Second Quiz — Model Answer

MATH 111 — Integral Calculus

Section 75291 • 25 October 2026 • Total 10 marks • Dr. Sumayah Mansour Bin Hazzaa

Marks for each step are shown on the right. Equivalent correct forms receive full marks. Every indefinite integral must carry the constant of integration.

Part (1) (2 marks) *the double-angle identity first*

Evaluate $\displaystyle\int \sin^2 x \cos^2 x \, dx$

Do not expand. Use $\sin x \cos x = \frac{1}{2}\sin 2x$ on the product:

$$\sin^2 x \cos^2 x = (\sin x \cos x)^2 = \left(\frac{1}{2}\sin 2x\right)^2 = \frac{1}{4}\sin^2 2x \qquad [0.5]$$

One square remains, so reduce it with $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$, taking $\theta = 2x$:

$$\frac{1}{4}\sin^2 2x = \frac{1}{4}\cdot\frac{1-\cos 4x}{2} = \frac{1}{8} - \frac{1}{8}\cos 4x \qquad [0.5]$$

$$\int \sin^2 x \cos^2 x \, dx = \frac{x}{8} - \frac{1}{32}\sin 4x + C \qquad [1]$$

Part (2) (2 marks) *by parts twice, then solve for the integral*

Evaluate $\displaystyle\int e^{3x}\cos x \, dx$

Let $I = \displaystyle\int e^{3x}\cos x \, dx$. Take $u = \cos x$, $dv = e^{3x}dx$:

$$I = \frac{1}{3}e^{3x}\cos x + \frac{1}{3}\int e^{3x}\sin x \, dx \qquad [0.5]$$

Apply parts again to the new integral, with $u = \sin x$, $dv = e^{3x}dx$:

$$\int e^{3x}\sin x \, dx = \frac{1}{3}e^{3x}\sin x - \frac{1}{3}\int e^{3x}\cos x \, dx = \frac{1}{3}e^{3x}\sin x - \frac{1}{3}I \qquad [0.5]$$

Substitute back; I now appears on both sides, so solve for it:

$$I = \frac{1}{3}e^{3x}\cos x + \frac{1}{9}e^{3x}\sin x - \frac{1}{9}I \qquad \Longrightarrow \qquad \frac{10}{9}I = e^{3x}\left(\frac{1}{3}\cos x + \frac{1}{9}\sin x\right) \qquad [0.5]$$

$$\int e^{3x}\cos x \, dx = \frac{e^{3x}}{10}(3\cos x + \sin x) + C \qquad [0.5]$$

Part (3) (2 marks) *the form $\int du/\sqrt{a^2+u^2}$*

Evaluate $\displaystyle\int \frac{dx}{\sqrt{9+4x^2}}$

Write the radical as $a^2 + u^2$: here $9 + 4x^2 = 3^2 + (2x)^2$, so put $u = 2x$, $du = 2\,dx$ and $a = 3$:

$$\int \frac{dx}{\sqrt{9+4x^2}} = \frac{1}{2}\int \frac{du}{\sqrt{3^2+u^2}} \qquad [1]$$

$$= \frac{1}{2}\sinh^{-1}\frac{u}{3} + C = \frac{1}{2}\sinh^{-1}\frac{2x}{3} + C \;=\; \frac{1}{2}\ln\left(2x + \sqrt{9+4x^2}\right) + C' \qquad [1]$$

Either form is accepted; the two differ by a constant.

Part (4) (2 marks) *save one $\sec^2 x$ for du*

Evaluate $\displaystyle\int \tan^4 x\, \sec^4 x\, dx$

The power of sec is even, so keep $\sec^2 x$ aside and convert the rest with $\sec^2 x = 1 + \tan^2 x$:

$$\int \tan^4 x\, \sec^4 x\, dx = \int \tan^4 x\,(1+\tan^2 x)\sec^2 x\, dx \qquad [0.5]$$

Now $u = \tan x$, $du = \sec^2 x\, dx$:

$$= \int u^4(1+u^2)\, du = \int (u^4+u^6)\, du = \frac{u^5}{5} + \frac{u^7}{7} + C \qquad [1]$$

$$\int \tan^4 x\, \sec^4 x\, dx = \frac{\tan^5 x}{5} + \frac{\tan^7 x}{7} + C \qquad [0.5]$$

Part (5) (2 marks) *by parts with $dv = dx$*

Evaluate $\displaystyle\int \tanh^{-1} x\, dx$

There is only one function, so take $u = \tanh^{-1} x$ and $dv = dx$. Then $du = \dfrac{dx}{1-x^2}$ and $v = x$:

$$\int \tanh^{-1} x\, dx = x\tanh^{-1} x - \int \frac{x}{1-x^2}\, dx \qquad [1]$$

The remaining integral is a substitution, $w = 1 - x^2$, $dw = -2x\, dx$:

$$\int \frac{x}{1-x^2}\, dx = -\frac{1}{2}\int \frac{dw}{w} = -\frac{1}{2}\ln|1-x^2| \qquad [0.5]$$

$$\int \tanh^{-1} x\, dx = x\tanh^{-1} x + \frac{1}{2}\ln(1-x^2) + C, \qquad |x| < 1 \qquad [0.5]$$

End of model answer — Total: 10 marks