

Name:..... Student Number:.....

The examination contain 2 pages.

Question 1: (1 mark) Determine the truth value of each the following propositions.

Proposition	Truth value
$(-1 + 2 = 5)$ if and only if $(-7 > 0)$	
$(1 > 5)$ only if $(7 < 2)$	
If $(3 + 1 = 4)$ or $(2 + 3 = 6)$, then $(2 + 4 = 6)$ and $(5 + 2 = 8)$	
$(3 = -3)$ whenever $(3 = 3)$	

Question 2: (1 mark) Give the correct answer in the following cases:

The conditional statement	is the ...
	converse of $p \rightarrow q$
	inverse of $p \rightarrow q$
	contrapositive of $p \rightarrow q$
	contrapositive of $\neg q \rightarrow p$

Question 3: (2 marks) Let $P(x, y)$ be the predicate " $x + 3y = 4x$ ". Determine the truth value of each the following statements.

Statement	Truth value
$P(1, 1)$	
$\exists x \in \mathbb{Z}, P(x, 1)$	
$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x, y)$	
$\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x, y)$	
$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x, y)$	
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x, y)$	

Question 4: (1 mark) Find the negation of the following propositions:

1. " $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}$ (if $x > 0 \wedge y > 0$, then $x.y > 0$)".

Answer:

2. " $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ ($x \star y = 0$)".

Answer:

3. " $\exists x \in \mathbb{R}, \forall y \in \mathbb{R}$ ($x \star y = y$)".

Answer:

Solutions

Question 1: (1 mark) Determine the truth value of each the following propositions.

Proposition	Truth value
$(-1 + 2 = 5)$ if and only if $(-7 > 0)$	True
$((1 > 5)$ only if $(7 < 2)) \equiv ((1 > 5) \rightarrow (7 < 2))$	True
If $(3 + 1 = 4)$ or $(2 + 3 = 6)$, then $(2 + 4 = 6)$ and $(5 + 2 = 8)$	False
$((3 = -3)$ whenever $(3 = 3)) \equiv ((3 = 3) \rightarrow (3 = -3))$	False

Question 2: (1 mark) Give the correct answer in the following cases:

The conditional statement	is the ...
$q \rightarrow p$	converse of $p \rightarrow q$
$\neg p \rightarrow \neg q$	inverse of $p \rightarrow q$
$\neg q \rightarrow \neg p$	contrapositive of $p \rightarrow q$
$\neg p \rightarrow q$	contrapositive of $\neg q \rightarrow p$

Question 3: (2 marks) Let $P(x, y)$ be the predicate " $x + 3y = 4x$ ". Determine the truth value of each the following statements.

Statement	Truth value
$P(1, 1) : 1 + 3(1) = 4(1)$	True
$\exists x \in \mathbb{Z}, P(x, 1)$	True
$\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x, y)$	True
$\exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, P(x, y)$	True
$\forall x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x, y)$	False
$\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, P(x, y)$	False

Question 4: (1 mark) Find the negation of the following propositions:

1. " $\forall x \in \mathbb{R}, \forall y \in \mathbb{R}$ (if $x > 0 \wedge y > 0$, then $x \cdot y > 0$)".

Answer: $\exists x \in \mathbb{R}, \exists y \in \mathbb{R} ((x > 0 \wedge y > 0) \wedge x \cdot y \leq 0)$

2. " $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} (x \star y = 0)$ ".

Answer: $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} (x \star y \neq 0)$

3. " $\exists x \in \mathbb{R}, \forall y \in \mathbb{R} (x \star y = y)$ ".

Answer: $\forall x \in \mathbb{R}, \exists y \in \mathbb{R} (x \star y \neq y)$