King Saud University, College of Science
Department of Mathematics
QUIZ 1447 H - (2025 − 2026 G)

First Quiz of Math 431**.**
Allotted time: Half hour

Exercise A security code consists of 4 digits, chosen from the digits 0 to 9.

1. How many different codes can be formed if repetition is allowed?
2. How many different codes can be formed if repetition is not allowed?
3. (a) How many different codes can be formed if repetition is not allowed and the code must be an even number?
   (b) How many different even codes can be formed if repetition is not allowed?

- **Solution:**

1. **Repetition allowed:**

$$10 \times 10 \times 10 \times 10 = 10^4 = 10,000 \text{ codes.}$$

2. **Repetition not allowed:**

$$P(10,4) = \frac{10!}{(10-4)!} = 10 \times 9 \times 8 \times 7 = 5,040 \text{ codes.}$$

3. **Repetition not allowed and even code:**
   The interpretation of "code" and "number" is crucial here, specifically regarding whether a leading zero is permitted.
   (a) (Interpretation A: "Even number" - Leading zero NOT allowed)
   When forming a 4-digit *number*, we assume the first digit cannot be zero (e.g., 0132 is treated as 132). We use case analysis based on the last digit:
   Case 1: The code ends in 0
   - 4th digit (last): 1 choice (0)
   - 1st digit: 9 choices (1-9)
   - 2nd digit: 8 choices (remaining)
   - 3rd digit: 7 choices (remaining)
   - Total Case 1: $9 \times 8 \times 7 \times 1 = 504$

   Case 2: The code ends in a non-zero even digit (2, 4, 6, or 8)
   - 4th digit (last): 4 choices (2, 4, 6, 8)
   - 1st digit: 8 choices (cannot be 0 and cannot be the last digit)
   - 2nd digit: 8 choices (remaining)
   - 3rd digit: 7 choices (remaining)
   - Total Case 2: $8 \times 8 \times 7 \times 4 = 1792$

   Total for this question (a): $504+1792 =$ **2296** different "even numbers" (codes).
   (b) (Interpretation B: "Even code" - Leading zero IS allowed)
   If the sequence is a "code" (like a PIN), leading zeros are usually permitted (e.g., 0132 is a valid code).
   - 4th digit (last): Must be even (0, 2, 4, 6, 8). 5 choices.
   - 1st digit: Any digit is allowed except the one used last. 9 choices.
   - 2nd digit: Two digits are used. 8 choices.
   - 3rd digit: Three digits are used. 7 choices.

   Total for this question (b): $9 \times 8 \times 7 \times 5 =$ **2520** different "even codes".