

Revision Sheet for Midterm

Math 481 – Riemann Integration and Sequences of Functions

Exercise 1:

Determine whether each function is Riemann integrable on $[0, 1]$.

1.

$$f(x) = \begin{cases} 1, & x < 1/2, \\ 0, & x \geq 1/2 \end{cases}$$

2.

$$f(x) = \begin{cases} 1, & x \in \mathbb{Q}, \\ 0, & x \notin \mathbb{Q} \end{cases}$$

3. $f(x) = x^2$

Exercise 2:

Write the following limits as integrals:

1. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{n^2}$

2. $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n} \frac{1}{1 + (k/n)^2}$

Exercise 3:

Determine the pointwise limit and uniform convergence:

1. $f_n(x) = x^n$ on $[0, 1]$

2. $f_n(x) = \frac{x}{n}$ on $[0, 1]$

3. $f_n(x) = \frac{1}{1+nx}$ on $[0, 1]$

Exercise 4:

Let

$$f_n(x) = \begin{cases} n, & 0 \leq x \leq \frac{1}{n} \\ 0, & \text{otherwise} \end{cases}$$

1. Find the pointwise limit.
2. Compute $\int_0^1 f_n(x) dx$.
3. Does $\int f_n \rightarrow \int f$?
4. Is the convergence uniform?