	-4	3.5
4					4	5.5	0	4.5	-1	6.5
7						7	1.5	6	0.5	8
-4							-4	0.5	-5	2.5
5								5	-0.5	7
-6									-6	4.5
9										9

$$U_{ij} = \frac{D_i + D_j}{2}$$

$$1 \le i \le j \le n$$

$$\text{K} = \text{T} + 1 = 8 + 1 = 9$$

1	2	3	4	5	6	7	8	9	…
-6	-5	-4	-4	-3	-2	-2	-1	-1	
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
…	47	48	49	50	51	52	53	54	55
	7	7	7	7.5	7.5	8	8	8.8	9

95% confidence that the difference number of activities lies between $(-1, 7)$

Exercise 3:

Twenty participants in an exercise program were measured on the number of sit-ups they could do before other physical exercise (first count) and the number they could do after they had done at least 45 min of other physical exercise (second count). Table 4 shows the results for 20 participants obtained during two separate physical exercise sessions. Determine the ES for a calculated z-score.

Participant	First count	Second count
1	18	28
2	19	18
3	20	28
4	29	20
5	15	30
6	22	25
7	21	28
8	30	18
9	22	27
10	11	30
11	20	24
12	21	27
13	21	10
14	20	40
15	18	20
16	27	14
17	24	29
18	13	30
19	10	24
20	10	36

Participant	1^{st} *count*	2^{nd} *count*	D	$\lvert D \rvert$	Sing	Rank $\lvert D \rvert$
1	18	28	10	10	+	11
2	19	18	-1	1	-	1
3	20	28	8	8	+	9
4	29	20	-9	9	-	10
5	15	30	15	15	+	16
6	22	25	3	3	+	3
7	21	28	7	7	+	8
8	30	18	-12	12	-	13
9	22	27	5	5	+	5.5
10	11	30	19	19	+	18
11	20	24	4	4	+	4
12	21	27	6	6	+	7
13	21	10	-11	11	-	12
14	20	40	20	20	+	19
15	18	20	2	2	+	2
16	27	14	-13	13	-	14
17	24	29	5	5	+	5.5
18	13	30	17	17	+	17
19	10	24	14	14	+	15
20	10	36	26	26	+	20

$$\sum R_- = 1 + 10 + 13 + 12 + 14 = 50$$
$$\sum R_+ = 11 + 9 + \cdots + 15 + 20 = 160$$

$$T = \min(\sum R_+, \sum R_-) = \min(160 \; , \; 50) = 50$$

$$\bar{x}_T = \frac{n(n+1)}{4} = \frac{20(20+1)}{4} = 105$$

$$S_T = \sqrt{\frac{n(n+1)(2n+1)}{24}} = \sqrt{\frac{20(20+1)(40+1)}{24}} = 26.786$$

$$Z = \frac{T - \bar{x}_T}{S_T} = \frac{50-105}{26.786} = -2.0533$$

Effect size:

$$ES = \frac{|Z|}{\sqrt{n}} = \frac{|-2.0533|}{\sqrt{20}} = 0.46$$ (Which indicates a strong measure of association.)

Exercise 4:

Consider a clinical investigation to assess the effectiveness of a new drug designed to reduce repetitive behaviors in children affected with autism. The data are shown below.

Child	1	2	3	4	5	6	7	8	9	10
Before Treatment	30	56	48	47	43	45	36	44	44	40
After 2 Weeks of Treatment	39	46	37	44	32	39	41	40	38	46

Use a one-tailed Wilcoxon signed rank test and a one-tailed sign test to assess the effectiveness of the drug (is there differences in behavior before and after taking the drug?). Use α = 0.05.

Child	Repetitive behaviors		Different	Rank	sign
	Before treatment	After treatment		without zero	
1	30	39	9	7	+
2	56	46	-10	8	-
3	48	37	-11	9.5	-
4	47	44	-3	1	-
5	43	32	-11	9.5	-
6	45	39	-6	5	-
7	36	41	5	3	+
8	44	40	-4	2	-
9	44	38	-6	5	-
10	40	46	6	5	+

1. Using Wilcoxon test:

   $H_0: \mu_D \geq 0$ (The effectiveness of a new drug does not reduce repetitive behaviors)

   $H_A: \mu_D < 0$ (The effectiveness of a new drug reduced repetitive behaviors)

$\sum R_- = 8 + 9.5 + 1 + 9.5 + 5 + 2 + 5 = 40$

$\sum R_+ = 7 + 3 + 5 = 15$

$T = \min(\sum R_+, \sum R_-)$

$= \min(40, 15) = 15$

TABLE B.3 Critical Values for the Wilcoxon Signed Rank Test Statistics *T*.

n	$\alpha_{\text{two-tailed}} \leq 0.10$ $\alpha_{\text{one-tailed}} \leq 0.05$	$\alpha_{\text{two-tailed}} \leq 0.05$ $\alpha_{\text{one-tailed}} \leq 0.025$	$\alpha_{\text{two-tailed}} \leq 0.02$ $\alpha_{\text{one-tailed}} \leq 0.01$	$\alpha_{\text{two-tailed}} \leq 0.01$ $\alpha_{\text{one-tailed}} \leq 0.005$
5	0			
6	2	0		
7	3	2	0	
8	5	3	1	0
9	8	5	3	1
10	10	8	5	3
11	13	10	7	5
12	17	13	9	7
13	21	17	12	9
14	25	21	15	12
15	30	25	19	15

$T = 15 > 10$ then we fail to reject H_0

(The effectiveness of a new drug does not reduce repetitive behaviors)

```
before=c(30,56,48,47,43,45,36,44,44,40)
after  =c(39,46,37,44,32,39,41,40,38,46)
wilcox.test(before,after, paired = TRUE, alternative = "greater")
            Wilcoxon signed rank test with continuity correction
data:  before and after
V = 40, p-value = 0.1099 <——— Accept H0
alternative hypothesis: true location shift is greater than 0
```

2. Using the sign test:

$H_0: P \geq 0.5$ (The effectiveness of a new drug does not reduce repetitive behaviors)
$H_A: P < 0.5$ (The effectiveness of a new drug reduced repetitive behaviors)

$$n = n_p + n_n = 10 < 25\ ,\ \text{then: } P(X) = \frac{n!}{(n-X)!\,X!}P^X(1-P)^{n-X}$$

$P(X=0) = \frac{10!}{(10-0)!0!}0.5^0(1-0.5)^{10-0} = 0.0010$

$P(X=1) = \frac{10!}{(10-1)!1!}0.5^1(1-0.5)^{10-1} = 0.0098$

$P(X=2) = \frac{10!}{(10-2)!2!}0.5^2(1-0.5)^{10-2} = 0.0439$

P-values for left tail,

$\text{P} - \text{value} = \text{P}(X \leq 3)$

X	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.001	0.0098	0.0439	0.1173	0.2051	0.1172	0.2051	0.1172	0.0439	0.0098	0.001

0.1719 median

$\text{P} - \text{value} = 0.1719 > 0.05$ We fail to reject H_0

(The effectiveness of a new drug does not reduce repetitive behaviors)

Using R

```
before=c(30,56,48,47,43,45,36,44,44,40)
after  =c(39,46,37,44,32,39,41,40,38,46)
d = before-after
d_nz=d[d!=0]
np=sum(d_nz>0)
nn=sum(d_nz<0)
#n=np+nn
n=length(d_nz)
binom.test(np,n,p=0.5,alternative = "greater")

#d = after-before
#binom.test(np,n,p=0.5,alternative = "less")

            Exact binomial test
data:  np and n
number of successes = 7, number of trials = 10, p-value = 0.1719  <----- Accept H0
alternative hypothesis: true probability of success is greater than 0.5
95 percent confidence interval:
 0.3933758 1.0000000
sample estimates:
probability of success
                 0.7
#or
library(BSDA)
SIGN.test(before_treatment,after_2_weeks,md=0,alternative = "two.sided")
```

Exercise 5:

Consider the following two independent samples:

Sample A	15	17	18	-	-	-	-
Sample B	14	16	19	19	20	22	23

The value of the test statistics for a right-tail Wilcoxon rank test is:

A	3	B	7	C	11	D	44

In testing for the difference between two populations, it is possible to use …………

A	The Wilcoxon Rank-Sum test	B	The Sign test
C	Either of (A) or (B)	D	None of these

Chapter 4

THE MANN−WHITNEY U-TEST AND THE KOLMOGOROV−SMIRNOV TWO-SAMPLE TEST

The Mann – Whitney U – test and The Kolmogorov – Smirnov two sample test:
For comparing two samples that are independent, or not related.
H_0: There is no significant different between the two methods.
H_A: There is a significant different between the two methods.
Parametric equivalent: *t*-Test for independent samples.

The sum of the ranks for group i	$\sum R_i$
The number of values from the i^{th} sample.	n_i
Test statistics is the smallest of	$U_i = n_1 n_2 + \frac{n_i(n_i+1)}{2} - \sum R_i$
The mean	$\overline{x}_U = \frac{n_1 n_2}{2}$
Standard deviation	$S_U = \sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}$
z-score	$Z = \frac{U_i - \overline{x}_U}{S_U}$
Effect size (ES): Determine the degree of association between the groups	$ES = \frac{\lvert Z \rvert}{\sqrt{n}}$; $0 < ES < 1$ ES closer to 0.10 small ES closer to 0.30 medium ES closer to 0.50 large

Critical value:
If $n \leq 20$, Use Table B.4 (page 245).
If $n > 20$, Compute a *z*-score and use a table with the normal distribution.

Exercise 1:

The following data were collected from a study comparing two methods being used to teach reading recovery in the 4th grade. Method 1 was a pull-out program in which the children were taken out of the classroom for 30 min a day, 4 days a week. Method 2 was a small group program in which children were taught in groups of four or five for 45 min a day in the classroom, 4 days a week. The students were tested using a reading comprehension test after 4 weeks of the program. The test results are shown in the table below.

Method 1	48	40	39	50	41	38	53
Method 2	14	18	20	10	12	102	17

1. Using Mann – Whitney U – test:

H_0: There is no significant different between the two methods.
H_A: There is a significant different between the two methods.

Rank	Score	Sample
1	10	Method 2
2	12	Method 2
3	14	Method 2
4	17	Method 2
5	18	Method 2
6	20	Method 2
7	38	Method 1
8	39	Method 1
9	40	Method 1
10	41	Method 1
11	48	Method 1
12	50	Method 1
13	53	Method 1
14	102	Method 2

For method 1:

$n_1 = 7$

$\sum R_1 = 7 + 8 + 9 + 10 + 11 + 12 + 13 = 70$

$$U_1 = n_1 n_2 + \frac{n_1(n_1+1)}{2} - \sum R_1$$

$$= 7 \times 7 + \frac{7(7+1)}{2} - 70 = 7$$

For method 2:

$n_2 = 7$

$\sum R_2 = 1 + 2 + 3 + 4 + 5 + 14 = 35$

$$U_2 = n_1 n_2 + \frac{n_2(n_2+1)}{2} - \sum R_2$$

$$= 7 \times 7 + \frac{7(7+1)}{2} - 35 = 42$$

$$U = \min(U_1, U_2) = \min(7, 42) = 7$$

To find the critical value for rejection, we use Table B.4 (p.245)

α	m \ n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
0.025	1																				
	2																				
	3																				
	4				0																
	5			0	1	2															
	6			1	2	3	5														
	7			1	3	5	6	8													
	8		0	2	4	6	8	10	13												
	9		0	2	4	7	10	12	15	17											
	10		0	3	5	8	11	14	17	20	23										
	11		0	3	6	9	13	16	19	23	26	30									
	12		1	4	7	11	14	18	22	26	29	33	37								
	13		1	4	8	12	16	20	24	28	33	37	41	45							
	14		1	5	9	13	17	22	26	31	36	40	45	50	55						
	15		1	5	10	14	19	24	29	34	39	44	49	54	59	64					
	16		1	6	11	15	21	26	31	37	42	47	53	59	64	70	75				
	17		2	6	11	17	22	28	34	39	45	51	57	63	69	75	81	87			
	18		2	7	12	18	24	30	36	42	48	55	61	67	74	80	86	93	99		
	19		2	7	13	19	25	32	38	45	52	58	65	72	78	85	92	99	106	113	
	20		2	8	14	20	27	34	41	48	55	62	69	76	83	90	98	105	112	119	127

$U = 7 < 8$, Reject H_0 (There is a significant different between the two methods).

Using R

```
method1 <- c(48, 40, 39, 50, 41, 38, 53)
method2 <- c(14, 18, 20, 10, 12, 102, 17)
wilcox.test(method1,method2, alternative = "two.sided")
                    Wilcoxon rank sum exact test

data:  method1 and method2
W = 42, p-value = 0.02622   <———— Reject H0
alternative hypothesis: true location shift is not equal to 0
```

Finding 95% confidence interval for the difference between two location parameters

2nd Sample (Y_j) \ 1st Sample (X_i)	38	39	40	41	48	50	53
10	28	29	30	31	38	40	43
12	26	27	28	29	36	38	41
14	24	25	26	27	34	36	39
17	21	22	23	24	31	33	36
18	20	21	22	23	30	32	35
20	18	19	20	21	28	30	33
102	-64	-63	-62	-61	-54	-52	-49

$D_{ij} = X_i - Y_j$

Lower limit is the 9th value from the bottom is 19
Upper limit is the 9th value from the top is 36

95% confidence interval is $(19\,,36)$

2. Using Kolmogorov – Smirnov two sample test

H_0: There is no significant different between the two methods.
[F(t) = G(t), for every t]

H_A: There is a significant different between the two methods.
[F(t) ≠ G(t) for at least one value of t]

$$F_m(t) = \frac{\text{number of observed X's} \le t}{m}$$

$$G_m(t) = \frac{\text{number of observed Y's} \le t}{n}$$

	Score Z_i	Sample	$F_7(Z_i)$	$G_7(Z_i)$	$\lvert F_7(Z_i) - G_7(Z_i)\rvert$
1	10	Method 2	0/7	1/7	1/7
2	12	Method 2	0/7	2/7	2/7
3	14	Method 2	0/7	3/7	3/7
4	17	Method 2	0/7	4/7	4/7
5	18	Method 2	0/7	5/7	5/7
6	20	Method 2	0/7	6/7	6/7
7	38	Method 1	1/7	6/7	5/7
8	39	Method 1	2/7	6/7	4/7
9	40	Method 1	3/7	6/7	3/7
10	41	Method 1	4/7	6/7	2/7
11	48	Method 1	5/7	6/7	1/7
12	50	Method 1	6/7	6/7	0/7
13	53	Method 1	7/7	6/7	1/7
14	102	Method 2	7/7	7/7	0/7

$$D_{max}(\text{largest divergence}) = 6/7$$

$$Z = D_{max}\sqrt{\frac{mn}{m+n}} = (0.86)\sqrt{\frac{(7)(7)}{7+7}} = 0.86 \times 1.87 = 1.604$$

Since $1 \leq Z < 3.2$, we use the p-value formula:

$$Q = e^{-2Z^2} = e^{-2\times(1.604)^2} = e^{-5.146} = 0.0058$$

$$p = 2(Q - Q^4 + Q^9 - Q^{16})$$
$$= 2(0.0058 - 0.0058^4 + 0.0058^9 - 0.0058^{16}) = 0.012$$

$p - \text{value} = 0.012 < \alpha = 0.05$

Then we reject H_0 (There is a difference in the percentages)

Using R

```
method1 = c(48, 40, 39, 50, 41, 38, 53)
method2 = c(14, 18, 20, 10, 12, 102, 17)
ks.test(method1, method2, alternative = "two.sided")

        Exact two-sample Kolmogorov-Smirnov test
data:  method1 and method2
D = 0.85714, p-value = 0.008159   <-------- Reject H0
alternative hypothesis: two-sided
```

Exercise 2:

Method 1

48	40	39	50	41
38	71	30	15	33
47	51	60	59	58
42	11	46	36	27
93	72	57	45	53

Method 2

14	18	20	10	12
102	21	19	100	23
16	82	13	25	24
97	28	9	34	52
70	22	26	8	17

For each of the following questions (1-4), determine which would be the simplest type of statistical analysis that would be appropriate to use. Use each type of analysis only once.

(A) Paired t test	(B) Two sample t-test	(C) ANOVA
(D) Kruskal-Wallis	(E) Wilcoxon Rank-Sum Test	

Compare the average number of hours per week spent on Facebook for Freshmen, Sophomore, Juniors and Seniors at UF, based on a random sample of 100 students.	(C) ANOVA
Compare the average number of hours per week spent on Facebook during the first week in April and the first week in May (finals week) for random students at UF, measured on the same 100 students.	(A) Paired t test

Compare the distribution of the number of hours per week spent on Facebook for male and female students at UF, based on a random sample of 10 students. There was an outlier in one of the groups.	(E) Wilcoxon Rank-Sum Test
Compare the average number of hours per week spent on Facebook for male and female students at UF, based on a random sample of 100 students.	(B) Two sample t-test

Exercise *:

The following data were obtained from a reading-level test for 1st-grade children. Compare the performance gains of the two different methods for teaching reading. Two different classes being taught a basic mathematics skills using two different methods.

Gain score (Method 1)	16	13	16	16	13	9	12	12	20	17
Gain score (Method 1)	11	2	10	4	9	8	5	6	4	16

Use two-tailed Mann–Whitney U and Kolmogorov–Smirnov two-sample tests to determine which method was better for teaching reading. Set $\alpha = 0.05$.

[1] The hypothesis associated with this test ....
[2] The calculated value of the test statistic is ....
[3] The critical value ....

The Mann-Whitney U test is preferred to a t-test when ....

A	Data are paired	B	Sample sizes are small
C	Sample are dependent	D	The assumption of normality is not met

Chapter 5

THE FRIEDMAN TEST