King Saud University, College of Science
Department of Mathematics
Second Semester 1447 H - (2025 – 2026 G)

First Quiz of Math 431.
Allotted time: Half hour

Exercise (5 points)

1. How many edges does a graph have if the sum of the vertex degrees is 48?
2. How many edges does a 2-regular graph on 14 vertices have?
3. A graph has 47 edges. What is the minimum possible number of vertices?
4. How many edges does the complement of $K_{2,3}$ have?
5. Let n and m be positive integers. How many edges does the complement of $K_{n,m}$ have?

Solutions

1. By the Handshaking Lemma, $\sum \deg(v) = 2|E|$.
   Thus, $2|E| = 48 \Rightarrow |E| = 24$.
2. In a 2-regular graph, each vertex has degree 2. Sum of degrees: $14 \times 2 = 28 = 2|E|$.
   Hence, $|E| = 14$.
3. The maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$. We need $\frac{n(n-1)}{2} \geq 47$.
   First method: Testing: $n = 9 \Rightarrow 36$ edges (not enough)
   $n = 10 \Rightarrow 45$ edges (not enough)
   $n = 11 \Rightarrow 55$ edges (enough).

   Minimum number of vertices: $\boxed{n = 11}$.

   Second method: The inequality
   $$\frac{n(n-1)}{2} \geq 47$$
   is equivalent to
   $$n^2 - n - 94 \geq 0.$$

   Solving the quadratic inequality
   $$n^2 - n - 94 \geq 0,$$
   we compute the discriminant:
   $$\Delta = (-1)^2 + 4 \cdot 94 = 377$$
   Since n must be a positive integer. Hence,
   $$n \geq \frac{1+\sqrt{377}}{2}.$$
   Therefore, the smallest possible value is $\boxed{n = 11}$.
   We can obtain such a graph by starting from the complete graph K_{11} and deleting $55-47 = 8$ edges. Hence,
   $$G = K_{11} \setminus \{e_1, \ldots, e_8\}.$$

4. The graph $K_{2,3}$ has $2 \times 3 = 6$ edges. Total vertices: 5.

   A complete graph on 5 vertices has $\binom{5}{2} = 10$ edges.

   Complement edges: $10 - 6 = 4$.

5. Total vertices: $n + m$. Edges in a complete graph: $\binom{n+m}{2}$.

   Edges in $K_{n,m}$: nm.
   Thus the complement of $K_{n,m}$ has the number of edges
   $\binom{n+m}{2} - nm = \frac{(n+m)(n+m-1)}{2} - nm = \frac{n(n-1)}{2} + \frac{m(m-1)}{2}$.