King Saud University Department of Mathematics

Quiz II 280-Math 1Semester (1443)H

Question 1 [2] Prove that $x2^x = 1$ for some $x \in (0,1)$.

Question 2[2] Let
$$f(x) = \begin{cases} x^2 \sin(\frac{1}{x}), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$

Use the definition of the derivative to prove that f is differentiable at 0.

Question 3 [3]

Let $f: (0,2) \to R$. Assume that $\lim_{x \to 1} f(x) = \frac{1}{2}$. Prove that there exists δ >0 such that if $x \in (0,2)$ and $|x-1| < \delta$ then f(x) > 0.

Question 4 [3]

Prove that $g(x) = \frac{1}{1+|x|}$ is uniformly continuous on R.