Mid Term I Examination, October, 2025
Dept. Math. College of Science
King Saud University
Math: 107 Full Marks: 25 Time: 90 Minutes

Q 1. [Marks: 4] Solve the following linear system using the Gauss-Jordan elimination methods.

$$\begin{aligned} x + 2y &= 3 \\ -x - 2z &= -5 \\ -3x - 5y + z &= -4 \end{aligned}$$

Q 2.[Marks: 4] Use Gaussian elimination method to find the values of α for which the following linear system has infinitely many solutions.

$$\begin{aligned} x - y + 2z &= 0 \\ 2x + 2y + z &= 0 \\ \alpha y - \frac{3}{2}z &= 0 \end{aligned}$$

Q 3. [Marks: 4+1=5] Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

Use elementary row operations to determine A^{-1}. Then use A^{-1} to compute $(A^T)^{-1}$.

Q 4. [Marks: 4] What conditions must a, b, and c satisfy in order for the system of equations

$$\begin{aligned} x - 2y + 5z &= a \\ 4x - 5y + 8z &= b \\ 3x - 3y + 3z &= c \end{aligned}$$

to be consistent?

Q 5. [Marks: 4] Find $det(A)$ (determinant of A) if

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 2 & 3 & 4 \\ 2 & 4 & -1 & 3 \\ 0 & 3 & -2 & 0 \end{bmatrix}$$

Q 6. [Marks: 4] For which values of k, A is invertible, where

$$A = \begin{bmatrix} 1 & k & k^2 \\ k & k & k^2 \\ 1 & 1 & k^2 \end{bmatrix}$$

SOLUTION MID TERM I, OCT 7, 2025

Dept. Math., College of Science, KSU

Math: 107 Full Mark: 25 Time: 90 Minutes

Q1. [4] Upon using augmented martrix of the given system, we get

$$\begin{pmatrix} 1 & 2 & 0 & 3 \\ -1 & 0 & -2 & -5 \\ -3 & -5 & 1 & -4 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{pmatrix}$$

$$\Rightarrow x = -1, y = 2, z = 3$$

Q2. [4] Considering the augmented matrix of the given system, we have

$$\begin{pmatrix} 1 & -1 & 2 & 0 \\ 2 & 2 & 1 & 0 \\ 0 & \alpha & -\frac{3}{2} & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -1 & 2 & 0 \\ 0 & 1 & -\frac{3}{4} & 0 \\ 0 & 0 & \frac{3\alpha-6}{4} & 0 \end{pmatrix}$$

$$\Rightarrow 3\alpha - 6 = 0 \Rightarrow \alpha = 2$$

Q3 [4+1=5] Considering the form $(A|I)$, we have the following

$$\begin{pmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 3 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 0 & -1/8 & 1/8 & 3/8 \\ 0 & 1 & 0 & 0 & 4/8 & 0 \\ 0 & 0 & 1 & 3/8 & -3/8 & -1/8 \end{pmatrix}$$

which is in the form $(I|A^{-1})$, where

$$A^{-1} = \frac{1}{8} \begin{pmatrix} -1 & 1 & 3 \\ 0 & 4 & 0 \\ 3 & -3 & -1 \end{pmatrix}$$

and

$$\left(A^T\right)^{-1} = (A^{-1})^T = \frac{1}{8} \begin{pmatrix} -1 & 0 & 3 \\ 1 & 4 & -3 \\ 3 & 0 & -1 \end{pmatrix}$$

Q4. [4]

$$\begin{pmatrix} 1 & -2 & 5 & a \\ 4 & -5 & 8 & b \\ 3 & -3 & 3 & c \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & -2 & 5 & a \\ 0 & 1 & -4 & \frac{-4a+b}{3} \\ 0 & 0 & 0 & a-b+c \end{pmatrix}$$

Therefore, the system of linear equations is consistent if and only if $b = a + c$

Q5. [4] $\det(A) = 22$.

Q6. [4] A is invertible if and only if $\det(A) \neq 0$ if and only if $k^2(k-1)(k-1) \neq 0$ if and only if $k \neq 0$, $k \neq 1$.