مع الإجابة الكاملة

Quiz N.1 M.318, Second semester 1444(2022)

Name ______________________.

University number ______________________.

Question 1(4): Prove that $\mu(x,y) = xy$ is an integrating factor for the differential equation

$(-xysinx + 2ycosx)dx + 2xcosxdy = 0$ and solve it, where $xy \neq 0$.

Question 2(4): Write the following differential equation in the form Bernoulli 's equation and solve it. $5xy^2y' + y^3 = 32(1 + lnx)y^{-2}$, $x > 0$ and $y \neq 0$ on some interval I.

Question3(2): Solve the differential equation: $x^2y' + x(x+2)y = e^x$, where $x > 0$.

Complete solution of Quiz 1. M. 318

Second semester 1444 (2022)

Question 1: $xy[(-xy\sin x+2y\cos x)dx+2x\cos x\,dy=0]$, $x\cdot y\neq 0$

(4)

$$(-x^2y^2\sin x+2y^2x\cos x)\,dx+2x^2y\cos x\,dy=0 \quad (1)$$

$$\underbrace{-x^2y^2\sin x+2y^2x\cos x}_{M}\qquad \underbrace{2x^2y\cos x}_{N}$$

(1) $\left\{\begin{array}{l}\dfrac{\partial M}{\partial y}=-2x^2y\sin x+4yx\cos x\\[2mm] \dfrac{\partial N}{\partial x}=4xy\cos x-2x^2y\sin x\end{array}\right\}$ Then the D.E is exact.

So there exists a function F of x and y s.t

$$\frac{\partial F}{\partial x}=M=-x^2y^2\sin x+2y^2x\cos x$$

$$\frac{\partial F}{\partial y}=N=2x^2y\cos x,$$

$$F(x,y)=\int\frac{\partial F}{\partial y}(x,y)\,dy=\int 2x^2y\cos x\,dy$$

$$=x^2y^2\cos x+\phi(x) \quad (1)$$

$$\frac{\partial F}{\partial x}=2xy^2\cos x-\cancel{x^2y^2\sin x}+\phi'(x)=-\cancel{x^2y^2\sin x}+2xy^2\cos x$$

$$\phi'(x)=0 \Rightarrow \phi(x)=c$$

Then the solution of the D.E. is (1)

$$\boxed{F(x,y)=x^2y^2\cos x+C=0}$$

Question 2

(4) $5xy^2y'+y^3=32(1+\ln x)y^{-2}$, $x>0$, $y\neq 0$ on $(a,b)=I$

(1) $y'+\frac{1}{5x}y=\frac{32}{5x}(1+\ln x)y^{-4}$ is Bernoulli's D.E., $n=-4$

$y^4y'+\frac{1}{5x}y^5=\frac{32}{5x}(1+\ln x)$, we put $y^5=u$, $\frac{u'}{5}=y^4y'$ (1)

(2) $\left\{\begin{array}{l}\dfrac{u'}{5}+\dfrac{1}{5x}u=\dfrac{32}{5x}(1+\ln x) \text{ or}\\[2mm] u'+\dfrac{1}{x}u=\dfrac{32}{x}(1+\ln x) \text{ is Linear D.E}\\[2mm] \mu(x)=e^{\int\frac{dx}{x}}=e^{\ln x}=x\end{array}\right.$

$$u\mu(x)=\int\frac{32}{x}(1+\ln x)\cdot x\,dx=32\int(1+\ln x)\,dx$$

$ux = 32(x + x\ln x - x) = 32x\ln x + c$

$y^5 x - 32x\ln x = c$ or $x(y^5 - 32\ln x) = c$

is the solution of the D.E.

Question 3 $\quad x^2 y' + x(x+2)y = e^x, \quad x>0$

(2)

$y' + (1 + \frac{2}{x})y = \frac{1}{x^2}e^x; \quad x>0$ is Linear D.E.

(1) $\mu(x) = e^{\int (1+\frac{2}{x})dx}$

$= e^{x + \ln x^2} = x^2 e^x$

$y\mu(x) = x^2 e^x y = \int \frac{1}{x^2} e^x \cdot e^x \cdot x^2 dx = \int e^{2x} dx$

$x^2 e^x y = \frac{1}{2} e^{2x} + c$

(1) or $\boxed{y = \frac{1}{2x^2} e^x + \frac{1}{x^2} e^{-x} c}$ is the general solution of the D.E.

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Quiz 2 M.318 , section 81599 , second semester 2023

Name ______________________:

University number ______________________:

Question 1:

Solve the following differential equation: $y'' + 6y' + 9y = \frac{1}{x^3}e^{-3x}$, $x > 0$.

Question 2:

Find only the form of the particular solution for the differential equation:

$$y''' - y'' + 4y' - 4y = (2x+3)e^{x} + 5e^{-x} + x^2\cos(2x).$$

Question 1 $\quad y'' + 6y' + 9y = \frac{1}{x^3}e^{-3x}, \; x>0$

(6)

1) $y'' + 6y' + 9y = 0 \Rightarrow m^2 + 6m + 9 = (m+3)^2 = 0, \; m = -3, -3$

$y_c = c_1 e^{-3x} + c_2 x e^{-3x}, \quad y_1 = e^{-3x}, \quad y_2 = x e^{-3x}$

(2) $y_p = y_1 u_1 + y_2 u_2 = e^{-3x} u_1 + x e^{-3x} u_2$

So that $\quad y_1 u_1' + y_2 u_2' = e^{-3x} u_1' + x e^{-3x} u_2' = 0$

$y_1' u_1' + y_2' u_2' = -3e^{-3x} u_1' + (e^{-3x} - 3x e^{-3x}) u_2' = \frac{1}{x^3} e^{-3x}$

$$W = \begin{vmatrix} e^{-3x} & x e^{-3x} \\ -3e^{-3x} & e^{-3x} - 3x e^{-3x} \end{vmatrix} = e^{-6x}$$

$$u_1' = \frac{\begin{vmatrix} 0 & x e^{-3x} \\ \frac{1}{x^3} e^{-3x} & e^{-3x} - 3x e^{-3x} \end{vmatrix}}{W} = \frac{-\frac{1}{x^2} e^{-6x}}{e^{-6x}} = \frac{-1}{x^2} \Rightarrow \boxed{u_1 = \frac{1}{x}} \quad (2)$$

$$u_2' = \frac{\begin{vmatrix} e^{-3x} & 0 \\ -3e^{-3x} & \frac{1}{x^3} e^{-3x} \end{vmatrix}}{e^{-6x}} = \frac{1}{x^3} = x^{-3}, \quad \boxed{u_2 = \frac{-1}{2}\frac{1}{x^2}}$$

$$y_p = e^{-3x} \cdot \frac{1}{x} + x e^{-3x}\left(\frac{-1}{2x^2}\right) = \boxed{\frac{1}{2x} e^{-3x}} \quad (2)$$

$y = y_c + y_p$ ...

Question 2.

4

$$\dddot{y} - \ddot{y} + 4\dot{y} - 4y = (2x+3)e^{x} + 5e^{-x} + x^2\cos(2x)$$

1) $\dddot{y} - \ddot{y} + 4\dot{y} - 4y = 0 \Rightarrow m^3 - m^2 + 4m - 4 = 0$

$m^2(m-1) + 4(m-4) = 0$

1

$(m-1)(m^2+4) = 0$

$m = 1, \quad m = \mp i$

$$\boxed{y_c = c_1 e^{x} + c_2 \cos 2x + c_3 \sin 2x}$$

1 1

$y_p = (Ax+B)xe^{x} + Ce^{-x} + (A_1 + A_2 x + A_3 x^2)x\cos 2x$

1

$+ (B_1 + B_2 x + B_3 x^2)x\sin(2x)$

مع الإجابة الكاملة

Midterm exam M. 318, second semester 1444(2022-23)

Name ______________________________ :

University number ______________________ :

Question1: (10)

(4) a) Find the solution of the following differential equation:

$$xy^4dx + (2+y^2)e^{-3x}dy = 0.$$

(6) b) Find the integrating factor for the differential equation and solve it:

$$(x^2 - y^2 + x)dx + 2xydy = 0, \text{ where } xy \neq 0.$$

(10) **Question2:** (10)

a) Find the general solution of the differential equation:
(5) $xy'' + (1-2x)y' + (x-1)y = 0,\ where\ x > 0$ and $y_1 = e^x$ is a particular solution of the differential equation.

b) A culture has initially y_0 number of bacteria. After one hour the number of bacteria is measured to be $\frac{5}{2}y_0$. If the rate of growth is proportional to the number of bacteria $y(t)$, present at time t, then determine the time necessary for the number of bacteria to be quadruple. (5)

Question3: (10)

a) Find the general solution of the differential equation:

(4) $$y^{(4)} + 5y'' - 36y = 0.$$

b) Find the solution of the initial value problem:

(6) $$\begin{cases} x^2y'' - xy' + y = 0, & x < 0. \\ y(-1) = 1, & y(-1) = 0. \end{cases}$$

Complete solutions of Midterm 318 M.
Second semester 1444 (2023)

Question 1: (a) $xy^4dx + (2+y^2)e^{-3x}dy = 0$

2 $xe^{3x}dx + \frac{2+y^2}{y^4}dy = 0$, $\int xe^{3x}dx + \int(2y^{-4}+y^{-2})dy = 0$; $y \neq 0$

$x\cdot\frac{1}{3}e^{3x} - \frac{1}{3}\int e^{3x}dx + \left(-\frac{2}{3}y^{-3} - \frac{1}{y}\right) = 0$

2 $\boxed{\frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} - \frac{2}{3}y^{-3} - \frac{1}{y} = C}$

(b) $(\underbrace{x^2-y^2+x}_{M})dx + \underbrace{2xy}_{N}\,dy = 0$; $xy \neq 0$

$\frac{\partial M}{\partial y} = -2y$, $\frac{\partial N}{\partial x} = 2y$

$\frac{\partial M/\partial y - \partial N/\partial x}{N} = \frac{-4y}{2xy} = \frac{-2}{x}$. $\mu(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x}$

2 $\mu(x) = e^{\ln x^{-2}} = \frac{1}{x^2}$ is an integrating factor for the D.E.

$\frac{1}{x^2}\left((x^2-y^2+x)dx + 2xy\,dy = 0\right)$

2 $\underbrace{\left(1 - \frac{y^2}{x^2} + \frac{1}{x}\right)}_{M}dx + \overbrace{\frac{2y}{x}}^{N}dy = 0$

$\frac{\partial M}{\partial y} = \frac{-2y}{x^2} = \frac{\partial N}{\partial x}$, then the D.E is exact

So $\exists$ function F of x and y s.t

2 $\frac{\partial F}{\partial x} = 1 - y^2/x^2 + 1/x \qquad \frac{\partial F}{\partial y} = \frac{2y}{x}$

$F(x,y) \int \frac{2y}{x}dy = \frac{y^2}{x} + \phi(x)$

$\frac{\partial F}{\partial x} = \frac{-y^2}{x^2} + \phi'(x) = 1 - y^2/x^2 + 1/x$

$\phi(x) = x + \ln|x| + c$

Then the solution of the D.E is $F(x,y) = \boxed{\frac{y^2}{x} + x + \ln|x| + c = 0}$

Question ②:

ⓐ $x\ddot{y} + (1-2x)y' + (x-1)y = 0, \quad x>0,$

$y_1 = e^x$ is a particular solution of the D.E

2

$$\ddot{y} + \left(\frac{1-2x}{x}\right)y' + \frac{x-1}{x}y = 0$$

$$\ddot{y} + \left(\frac{1}{x}-2\right)y' + \frac{x-1}{x}y = 0, \quad P(x) = \left(\frac{1}{x}-2\right)$$

1

$$y_2 = y_1 \int \frac{e^{-\int P(x)dx}}{y_1^2}\,dx, \qquad e^{-\int P(x)dx} = e^{\int(-\frac{1}{x}+2)dx} = e^{\ln x^{-1} + 2x} = e^{2x}\cdot\frac{1}{x}$$

2

$$y_2 = e^x \int \frac{e^{2x}\cdot\frac{1}{x}dx}{e^{2x}} = e^x \int \frac{1}{x}dx = e^x \ln x$$

Then the general solution of the D.E. is given by

$$\boxed{y = c_1 e^x + c_2 e^x \ln x}$$

ⓑ $\frac{dy}{y} = k\,dt \Rightarrow \quad y(t) = c e^{kt}$

Let $y(0) = y_0$, then $y(1) = \frac{5}{2}y_0$

1 ✓ $y_0 = y(0) = c \Rightarrow y(t) = y_0 e^{kt}$

2 $y(1) = \frac{5}{2}y_0 = y_0 e^k \Rightarrow \frac{5}{2} = e^k$ or $k = \ln(5/2)$

$$\boxed{y(t) = y_0 e^{(\ln(5/2))t}}$$

Now, we have to find t so that $y(t) = 4y_0$

2

$$y(t) = 4y_0 = y_0 e^{(\ln(5/2))t}$$

$$\ln 4 = t\ln(5/2) \text{ or } t = \frac{\ln 4}{\ln(5/2)} \simeq \frac{1.386}{0.916}$$

$$t \simeq 1.513$$

$$\boxed{t \simeq 1.5 \text{ hour}}$$

Question ③ :

ⓐ $y^{(4)} + 5y'' - 36y = 0, \quad y = e^{mx}$

(2) $m^4 + 5m^2 - 36 = (m^2+9)(m^2-4) = 0$

$m^2 + 9 = 0 \Rightarrow m = \mp 3i, \quad m = \mp 2$

(2) $\boxed{y = c_1 \cos(3x) + c_2 \sin(3x) + c_3 e^{2x} + c_4 e^{-2x}}$

is the general solution of the D.E.

ⓑ $\begin{cases} x^2 y'' - x y' + y = 0 ; & x < 0 \\ y(-1) = 1, \quad y'(-1) = 0 \end{cases}$

$x^2 y'' - x y' + y = 0, \quad y = x^m, \quad x > 0$

$m(m-1) - m + 1 = 0$

(3) $m^2 - 2m + 1 = (m-1)^2 = 0, \quad m = 1, 1$

$y = c_1 x + c_2 x \ln x ; \quad x > 0$

(1) $y = c_1(-x) + c_2(-x)\ln(-x)$ or $\boxed{y = c_3 x + c_4 x \ln(-x) ; \quad x < 0}$

is the G. Solution of the D.E. for $x < 0$

$y' = c_3 + c_4 \ln(-x) + c_4$

But: $y(-1) = 1 \Rightarrow \quad -c_3 + 0 = 1 \Rightarrow c_3 = -1$ (1)

$y'(-1) = c_3 + 0 + c_4 = 0 \Rightarrow c_4 = -c_3 = 1 \Rightarrow c_4 = 1$ (1)

So the unique solution of the D.E is the curve

$$y = -x + x\ln(-x)$$

$$\boxed{y = x(\ln(-x) - 1)}$$
