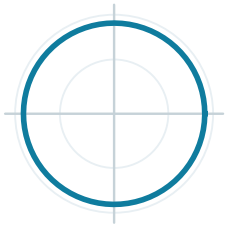
Polar Curves — A Reference Gallery

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Lines and circles

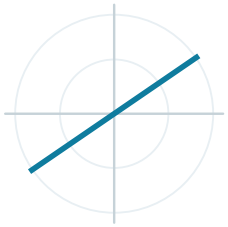
the simplest polar equations

$r = a$



radius $|a|$, centre at the pole

$\theta = \alpha$



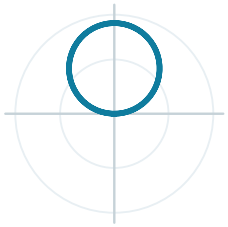
a whole line, not a ray

$r = 2a\cos\theta$



radius $|a|$, centre $(a, 0)$

$r = 2a\sin\theta$



radius $|a|$, centre $(0, a)$

Cardioids

$r = a(1 \pm \cos\theta)$ *and* $r = a(1 \pm \sin\theta)$ — *the cusp sits at the pole*

$r = a(1 - \cos\theta)$



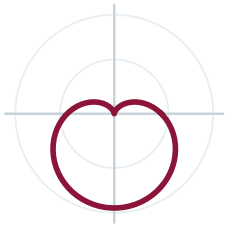
opens left

$r = a(1 + \cos\theta)$



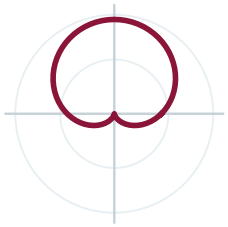
opens right

$r = a(1 - \sin\theta)$



opens down

$r = a(1 + \sin\theta)$

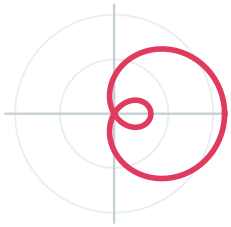


opens up

Limaçons

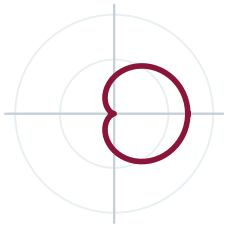
$r = a + b\cos\theta$ *and* $r = a + b\sin\theta$ — *the ratio $|a/b|$ decides the shape*

$|a/b| < 1$



inner loop

$|a/b| = 1$



cardioid

$1 < |a/b| < 2$



dimpled

$|a/b| \geq 2$



convex, no dimple

Reading an equation before you draw it

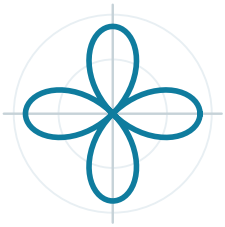
$r = a$	circle about the pole
$\theta = \alpha$	line through the pole
$r = 2a\cos\theta,\ r = 2a\sin\theta$	circle through the pole
$r = a \pm a\cos\theta,\ r = a \pm a\sin\theta$	cardioid
$r = a \pm b\cos\theta,\ r = a \pm b\sin\theta$	limaçon (loop when $\|a\| < \|b\|$)
$r = a\cos n\theta,\ r = a\sin n\theta$	rose: n petals if n odd, $2n$ if n even
$r^2 = a^2\cos 2\theta,\ r^2 = a^2\sin 2\theta$	lemniscate
$r = a\theta$	spiral of Archimedes

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Roses

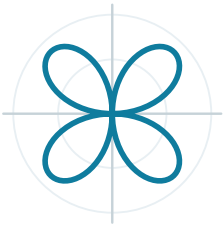
$r = a\cos n\theta$ and $r = a\sin n\theta$ — n petals if n is odd, $2n$ petals if n is even

$r = a\cos 2\theta$



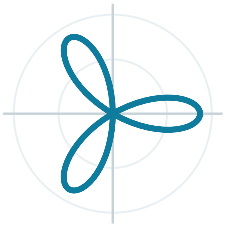
4 petals

$r = a\sin 2\theta$



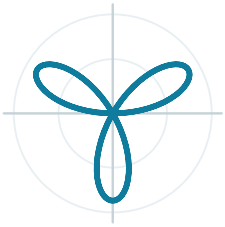
4 petals, rotated

$r = a\cos 3\theta$



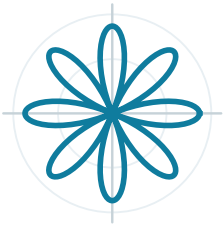
3 petals

$r = a\sin 3\theta$



3 petals, rotated

$r = a\cos 4\theta$



8 petals

$r = a\sin 5\theta$

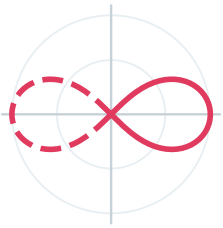


5 petals

Lemniscates and spirals

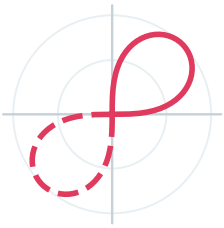
a lemniscate has no points where the right-hand side is negative

$r^2 = a^2\cos 2\theta$



along the x-axis

$r^2 = a^2\sin 2\theta$



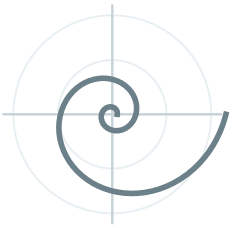
along $\theta = \pi/4$

$r = \theta,\ \theta \geq 0$



spiral of Archimedes

$r = e^{\theta/4}$



logarithmic (for interest)

Symmetry tests

a test that passes proves the symmetry; a test that fails proves nothing

replace θ by $-\theta$	symmetric about the x-axis
replace θ by $\pi - \theta$	symmetric about the y-axis
replace r by $-r$	symmetric about the pole

Recognising the four examples

$r = 4\sin\theta$	circle, radius 2, centre (0, 2)	$r = 2 + 4\cos\theta$	limaçon with an inner loop
$r = 2 + 2\cos\theta$	cardioid, opening right	$r = a\sin 2\theta$	four-leafed rose

To find the tangent lines at the pole, set r = 0 and solve for θ