Problem 1 : Consider the following matrices:

$$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} ; \; B = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -3 \end{bmatrix} ; C = \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 0 & -2 \end{bmatrix} ; \; D = \begin{bmatrix} 1 & -3 \\ -1 & -4 \end{bmatrix}$$

Compute the following :

- (AC)
- (BC)
- (D – A^T)

Solution 1 :

$$AC = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 0 & -2 \end{bmatrix} \; \textit{not possible because } \; A_{2x2} \; \textit{and} \; C_{3x2}$$

$$BC = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 3 & -3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 6 & 9 \end{bmatrix}$$

$$D - A^T = \begin{bmatrix} 1 & -3 \\ -1 & -4 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} -2 & 9 \\ 0 & -2 \end{bmatrix}$$

Problem 2 : Express the inverse of the following nonsingular matrix as products of elementary matrices:

$$\begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & 4 \\ 5 & 8 \end{bmatrix} \overset{(1/2)R_1}{\sim} \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix} \underset{R_2 - 5R_1}{\sim} \begin{bmatrix} 1 & 2 \\ 0 & -2 \end{bmatrix} (-1/2)R_2$$
$$\sim \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \overset{R_1 - 2R_2}{\sim} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then we can construct a sequence of elementary matrices $E_4, \ldots, E_1$ such that $E_4 \cdots E_1 A = I$ as follows:

The first row operation is $(1/2)R_1$, so $E_1 = \begin{bmatrix} 1/2 & 0 \\ 0 & 1 \end{bmatrix}$.

The second row operation is $R_2 - 5R_1$, so $E_2 = \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}$.

The third row operation is $(-1/2)R_2$, so $E_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix}$.

The fourth row operation is $R_1 - 2R_2$, so $E_4 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

Then $A^{-1} = E_4 E_3 E_2 E_1$

Problem 3 : Use the inversion method to find the inverse of the following nonsingular matrix:

$$\begin{bmatrix} -2 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution:

$$\left[\begin{array}{ccc|ccc} -2 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$ ← Adjoined the identity matrix to the coefficient matrix

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ -2 & 0 & 1 & 1 & 0 & 0 \end{array}\right]$$ ← Interchanged the first and third rows

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 2 \end{array}\right]$$ ← Added twice the first row to the third row

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 2 & 1 & 1 & 0 & 2 \end{array}\right]$$ ← Multiplied the second row by -1

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & -1 & 1 & 2 & 2 \end{array}\right]$$ ← Added -2 times the second row to the third row

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & -2 & -2 \end{array}\right]$$ ← Multiplied the third row by -1

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 & -2 & -2 \end{array}\right]$$ ← Added -1 times the third row to the second row

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & -1 & -2 & -2 \end{array}\right]$$ ← Added -1 times the second row to the first row

Thus the inverse is $\begin{bmatrix} -1 & -1 & -1 \\ 1 & 1 & 2 \\ -1 & -2 & -2 \end{bmatrix}$.

Question 4: Choose the correct answer

1- $If\ \ A = \begin{bmatrix} 5 \\ 3 \\ -2 \end{bmatrix}\ and\ \ B = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}\ \ \rightarrow (A + 2B)^T$ is

a) $\begin{bmatrix} 0 & 4 & 0 \end{bmatrix}$
b) $\begin{bmatrix} 3 & 7 & 0 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$
d) $\begin{bmatrix} 1 & 2 & -2 \end{bmatrix}$

2- If AB=I, So B must equal to

a) -A
b) A^T
c) A^{-1}
d) I

3- $If\ \ A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{bmatrix}\ and\ \ B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}\ \ \rightarrow (AB)\ is$

a) $\begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}$
b) $\begin{bmatrix} 3 \\ 7 \\ -3 \end{bmatrix}$
c) $\begin{bmatrix} 1 & 7 & -3 \end{bmatrix}$
d) $\begin{bmatrix} 0 & 2 & -1 \end{bmatrix}$

4- $If\ \ A = \begin{bmatrix} -2 & 4 \\ 0 & 1 \end{bmatrix}\ \rightarrow\ A^{-1}$ is

a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
b) $\begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix}$
c) The matrix A is singular
d) $\begin{bmatrix} -\frac{1}{2} & 2 \\ 0 & 1 \end{bmatrix}$