Review CH5, CH6 and CH7

Physics 103: Classical Mechanics

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2025

Version 1

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1. CH5 The Laws of Motion

2. CH6 Circular Motion and Newton’s Laws

3. CH7 Energy and Energy Transfer

1.1 Some important formulas

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$$\boxed{\Sigma\vec{\boldsymbol{F}} = m\vec{\boldsymbol{a}}}$$

$$\boxed{\Sigma F_x = ma_x, \quad \Sigma F_y = ma_y, \quad \Sigma F_z = ma_z}$$

$$\boxed{\overrightarrow{\boldsymbol{W}} \equiv \vec{\boldsymbol{F}}_g = m\vec{\boldsymbol{g}}}$$

1.2 Some General Examples

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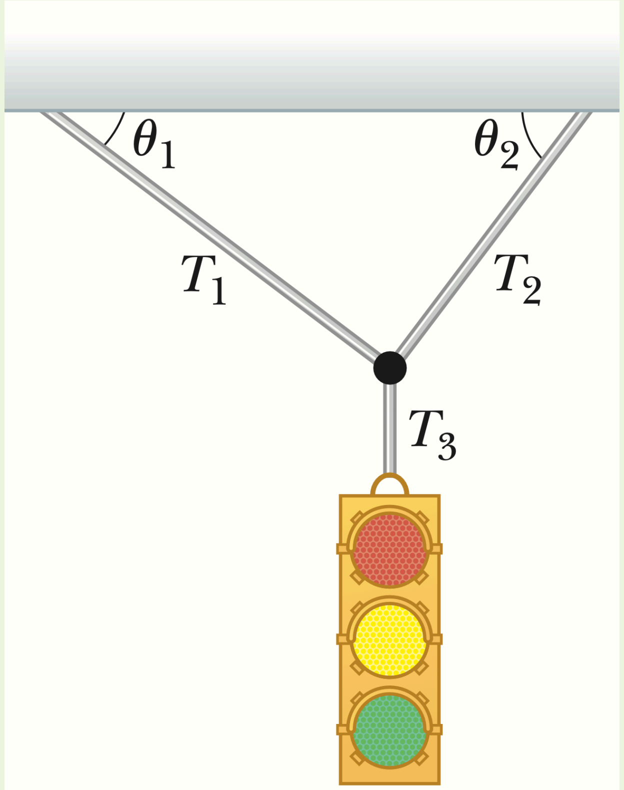
Example 1.1

A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support, as in the Figure. The upper cables make angles of 37° and 53° with the horizontal. These upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds 100 N. Will the traffic light remain hanging in this situation, or will one of the cables break?

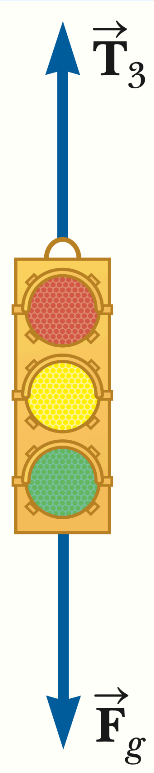


Solution 1.1

- For simplicity, we first consider only the vertical cable.

$$\sum F_y = T_3 - F_g = 0$$

$$\Rightarrow \quad T_3 = F_g = 122N$$



1.2 Some General Examples

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Solution 1.1

- Next, we consider the two angled cables,

(1) $\sum F_x = T_2 \cos\theta_2 - T_1 \cos\theta_1 = 0$

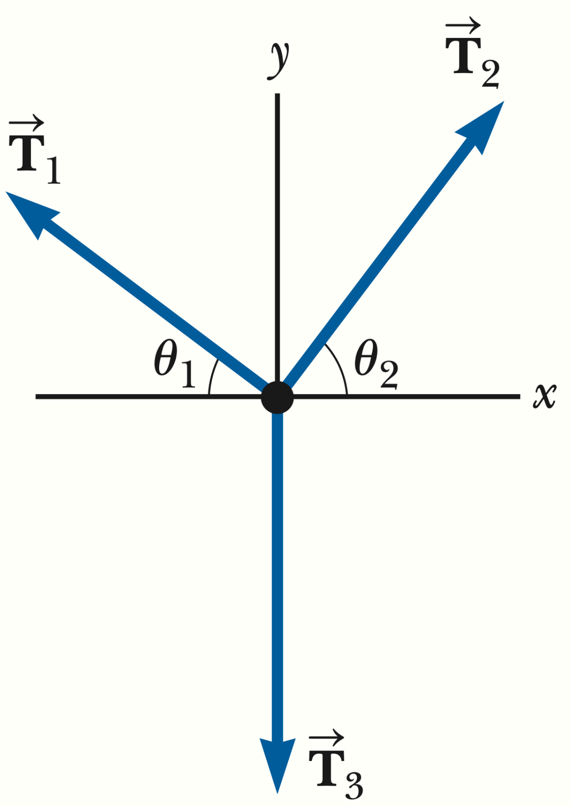
(2) $\sum F_y = T_2 \sin\theta_2 + T_1 \sin\theta_1 - F_g = 0$

- We have two equations and two unknowns, T_1 and T_2. From equation (1), we have

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} \tag{3}$$

- Substituting this into equation (2) gives

$$\left(T_1 \frac{\cos\theta_1}{\cos\theta_2}\right) \sin\theta_2 + T_1 \sin\theta_1 = F_g$$



1.2 Some General Examples

$$T_1(\cos\theta_1 \tan\theta_2 + \sin\theta_1) = F_g$$

- Solving for T_1 gives

$$T_1 = \frac{F_g}{\cos\theta_1 \tan\theta_2 + \sin\theta_1} = \frac{122}{\cos 37° + \tan 53° + \sin 37°} = 73.4 \text{ N}$$

- To find T_2, we substitute the value of T_1 into Eq(3):

$$T_2 = T_1 \frac{\cos\theta_1}{\cos\theta_2} = 73.4 \frac{\cos 37°}{\cos 53°} = 97.4 \text{ N}$$

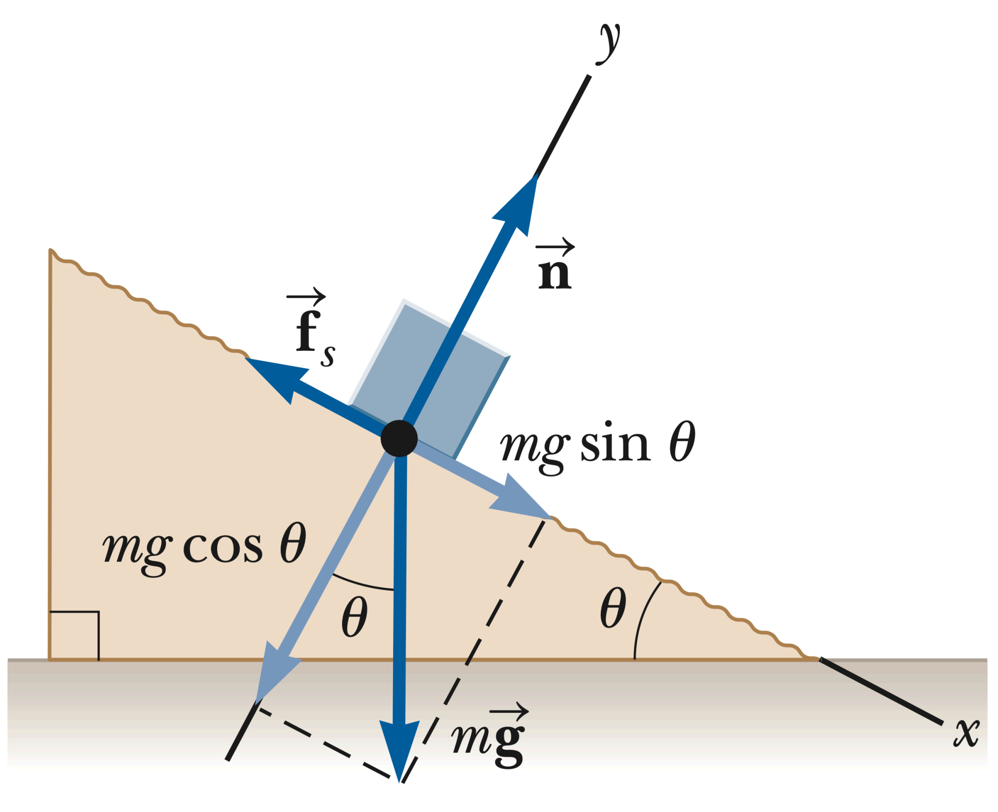
- Since neither tension exceeds 100 N, the traffic light will remain hanging in this situation.

1.2 Some General Examples

Example 1.2

The following is a simple method of measuring coefficients of friction. Suppose a block is placed on a rough surface inclined relative to the horizontal as shown in the Figure. The incline angle is increased until the block starts to move.

Show that you can obtain μ_s by measuring the critical angle θ_c at which this slipping just occurs.



1.2 Some General Examples

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Solution 1.2

$$\sum F_x = ma$$

$$mg\sin\theta_c - f_s = 0$$

$$\implies f_s = mg\sin\theta_c \qquad (1)$$

$$\sum F_y = n - mg\cos\theta_c = 0$$

$$\implies n = mg\cos\theta_c \qquad (2)$$

- Therefore,

$$\frac{f_s}{n} = \tan\theta_c$$

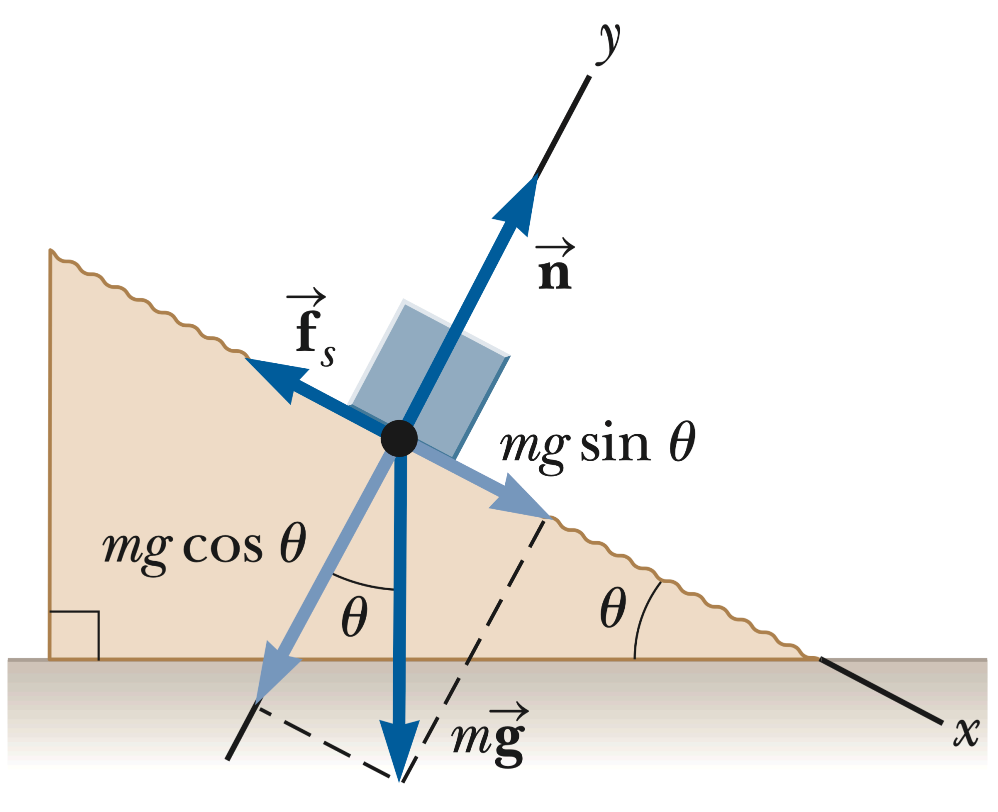
- Using ($f_s = \mu_s n$), we get

$$\mu_s = \tan\theta_c$$

1.2 Some General Examples

For example, if the block starts to slip when $\theta_c = 20°$, then

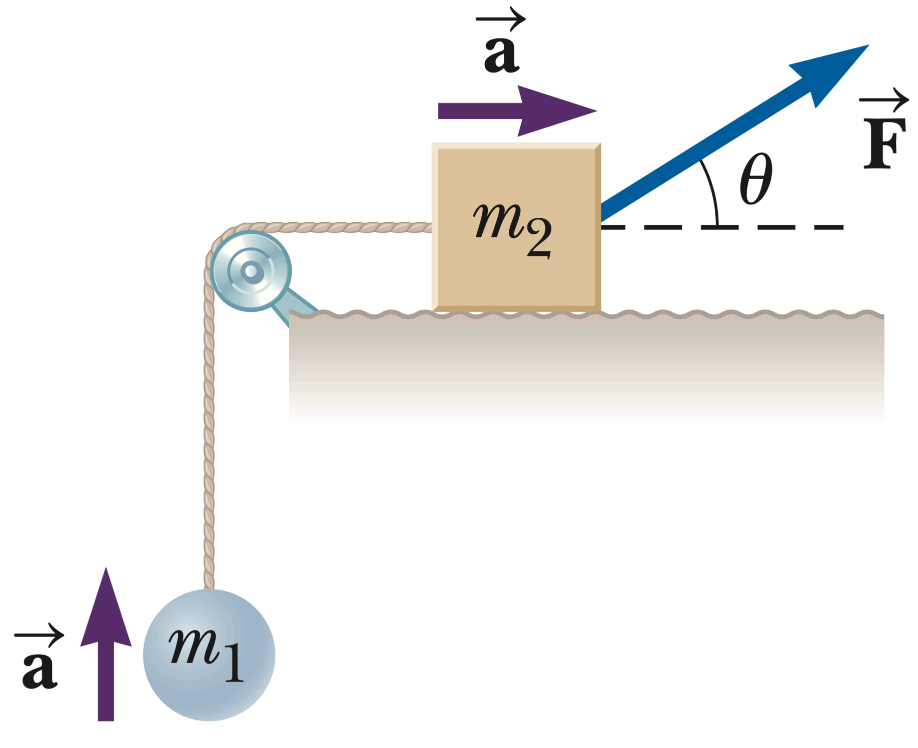
$$\mu_s = \tan 20° = 0.364$$



1.2 Some General Examples

Example 1.3

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley, as shown in the Figure. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown. The coefficient of kinetic friction between the block and surface is μ_k. *Determine* the magnitude of the acceleration of the two objects.



1.2 Some General Examples

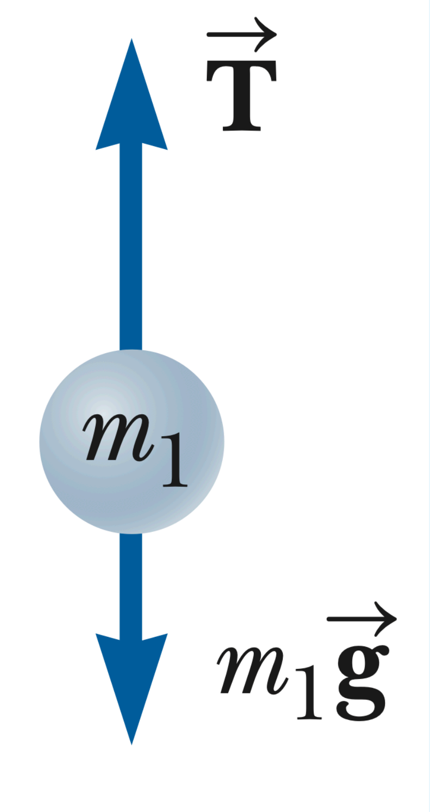
Solution 1.3

- We set the positive direction to be clockwise.
- Applying Newton's second law to m_1:

$$\sum F_{1y} = T - m_1 g = m_1 a \tag{1}$$

- Therefore,

$$\begin{aligned} T &= m_1 g + m_1 a \\ &= m_1 (g + a) \end{aligned} \tag{2}$$



1.2 Some General Examples

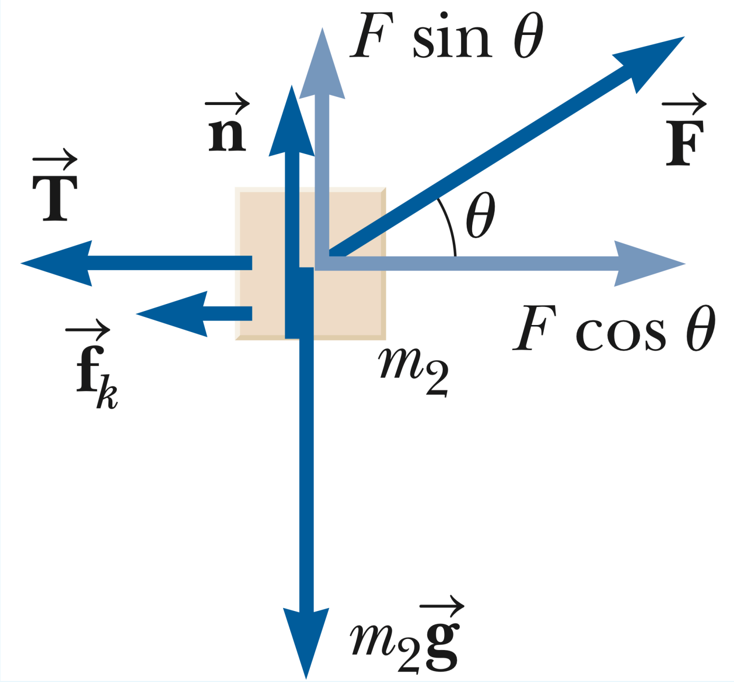
- Applying Newton's second law to m_2:

$$\sum F_{2y} = n + F\sin\theta - m_2 g = 0 \tag{3}$$

$$\implies n = m_2 g - F\sin\theta \tag{4}$$

$$\sum F_{2x} = F\cos\theta - T - f_k = m_2 a \tag{5}$$

$$\begin{aligned} \implies m_2 a &= F\cos\theta - T - (\mu_k \mathrm{n}) \\ &= F\cos\theta - (m_1 g + m_1 a) \\ &\quad -\mu_k (m_2 g - F\sin\theta) \end{aligned} \tag{6}$$



1.2 Some General Examples

$$m_2 a + m_1 a = F\cos\theta - m_1 g - \mu_k m_2 g + \mu_k F\sin\theta$$

$$(m_1 + m_2)a = F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g$$

- Solving for a gives:

$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2)g}{m_1 + m_2} \tag{7}$$

1.2 Some General Examples

- Note: If a is negative, then the direction of the kinetic friction f_k in Eq (5) has to be reversed, to be opposite to the direction of motion.
  Therefore, $\mu_k \Longrightarrow -\mu_k$ in Eq (7).

1. CH5 The Laws of Motion

2. CH6 Circular Motion and Newton's Laws

3. CH7 Energy and Energy Transfer

2.1 Some important formulas

$$\boxed{\sum F = ma_c = m\frac{v^2}{r}}$$

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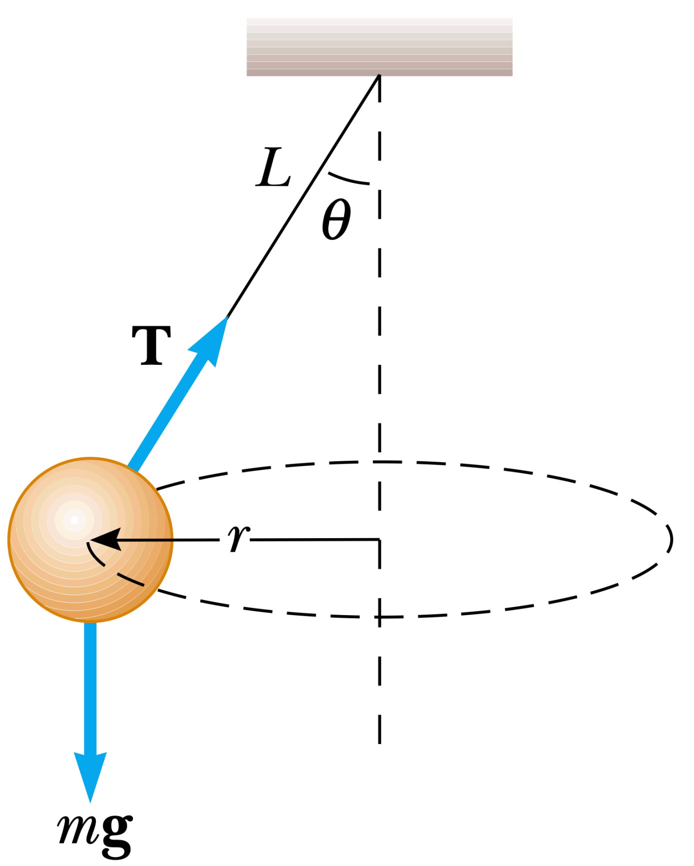
2.2 Some General Examples

A small object of mass m is suspended from a string of length L. The object revolves with constant speed v in a horizontal circle of radius r, as shown in the Figure. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.)

Find an expression for v.



2.2 Some General Examples

Applying Newton's second law:

$$\sum \vec{\boldsymbol{F}}_y = ma$$

$$T\cos\theta - mg = 0$$

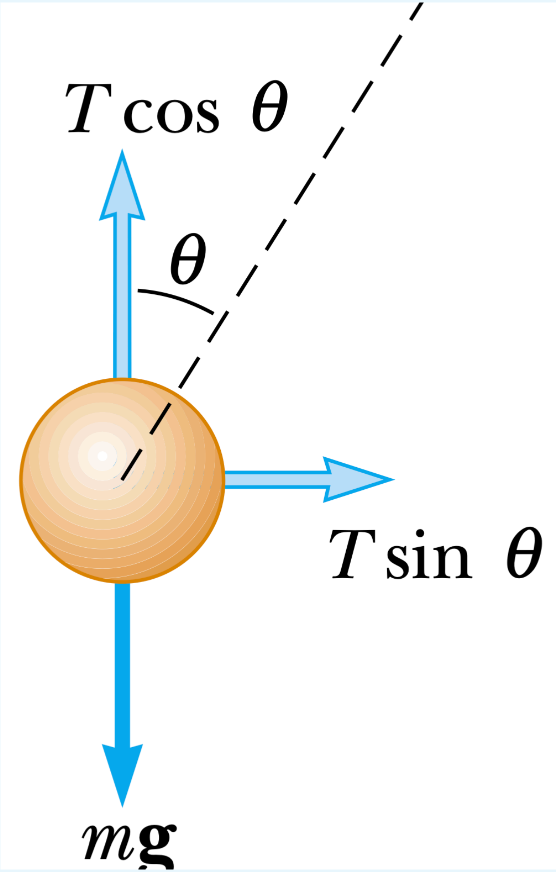
$$\implies T\cos\theta = mg \tag{8}$$

$$\sum F_r = m\frac{v^2}{r}$$

$$\implies T\sin\theta = m\frac{v^2}{r} \tag{9}$$

- Dividing Eq (2) by Eq (1), we get:

$$\tan\theta = \frac{v^2}{rg}$$



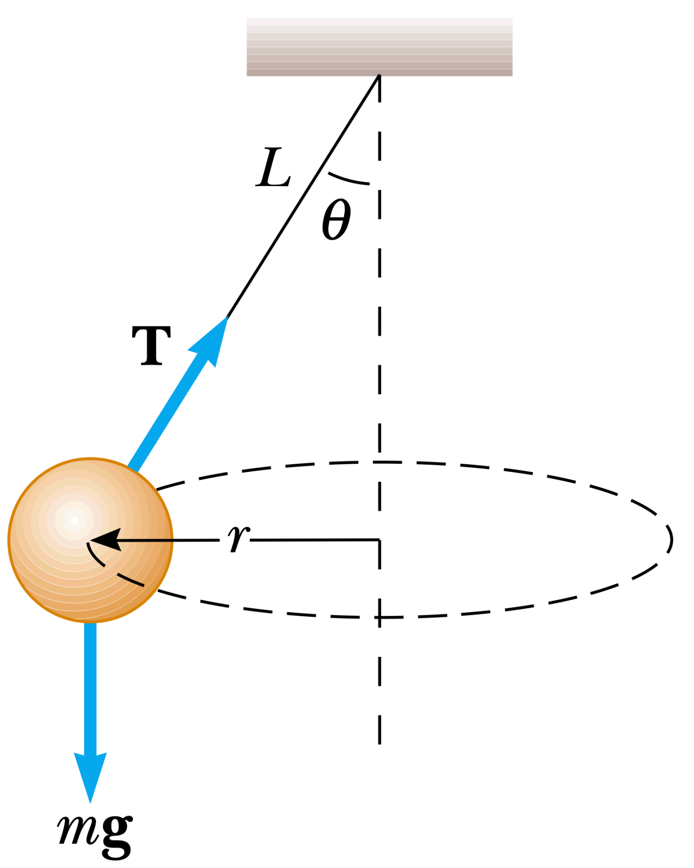
2.2 Some General Examples

$$\Longrightarrow v = \sqrt{rg\tan\theta}$$

- Using $r = L\sin\theta$, we get:

$$\boxed{v = \sqrt{\tan\theta\sin\theta\ g\ L}}$$

- Note that the velocity of the object does not depend on its mass.



1. CH5 The Laws of Motion

2. CH6 Circular Motion and Newton's Laws

3. CH7 Energy and Energy Transfer

3.1 Some important formulas

$$\vec{\boldsymbol{A}} = A_x \hat{\boldsymbol{\imath}} + A_y \hat{\boldsymbol{\jmath}} + A_z \hat{\boldsymbol{k}}$$

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |\vec{\boldsymbol{A}}| \, |\vec{\boldsymbol{B}}| \cos\theta$$

$$|\vec{\boldsymbol{A}}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = A_x B_x + A_y B_y + A_z B_z$$

3.1 Some important formulas

$$W = \vec{\boldsymbol{F}} \cdot \Delta \vec{\boldsymbol{r}} = F \Delta r \cos \theta$$

$$W = \int_{x_i}^{x_f} F_x \, \mathrm{d}x$$

$$W_s = \int_{x_i}^{x_f} (-kx) \, \mathrm{d}x = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

3.1 Some important formulas

$$\boxed{W_{\text{ext}} = \int_{x_i}^{x_f} (kx)\,\mathrm{d}x = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2}$$

$$\boxed{\Delta K = +W - f_k d}$$

$$\boxed{\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + F_{\text{net}}d - f_k d}$$

3.1 Some important formulas

$$\boxed{\bar{P} = \frac{W}{\Delta t}}$$

$$\boxed{P = \frac{\mathrm{d}W}{\mathrm{d}t}}$$

$$\boxed{P = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}}}$$

3.2 Some General Examples

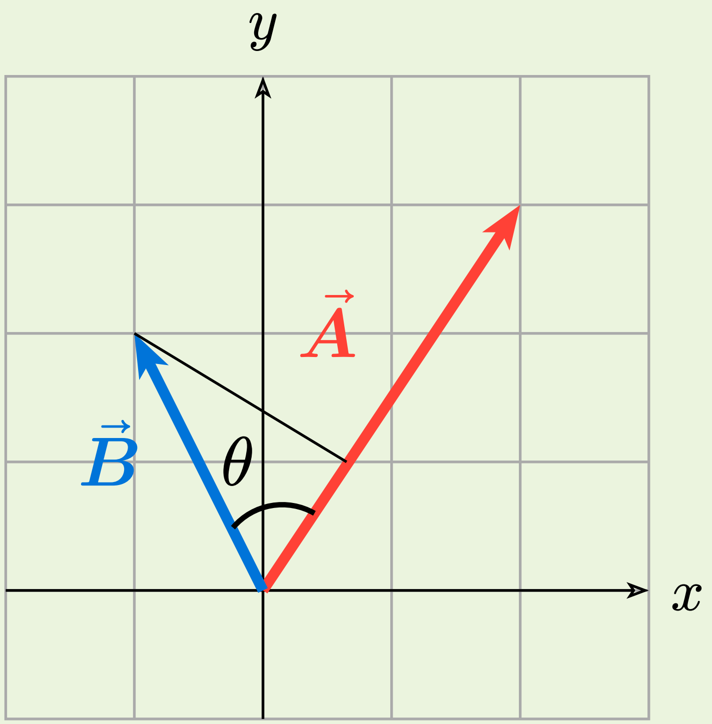
Example 3.5

The vectors $\vec{A}$ and $\vec{B}$ are given by

$$\vec{A} = 2\hat{\imath} + 3\hat{\jmath}$$
$$\vec{B} = -\hat{\imath} + 2\hat{\jmath}$$

(A) Determine the scalar product $\vec{A} \cdot \vec{B}$

(B) Find the angle θ between $\vec{A}$ and $\vec{B}$.



3.2 Some General Examples

Solution 3.5

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = (2)(-1) + (3)(2) = -2 + 6 = 4$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\cos\theta = \frac{4}{\sqrt{2^2 + 3^2} \times \sqrt{(-1)^2 + 2^2}}$$

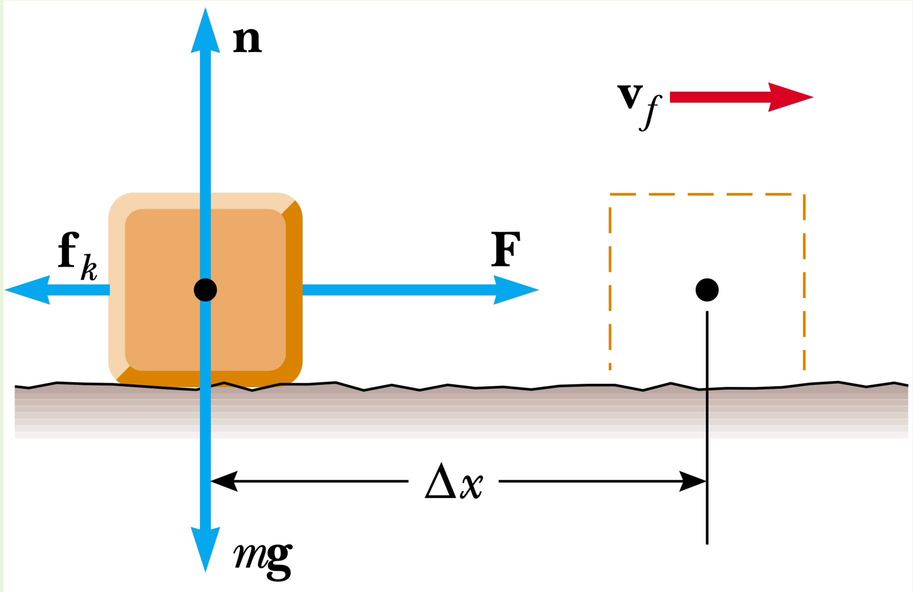
$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{65}}\right) = 60.3^\circ$$

3.2 Some General Examples

Example 3.6

A 6 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.



3.2 Some General Examples

Solution 3.6

$$\begin{aligned}\frac{1}{2}mv_f^2 &= \frac{1}{2}mv_i^2 + F_{\text{net}}d - f_k d\\ &= 0 + Fd - (\mu_k mg)d\end{aligned}$$

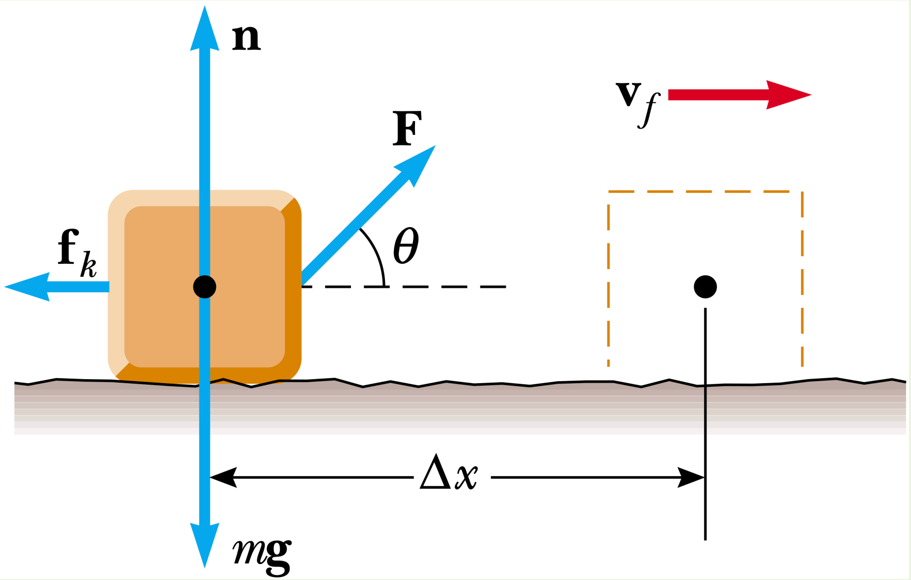
$$\frac{1}{2}(6\ \text{kg})v_f^2 = 0 + (12\ \text{N})(3\ \text{m}) - (0.15)(6\ \text{kg})(9.8\ \text{m/s}^2)(3\ \text{m})$$

$$\implies v_f = 1.8\ \text{m/s}$$

3.2 Some General Examples

Example 3.6

(B) Suppose the force F is applied at an angle as shown in the Figure. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?



3.2 Some General Examples

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Solution 3.6

- To find the angle that maximizes the final speed, we first write the equation for the final speed in terms of the angle θ:

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (F\cos\theta)d - f_k d$$

- To find the frictional force, we apply Newton's second law in the vertical direction:

$$n + F\sin\theta - mg = 0$$
$$\Rightarrow n = mg - F\sin\theta$$
$$\Rightarrow f_k = \mu_k n = \mu_k(mg - F\sin\theta)$$

3.2 Some General Examples

Using this expression for f_k and with $v_i = 0$, we get:

$$\frac{1}{2}mv_f^2 = (F\cos\theta)\ d - \mu_k(mg - F\sin\theta)\ d$$

- To maximize $\frac{1}{2}mv_f^2$, we take the derivative with respect to θ and set it to zero:

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\frac{1}{2}mv_f^2\right) = \frac{d}{d\theta}\Big[(F\cos\theta)\ d - \mu_k(mg - F\sin\theta)\ d\Big] = 0$$

$$= -(F\sin\theta)d - \mu_k(0 - F\cos\theta)d = 0$$

$$\Rightarrow -\cancel{Fd}\sin\theta + \mu_k\cancel{Fd}\cos\theta = 0$$

$$\Rightarrow \sin\theta = \mu_k\cos\theta$$

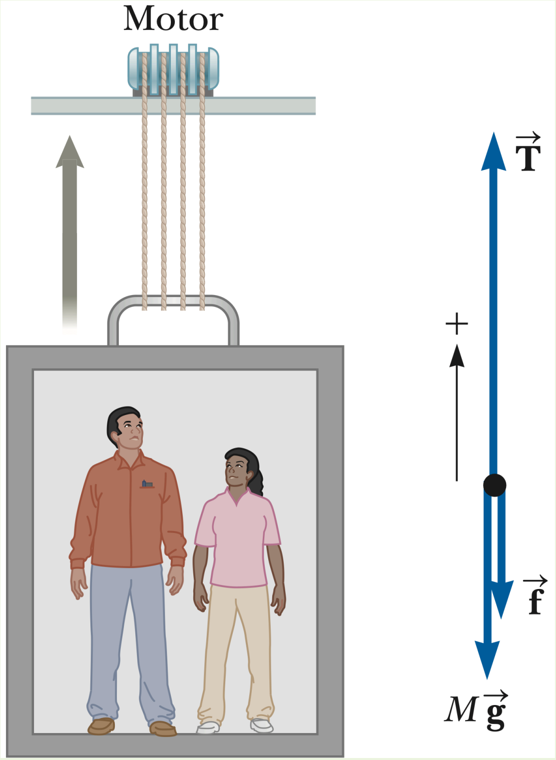
$$\Rightarrow \tan\theta = \mu_k$$

$$\Rightarrow \theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15) = 8.5°$$

Example 3.7

An elevator car has a mass of 1600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4000 N retards its motion upward, as shown in the Figure.

(A) What power delivered by the motor is required to lift the elevator car at a constant speed of 3 m/s?



3.2 Some General Examples

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Solution 3.7

- The motor must supply the force of magnitude T that pulls the elevator car upward.
- Using Newton's second law in the vertical direction, we find:

$$\sum F = T - f - \left(m_{\text{car}} + m_{\text{pass}}\right)g = 0$$

$$T = f + \left(m_{\text{car}} + m_{\text{pass}}\right)g$$

$$T = (4000\ \text{N}) + (1600\text{kg} + 200\text{kg})(9.8\ \text{m/s}^2) = 2.16 \times 10^4\ \text{N}$$

- The power delivered by the motor is:

$$P = \vec{T} \cdot \vec{v} = (2.16 \times 10^4\ \text{N})(3\ \text{m/s}) = 6.48 \times 10^4\ \text{W}$$

Example 3.7

(B) What power must the motor deliver at the instant the speed of the elevator is v if the motor is designed to provide the elevator car with an upward acceleration of $1\ \mathrm{m/s}$?

3.2 Some General Examples

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Solution 3.7

- Similar to part (A), we apply Newton's second law in the vertical direction to find the tension in the cable:

$$\sum F_y = T - f - M_{\text{tot}} g = M_{\text{tot}} a$$

$$\Rightarrow T = M_{\text{tot}}(g + a) + f$$

$$\Rightarrow T = (1800\text{kg})(9.8\ \text{m/s}^2 + 1\ \text{m/s}^2) + 4000\ \text{N} = 2.34 \times 10^4\ \text{N}$$

- The power delivered by the motor at v is:

$$P = \vec{\boldsymbol{T}} \cdot \vec{\boldsymbol{v}} = 2.34 \times 10^4\ v$$