

Q#5:

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{bmatrix}$. Show that $\det(B) = -\det(A)$.

Answer:

$$\begin{aligned} |B| &= \begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & c \end{vmatrix} \xrightarrow[\substack{aR_1 \\ bR_2 \\ cR_3}]{\frac{1}{abc}} \begin{vmatrix} a & abc & a^2 \\ b & bca & b^2 \\ c & abc & c^2 \end{vmatrix} \\ &= \frac{1}{abc} \begin{vmatrix} a & b & c \\ abc & abc & abc \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow{\frac{1}{abc} R_2} \\ &= \frac{1}{abc} \begin{vmatrix} a & b & c \\ 1 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix} \xrightarrow{R_{12}} \\ &= - \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = -|A| \end{aligned}$$