

Quiz 3 with answer CSC 281

Question 1: (1 mark) Suppose that A and B are two nonempty and distinct sets. Determine the truth value of each of the following expressions.

Expression	True	False
$A \cap (B \cup \emptyset) = (A \cap B) \cup \emptyset$	✓	
$A - (B \cup \emptyset) = (A - B) \cap (A - \emptyset)$	✓	
If $A \subseteq B$, then $A \cap B = B$		✓
$A \times \{1\} = \{\{a, 1\}, a \in A\}$		✓

Question 2: (1 mark) Let $A = \{1, 2\}$. Determine the truth value of each of the following expressions.

Expression	True	False
$\{\{\}\} \in P(A)$		✓
$\emptyset \in P(A)$	✓	
$\{\{\}, \{2\}\} \subseteq P(A)$	✓	
$\emptyset \subseteq P(A)$	✓	

Question 3: (1.5 mark) Determine the solution(s) for each of the following recurrence relations.

1.

$$a_0 = 3, \quad a_n = 2a_{n-1}, \quad n \geq 1$$

- $a_n = 3 \cdot (n + 1)^2$
- $a_n = n^2 + 3$
- $a_n = 3 \cdot 2^n$ ✓
- $a_n = (n + 1)^2 + 2$

2.

$$a_n = n + 2, \quad n \geq 0$$

- $\sum_{k=0}^n a_k = \frac{(n+1)(n+3)}{2}$
- $\sum_{k=0}^n a_k = \frac{(n+1)(n+4)}{2} \checkmark$
- $\sum_{k=0}^n a_k = 2(n+1) + \frac{(n)(n+1)}{2} \checkmark$
- $\sum_{k=0}^n a_k = \frac{(n+2)(n+3)}{2}$

3.

$$a_0 = 4, \quad a_n = 2a_{n-1}, \quad n \geq 1$$

- $\sum_{k=0}^n a_k = 4(2^{n+1} - 1) \checkmark$
- $\sum_{k=0}^n a_k = 2(2^{n+1} - 2)$
- $\sum_{k=0}^n a_k = 2^{n+3} - 4 \checkmark$
- $\sum_{k=0}^n a_k = 2^{n+3} - 1$

Question 4: (1.5 mark) Select the correct answer for each of the following questions.

1. $f : \mathbb{N} \rightarrow \mathbb{N}, f(x) = x^2 + x$
 - one-to-one only \checkmark
 - onto only
 - one-to-one correspondence
 - neither
2. $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x^3 + 1$
 - one-to-one only
 - onto only
 - one-to-one correspondence \checkmark
 - neither
3. $f : \mathbb{Z} \rightarrow \mathbb{Z}, f(n) = n^3 + 1$
 - one-to-one only \checkmark
 - onto only
 - one-to-one correspondence
 - neither