Quiz 1 Model answer

$\boxed{1}$ $\underbrace{2+2=4}_{T}$ or $\underbrace{1+3=5}_{F}$

$T \vee F \equiv T$ (a)

$\boxed{2}$ If conditional statement is true, then (b) its contrapositive is true.

as $p \to q \equiv \neg q \to \neg p$

$\boxed{3}$ $\neg p \vee (p \vee q) \to q$

(c) Contingency

either by truth table (which is better and clear)

or

$$\neg p \vee (p \vee q) \to q \equiv (\neg p \vee p) \vee q \to q$$

$$\equiv (T \vee q) \to q$$

$$\equiv T \to q$$

$$\equiv \neg T \vee q$$

$$\equiv F \vee q$$

$$\equiv q$$

this contains T and F

So contingency.

$\boxed{4}$ $(p \wedge q) \to r$

$\equiv \neg(p \wedge q) \vee r$

$\equiv \neg p \vee \neg q \vee r$

$\equiv \neg p \vee \underbrace{(\neg q \vee r)}$

$\equiv p \longrightarrow (\neg q \vee r)$ (b)

$\boxed{5}$ $\neg\left[\forall x \in \mathbb{R}\ (x^2 \geq x \ \vee |x| < 1)\right]$

$\equiv \exists x \in \mathbb{R}\ (x^2 < x \ \wedge |x| \geq 1)$ (c)

$\boxed{6}$ $Q(x, y): \ x^2 = 2y^2 + 1$

$Q(3,2): \ 3^2 = 2(2)^2 + 1$

$9 = 2(4) + 1$ True (b)

$\boxed{7}$ $P(n)$: If n is non-negative, then $n^2 \geq n+1$

$\underline{P(0)}$: $0^2 = 0 \not\geq 0+1$

False (b)

$\boxed{8}$ If $\underbrace{x+y \geq z}_{p}$, then $\underbrace{x \geq 4}_{q}$

Inverse : $\neg p \longrightarrow \neg q$

If $x+y<z$, then $x<4$.

$\boxed{9}$ The statement "If $\underbrace{n \text{ is odd}}_{p}$, $\xrightarrow{}$ then $\underbrace{n^2 \text{ is odd}}_{q}$"

another way:

A necessary condition for $\underbrace{n \text{ is od}}_{p}$, is n^2 $\underbrace{\text{is odd}}_{q}$

(revise all equivalent expressions for $p \to q$ from Lecture).

$\boxed{10}$ $\forall n \in \mathbb{Z}$, [n is perfect square $\longrightarrow$ $(n+2)$ is a perfect square]

False (b)

Counter example

$n = 4$

$4 = 2^2$, So 4 is a perfect square.

but

$n+2 = 4+2 = 6$ is not a perfect square.

we can't write $6 = a^2$, $a \in \mathbb{Z}$.