STAT 333
--- Section 2.1: Basic Inference

Basic Definitions

Population: The collection of all the individuals of interest.
• This collection may be _large_ or even infinite.

Sample: A collection of elements of the population.

• Suppose our population consists of a finite number (say, *N*) of elements.

Random Sample: A sample of size n from a finite population such that each of the possible samples of size n was equally likely to have been obtained .

Another definition:

Random Sample: A sample of size n forming a sequence of n independent and identically distributed (iid) random variables $X_1, X_2, \ldots, X_n$

•Note these definitions are equivalent only if the elements are drawn with replacement from the population.

• If the population size is very large, whether the sampling was done with or without replacement makes little practical difference.

Multivariate Data

- **Sometimes each individual may have more than one variable measured on it.**

- Each observation is then a multivariate random variable (or random vector)

$$\underline{X}_i = (y_{i1}, y_{i2}, \ldots\ldots, y_{ik})$$

Example: If the weight and height of a sample of 8 people are measured, our multivariate data are:

$\underline{X}_1 = (y_{11}, y_{12})$
$\underline{X}_2 = (y_{21}, y_{22})$
$\underline{X}_3 = (y_{31}, y_{32})$ where, y_{i1} : weight , y_{i2}: height
$\underline{X}_4 = (y_{41}, y_{42})$ $i=1,2,\ldots\ldots 8$
$\underline{X}_5 = (y_{51}, y_{52})$
$\underline{X}_6 = (y_{61}, y_{62})$
$\underline{X}_7 = (y_{71}, y_{72})$
$\underline{X}_8 = (y_{81}, y_{82})$

- If the sample is random, then the components Y_{i1} and Y_{i2} might not be independent, but the vectors $\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_8$ will still be independent and identically distributed.

- That is, knowledge of the value of $\underline{X}_1$, say, does not alter the probability distribution of $\underline{X}_2$.

Measurement Scales

Nominal Scale:

If a variable simply places an individual into one of several (unordered) categories, the variable is measured on a **nominal** scale.

Examples:

Hair color ,Gender , Nationality, Major

Ordinal Scale:

If the variable is categorical but the categories have a meaningful ordering, the variable is on the **ordinal** scale.

Examples:

Grades of students, Rating of movies, Education level , Likerty-Type scale (Strongly agree, agree ,….)

Interval Scale:

If the variable is numerical and the value of zero is arbitrary rather than meaningful, then the variable is on the interval scale.

Examples:

Temperature in C^o
Temperature in F^o

Note: For interval data, the interval (difference) between two values is meaningful, but ratios between two values are not meaningful.

Ratio Scale:

If the variable is numerical and there is a meaningful zero, the variable is on the ratio scale.

Examples:

Height , Speed ,Age, Weight loss ,height

• With ratio measurements, the ratio between two values has meaning.

Weaker ←---------------------------------→ Stronger

Nominal ordinal interval ratio

Note:

• Most classical parametric methods require the scale of measurement of the data to be interval (or stronger).

• Some nonparametric methods require ordinal (or stronger) data; others can work for data on any scale.

• **A parameter** is a characteristic of a population.

Examples of parameter :

Population mean (μ)
Population standard deviation (σ)
Population proportion (P)
Population median

• Typically a parameter cannot be calculated from sample data.

• A **statistic** is a function of random variables.

• **Given the data**, we can calculate the value of a statistic.

Examples of statistic :

Sample mean
Sample standard deviation (S)
Sample proportion ($\hat{p}$)
Sample median

Order Statistics

• The *k*-th order statistic for a sample $X_1, X_2, \ldots, X_n$ is denoted $X^{(k)}$ and is the *k*-th smallest value in the sample.

• The values $X^{(1)} \leq X^{(2)} \leq \ldots \leq X^{(n)}$ are called the ordered random sample.

Example:

If our sample is: 14, 7, 9, 2, 16, 18

then $X^{(3)} =$

$X^{(5)} =$

<u>Section 2.2: Estimation</u>

• Often we use a statistic to **<u>estimate</u>** some aspect of a population of interest.

• A statistic used to estimate is called an **<u>estimator</u>**.

<u>Familiar Examples</u>:

• The sample mean:

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

• The sample variance:

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

• The sample standard deviation:

$$S = \sqrt{S^2}$$

• These are **point estimates** (single numbers).

• An **interval estimate (confidence interval)** is an interval of numbers that is designed to contain the parameter value.

• **A 95% confidence interval** is constructed via a formula that has 0.95 probability (over repeated samples) of containing the true parameter value.

Familiar large-sample formula for CI for μ:

$$(\bar{X} - Z_{1-\frac{\propto}{2}}\frac{s}{\sqrt{n}}, \bar{X} + Z_{1-\frac{\propto}{2}}\frac{s}{\sqrt{n}})$$

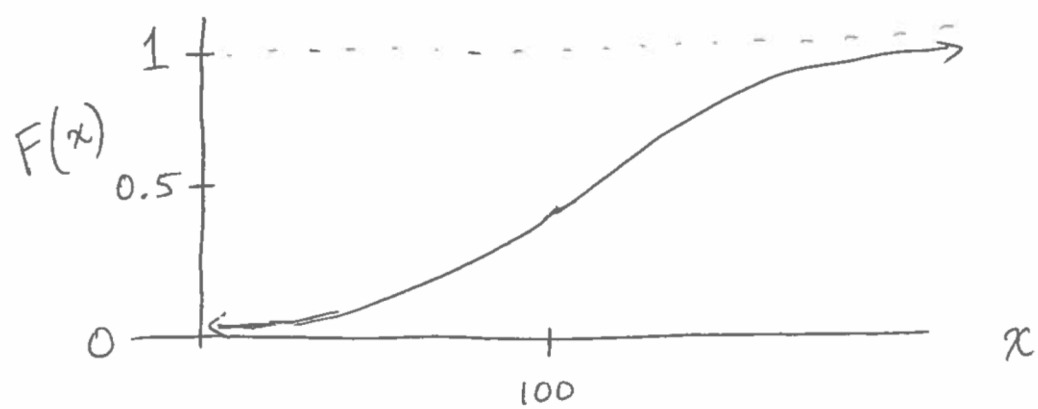
<u>Some Less Familiar Estimators</u>

• The **cumulative distribution function** (c.d.f.) of a random variable is denoted by F(x):

$$\mathbf{F(\mathit{x}) = P(\mathit{X} \leq \mathit{x})}$$

• **This is** $\int_{-\infty}^{x} f(t)dt$ **when *X* is a continuous r.v.**

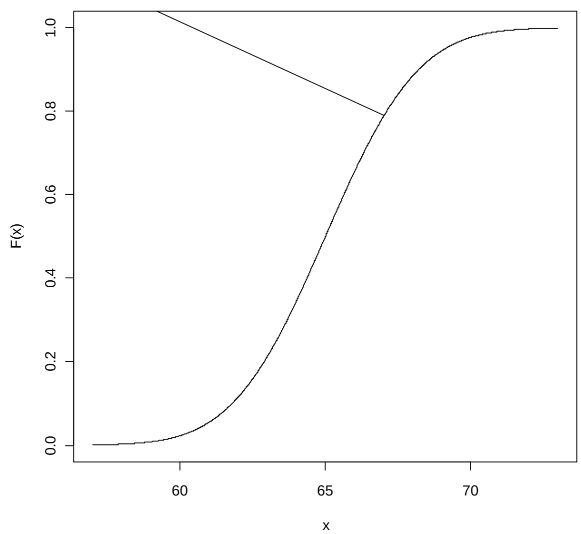
<u>Example:</u>
If X is a normal variable with mean 100, its c.d.f. F(x) should look like:



• Sometimes **we do not know the distribution** of our variable of interest.

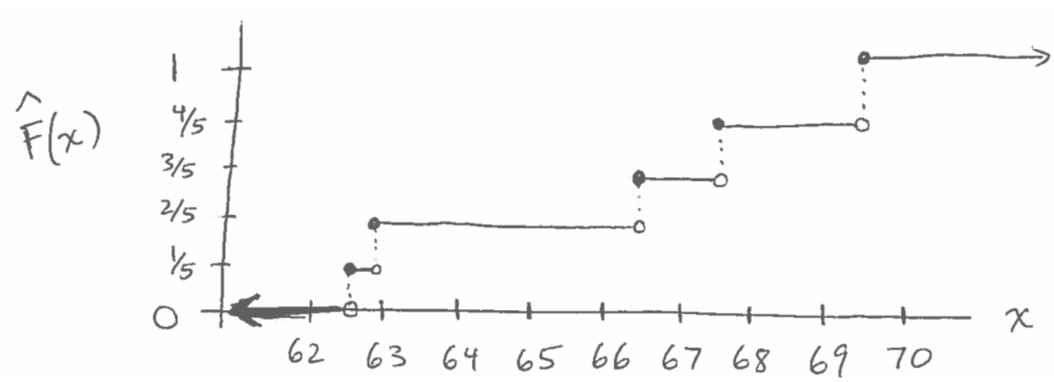
• The **empirical distribution function** (e.d.f.) is an estimator of the true c.d.f. – it can be calculated from the sample data.

Example: Suppose heights of adult females have normal distribution with mean 65 inches and standard deviation 2.5 inches. The c.d.f. of this distribution is:



• Now suppose we do NOT know the true height distribution. We randomly sample 5 females and measure their heights as: 69.3, 66.3, 62.6, 62.9, 67.4

e.d.f.:



• The **survival function** is defined as **1 – F(*x*)**, which is the probability that the random variable **takes a value greater than *x***.

• This is useful in reliability/survival analysis, when it is the probability of the item surviving past time *x*.

• The **Kaplan-Meier estimator** (p. 89-91) is a way to estimate the survival function when the survival time is observed for only some of the data values.

The Bootstrap

• The **nonparametric bootstrap** is a method of estimating characteristics (like expected values and standard errors) of summary statistics.

• This is especially useful when the true population distribution is unknown.

• The **nonparametric bootstrap is based on the e.d.f**. rather than the true (and perhaps unknown) c.d.f.

Method: Resample data (randomly select *n* values from the original sample, with replacement) *m* times.

• These "bootstrap samples" together mimic the population.

• For each of the *m* bootstrap samples, calculate the statistic of interest.

• These *m* values will approximate the sampling distribution.

• From these bootstrap samples, we can estimate the:

(1) expected value of the statistic
(2) standard error of the statistic
(3) confidence interval of a corresponding parameter

Example: We wish to estimate the 85th percentile of the population of BMI measurements of SC high schoolers.

• We take a random sample of 20 SC high school students and measure their BMI.

• See code on course web page for bootstrap computations:

Estimated standard error of sample 85th percentile is 1.65

A 95% bootstrap CI for the population 85th percentile is :

(26.6 , 30.65)