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What conditions must a , b , c , and d satisfy for the following system to be consistent?

$$\begin{aligned}x_1 + x_2 - x_4 &= a \\x_2 - x_3 - 2x_4 &= b \\2x_1 + 2x_3 + 2x_4 &= c \\2x_1 + x_2 + x_3 &= d.\end{aligned}$$

Solution: Since $\left[\begin{array}{cccc|c} 1 & 1 & 0 & -1 & a \\ 0 & 1 & -1 & -2 & b \\ 2 & 0 & 2 & 2 & c \\ 2 & 1 & 1 & 0 & d \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & -1 & a \\ 0 & 1 & -1 & -2 & b \\ 0 & 0 & 0 & 0 & c - 2a + 2b \\ 0 & 0 & 0 & 0 & d - 2a + b \end{array} \right]$, the given system would be consistent for real numbers a, b, c, d satisfying the conditions $c = 2(a - b)$; $d = c + b$.