### **Band Theory and Electronic Properties of Solids**

## Phys 674

Physics & Astronomy King Saud University 2<sup>nd</sup> Term: 2025

Week No. 02

The Drude Theory of Metals Part II

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## The Drude Theory of Metals AC ELECTRICAL CONDUCTIVITY OF A METAL \* we know that E(F) E(t) = E.cos(W++4) - - () Lamplitude Laphase angle $\mathcal{A}t \quad cos(wt+\phi) = Re\left[e^{i(wt+\phi)}\right] - - \Im$ =) E(H) = Re[E(W) e] -- (3) only real Partran Correspond to measurable quantities.

" E is  $f(t) \Rightarrow$ Ê = - e E(t) - -(4) $V = \frac{dP}{P} = - - - C$  $\Rightarrow \frac{de}{dt} = -e E(t) - -E$ Now: add Cellisrens effect -> momentum Relaxation:  $= \int \frac{dP}{dF} = -\frac{P}{T} - e\vec{E}(t) - -\vec{P}$ 

To calculate the current induced in a metal by a time-dependent electric field, we write the field in the form:

 $\mathbf{E}(t) = \operatorname{Re}\left[\mathbf{E}(\omega) e^{-i\omega t}\right]$ (1.23) $\therefore \frac{d\mathbf{p}}{dt} = -\frac{\mathbf{p}}{\tau} - e\mathbf{E}$ (1.24)with steady state solution :  $\mathbf{p}(t) = \operatorname{Re}\left[\mathbf{p}(\omega) e^{-i\omega t}\right]$ (1.25)put complex quantities:  $\mathbf{p}(\omega) e^{-i\omega t}$ and  $\mathbf{E}(\omega) e^{-i\omega t}$  in (1.24):  $-i\,\omega\mathbf{p}(\omega)\,\mathrm{e}^{-i\,\omega t}=-\frac{\mathbf{p}(\omega)}{\tau}\,\mathrm{e}^{-i\,\omega t}-e\mathbf{E}(\omega)\,\mathrm{e}^{-i\,\omega t}$  $\therefore i \,\omega \mathbf{p}(\omega) = \frac{\mathbf{p}(\omega)}{\tau} + e\mathbf{E}(\omega)$ (1.26)

(1.27)

(1.28)

• we then express the current density **j**:

$$\therefore \mathbf{j} = -\frac{ne\mathbf{p}}{m}$$
  

$$\mathbf{j}(t) = \operatorname{Re}\left[\mathbf{j}(\omega) e^{-i\omega t}\right]$$
  

$$\therefore \mathbf{j}(\omega) = -\frac{ne\mathbf{p}(\omega)}{m}$$
  

$$\therefore i\omega\mathbf{p}(\omega) = \frac{\mathbf{p}(\omega)}{\tau} + e\mathbf{E}(\omega)$$
  

$$\Rightarrow \mathbf{p}(\omega)\left[i\omega - \frac{1}{\tau}\right] = e\mathbf{E}(\omega)$$
  

$$\Rightarrow \mathbf{p}(\omega) = \frac{e\tau \mathbf{E}(\omega)}{i\omega\tau - 1}$$
  

$$\therefore \mathbf{j}(\omega) = -\frac{ne}{m}\left(\frac{e\mathbf{E}(\omega)}{i\omega - \frac{1}{\tau}}\right) = \frac{\tau ne^{2}\mathbf{E}(\omega)}{m(1 - i\omega\tau)}$$

### • One can rewrite (1.29)as:

$$\mathbf{j}(\omega) = \sigma(\omega)\mathbf{E}(\omega) \qquad (1.28)$$

$$\sigma(\omega) = \frac{\tau n e^2}{m(1 - i\omega\tau)} = \frac{\sigma_o}{1 - i\omega\tau} \qquad (1.29)$$

$$\mathbf{j} = \frac{\eta e^2 \tau}{\omega} \in (DC)$$

So, this result lead to (1.5) when ω=0 (DC current)
 Therefore if the field does not vary appreciably over distances comparable to the electronic mean free path, we may correctly calculate j(r, t), the current density at r, by taking the field everywhere in space to be given by its value E(r, t) at the point r.

 $\mathbf{j}(\mathbf{r},\omega) = \sigma(\omega)\mathbf{E}(\mathbf{r},\omega) \tag{1.30}$ 

 $\Box$  is therefore valid if  $\lambda$  of the field is large compared to the electronic mean free path  $\ell$ .

Assuming, that the wavelength is large compared to the mean free path, we may write Maxwell's equations as:

 $\nabla \cdot \mathbf{E} = 0 \qquad (1.32)$   $\nabla \cdot \mathbf{H} = 0 \qquad (1.33)$   $\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} \qquad (1.34)$   $\nabla \times \mathbf{H} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \qquad (1.35)$ 

 $\Box$  We look for a solution with time dependent  $e^{-i\omega t}$  and use (1.30)

$$\nabla \times (\nabla \times \mathbf{E}) = -\nabla^2 \mathbf{E} = -\frac{1}{c} \frac{\partial}{\partial t} \nabla \times \mathbf{H}$$
$$= -\frac{1}{c} \frac{\partial}{\partial t} \left( \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \right)$$
(1.36)

## AC ELECTRICAL CONDUCTIVITY OF A METAL

$$\therefore -\nabla^{2}\mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t} \left(\frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}\right)$$
$$= -\frac{1}{c}\frac{\partial}{\partial t} \left[\frac{4\pi}{c}\sigma(\omega)\mathbf{E}(\omega) + \frac{1}{c}\frac{\partial\mathbf{E}(\omega)}{\partial t}\right]$$

using  $e^{-i\omega t}$  for **E**:

$$\therefore -\nabla^{2} \mathbf{E} e^{-i\omega t} = -\frac{4\pi}{c^{2}} \sigma(\omega) \frac{\partial}{\partial t} \mathbf{E}(\omega) e^{-i\omega t} - \frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \mathbf{E} e^{-i\omega t}$$

$$= \left[ \frac{4\pi i \sigma \omega}{c^{2}} \mathbf{E} + \frac{\omega^{2}}{c^{2}} \mathbf{E} \right] e^{-i\omega t}$$

$$\therefore -\nabla^{2} \mathbf{E} = \frac{\omega^{2}}{c^{2}} \left[ 1 + \frac{4\pi i \sigma}{\omega} \right] \mathbf{E}$$

$$(1.37)$$

$$\det \left( 1 + \frac{4\pi i \sigma}{\omega} \right) = \varepsilon(\omega)$$

$$(1.38)$$

$$\therefore -\nabla^{2} \mathbf{E} = \frac{\omega^{2}}{c^{2}} \varepsilon(\omega) \mathbf{E}$$

$$(1.39)$$

 $\epsilon$  is the dielectric constant of the medium

**\Box** For high frequency:  $\omega \tau >> 1$ :

$$\because \sigma(\omega) = \frac{\tau n e^2}{m(1 - i\omega\tau)} = \frac{\sigma_o}{1 - i\omega\tau}$$

$$(1.30)$$

$$\because \varepsilon(\omega) = 1 + \frac{4\pi i\sigma}{\omega}$$

$$\therefore \varepsilon(\omega) = 1 + \frac{4\pi i}{\omega} \frac{\tau n e^2}{m(1 - i\omega\tau)} = 1 + \frac{4\pi n e^2}{m} \frac{i\tau}{\omega(1 - i\omega\tau)} = 1 + \frac{4\pi n e^2}{m} \frac{i\tau}{\omega - i\omega^2\tau}$$

$$\varepsilon(\omega) \approx 1 - \frac{4\pi n e^2}{m} \frac{1}{\omega^2}$$

$$OR : \varepsilon(\omega) \approx 1 - \frac{\omega_p^2}{\omega^2}$$

$$(1.40)$$

$$with : \omega_p^2 = \frac{4\pi n e^2}{m}$$

$$(1.41)$$

### $\Box$ where $w_p$ is the plasma frequency.

- When ε is real and negative (ω < ω<sub>p</sub>) the solutions to (1.39) decay exponentially in space: i.e., no radiation can propagate
   when ε is positive (ω > ω<sub>p</sub>) the solutions to (1.39) become oscillatory, radiation can propagate, and the metal should become transparent.
- □ A second important consequence of (1.40) is that the electron gas can sustain charge density oscillations. By this we mean a disturbance in which the electric charge density has an oscillatory time dependence  $e^{-i\omega t}$

□ From the equation of continuity:

$$:\nabla \cdot \mathbf{j} = -\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial t} \rho(\omega) e^{-i\omega t}$$
$$\therefore \nabla \cdot \mathbf{j} = i \,\omega \rho(\omega)$$

(1.42)

 $E(\bar{r},t) = E_{o} e^{i(\mu r - \omega t)}$ 

- r: direction of propagation E: Ourplitude
- W: ang. Rreg.
- Laplación solution to D.

$$\nabla^2 E = -\kappa^2 E - - \cdot 2$$

$$eq. (1.39) \circ - \nabla^2 E = \frac{\omega^2}{c_2} \in (\omega) E - \cdot 3$$

() in () [1.35] ?  $W^{2}E = \frac{\omega^{2}}{C} \mathcal{E}(\omega)E$ - - (4  $\implies k^2 = \frac{\omega}{c^2} \in (\omega) - \bigcirc$ we have  $E(cw) = 1 - \frac{wp}{w} - - G[1.4c]$ if we will a second on -- (7) ⇒ K' is -true = K is purely imaginary  $\chi = \sqrt{-4^2}$ -(8)K=id



## AC ELECTRICAL CONDUCTIVITY OF A METAL

### • with Gauss's law:

$\nabla \mathbf{E}(\omega) = 4\pi\rho(\omega)$	(1.43)
$\because \mathbf{j}(\boldsymbol{\omega}) = \boldsymbol{\sigma}(\boldsymbol{\omega}) \mathbf{E}(\boldsymbol{\omega})$	(1.30)
$:: \nabla . \mathbf{j} = i  \omega \rho(\omega)$	(1.42)

 $\therefore \nabla \cdot \mathbf{j} = \sigma(\omega) \nabla \cdot \mathbf{E}$   $= \sigma(\omega) 4\pi\rho(\omega)$   $\therefore i \, \omega \rho(\omega) = \sigma(\omega) 4\pi\rho(\omega)$   $\therefore i \, \omega = 4\pi\sigma(\omega)$   $\Rightarrow i \, \omega - 4\pi\sigma(\omega) = 0$   $\Rightarrow 1 + \frac{4\pi i \, \sigma(\omega)}{\omega} = 0$ 

(1.44)

All quantities in (1.44) are constants except σ (reciprocal of which is the charge density). Hence, this eq. means that charge density wave is propagating (same condition as above)

As final Points in AC conductivity: \* He agree with experiments : \* ()  $O(w) \neq \frac{1}{1+w^2 \tau^2}$ He agree with ex. in moderate to high freq. W @ Real @ Imaginary parts:  $G(\omega) = \frac{\sigma_0}{1+\omega^2 c^2} + i \frac{\sigma_0 \omega t}{1+\omega^2 c^2}$ Resistive Reactive agree with exp. 3 High- Rieg. at  $w >> \frac{1}{\tau} = 3 G(w) \propto \frac{1}{c_{12}}$ 

## AC ELECTRICAL CONDUCTIVITY OF A METAL

### **Summary of Agreement and Disagreement**

Aspect	Agreement	Disagreement
Frequency Dependence	Predicts $\sigma(\omega) \propto rac{1}{1+\omega^2  au^2}$ , matching experiments at moderate frequencies.	Fails to capture interband transitions and quantum effects at high frequencies.
Complex Conductivity	Separates real and imaginary components, consistent with experimental trends.	Fails to fully explain the detailed phase and magnitude in complex systems.
Low-Temperature Behavior	Provides a qualitative explanation of reduced scattering.	Does not explain the dramatic increase in conductivity due to reduced phonon scattering.
High-Frequency Behavior	Correctly predicts $\sigma(\omega) \propto rac{1}{\omega^2}$ at very high frequencies.	Cannot explain plasma frequency effects or quantum effects in the optical regime.
Applicability to Simple Metals	Works reasonably well for alkali metals.	Fails for transition metals, correlated systems, or materials with complex band structures.

The empirical law of Wiedemann and Franz is a fundamental observation in solid-state physics regarding the relationship between the thermal and electrical conductivities of metals. It states that:

**□**"The ratio of the thermal conductivity ( $\kappa$ ) to the electrical conductivity ( $\sigma$ ) of a metal is directly proportional to its absolute temperature (T)":

$$\frac{\kappa}{\sigma} = LT$$

**L:** Lorenz number, a proportionality constant. Lorenz Number (L): For many metals, the Lorenz number is approximately:

$$L = \frac{\pi^2 k_B^2}{3e^2} \approx 2.44 \times 10^{-8} \,\mathrm{W} \cdot \Omega / \mathrm{K}^2$$

The Wiedemann-Franz law highlights that in metals:

Both heat and electrical conduction are dominated by free electrons.
 This law holds well at high temperatures where the classical Drude model is applicable. However, deviations occur at very low temperatures due to quantum effects.

## The Drude Theory of Metals Thermal Conductivity of A METAL

- □ The Drude model successfully explained the empirical law of Wiedemann and Franz (1853).
- The Wiedemann-Franz law states that the ratio of thermal conductivity (κ) to electrical conductivity (σ) is proportional to temperature (T).
- This proportionality constant  $(\frac{\kappa}{\sigma}T)$  is approximately the same for all metals, known as the Lorenz number.
- □In metals, the majority of the thermal current is carried by conduction electrons, not ions.
- This is based on the observation that metals conduct heat much better than insulators, where ions dominate thermal conduction.

# The Drude Theory of Metals Thermal Conductivity of A METAL

Table 1.6 provides measured thermal conductivities for several metals at 273 K and 373 K, along with their Lorenz numbers. EXPERIMENTAL THERMAL CONDUCTIVITIES AND LORENZ NUMBERS OF SELECTED METALS

	273 K		373 K	
ELEMENT	κ (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K <sup>2</sup> )	к (watt/cm-K)	$\kappa/\sigma T$ (watt-ohm/K <sup>2</sup> )
Li	0.71	$2.22 \times 10^{-8}$	0.73	$2.43 \times 10^{-8}$
Na	1.38	2.12		
Κ	1.0	2.23		
Rb	0.6	2.42		
Cu	3.85	2.20	3.82	2.29
Ag	4.18	2.31	4.17	2.38
Au	3.1	2.32	3.1	2.36
Be	2.3	2.36	1.7	2.42
Mg	1.5	2.14	1.5	2.25
Nb	0.52	2.90	0.54	2.78
Fe	0.80	2.61	0.73	2.88
Zn	1.13	2.28	1.1	2.30
Cd	1.0	2.49	1.0	
Al	2.38	2.14	2.30	2.19
- In	0.88	2.58	0.80	2.60
TI	0.5	2.75	0.45	2.75
Sn	0.64	2.48	0.60	2.54
Pb	0.38	2.64	0.35	2.53
Bi	0.09	3.53	0.08	3.35
Sb	0.18	2.57	0.17	2.69

Source: G. W. C. Kaye and T. H. Laby, *Table of Physical and Chemical Constants*, Longmans Green, London, 1966.

# The Drude Theory of Metals Thermal Conductivity of A METAL

### Fourier's Law of Heat Conduction:

Fourier's law describes the heat conduction process and states that the heat flux (q) in a material is proportional to the negative gradient of temperature ( $\nabla$ T). Mathematically, it is expressed as:

$$q = -\kappa \nabla T$$

#### Where:

- q: Heat flux (thermal current density), measured in  $W/m^2$  (watts per square meter).
- $\kappa$ : Thermal conductivity of the material (W/m·K).
- VT: Temperature gradient in K/m.
- The negative sign indicates that heat flows from regions of higher temperature to lower temperature.

### **Physical Meaning:**

- Heat flows in the direction opposite to the temperature gradient.
- κ represents the material's ability to conduct heat—higher values indicate better thermal conduction.

# The Drude Theory of Metals Thermal Conductivity OF A METAL

- □ For a steady-state thermal conduction scenario, a temperature gradient results in a thermal current flowing opposite to the gradient.
- Heat flow can be maintained by supplying heat to the hot end of a metal bar at the same rate it is lost, resulting in a uniform thermal energy flow.
- Thermal Current Density, defined as a vector parallel to the heat flow direction, representing the thermal energy crossing a unit area per unit time.
- □Thermal current is proportional to the temperature gradient (∇T), according to Fourier's law.
- □ Thermal Current Density:

$$\vec{j_q} = -\kappa \nabla T \tag{1.47}$$

## **Thermal Conductivity OF A METAL**

- **Thermal Current in Steady State:** In a steady state, the thermal current flows along the x-direction and is proportional to the negative of the temperature gradient:  $j_q = -\kappa dT/dx$
- **Electron Contribution to Thermal Current**: Electrons play a major role in thermal conduction, especially in metals.
- After a collision, electrons emerge with energy corresponding to the local temperature.
- □ Electrons from the high-temp. region are more energetic than those from the low-temp. region, leading to a net energy transfer.
- □ Simplified One-Dimensional Model: Assumes electrons only move along the x-axis.

At a given point x: <sup>1</sup>/<sub>2</sub> of the electrons come from the high-temperature side (x-vt). <sup>1</sup>/<sub>2</sub> come from the low-temp. side (x+vt).
 The thermal energy carried by electrons depends on their last collision point.

## **Thermal Conductivity OF A METAL**

Contributions to Thermal Current:
 From high-temperature electrons:

$$\frac{n}{2}v\varepsilon[T(x-vt)]$$

From low-temperature electrons:

$$\frac{n}{2}(-v)\varepsilon[T(x+vt)]$$



 $\Box$   $\epsilon$  is thermal energy/e

The net thermal current arises from the difference between these contributions.

The figure 1.6 shows the schematic highlights on how electrons originating from higher-temperature regions carry more energy, resulting in a flow of thermal energy toward lower-temperature regions.

moving from eq. 1.48 -> 1.49° () expand E[T(X-ut)]? (Tylor expansion)  $\alpha \in [T(x)] - \frac{de}{dT} \frac{dT}{dx} (xeT) - \cdots O$ ( expand ELT(X+20)]  $\gamma \in [T(x)] + \frac{d \in dT}{dT} \frac{dT}{dx} (20T) - 0$ : 1. 48 =  $j^{9} = \frac{1}{2} h 28 \int E[T(x)] - \frac{dE}{dT} \frac{dT}{dx} (207)$  $-e[\tau(x)] + \frac{d\epsilon}{d\tau} \frac{d\tau}{dx}(\tau_0\tau)'$  $\rightarrow j^{q} = \frac{1}{2} n 2 e \left[ -2 \frac{d e}{dT} \frac{dT}{dx} (2 v \overline{L}) \right] - - - \overline{3}$ 

 $) j_{=}^{9} - \frac{1}{2}hne^{7} T \frac{de}{dT} \frac{dT}{dX}$  $= \pm h \sqrt{2} \tau \frac{d \zeta}{d \zeta} \left( - \frac{d'}{d \chi} \right) - - \left( \frac{q'}{q'} \right)$ 

### 1. Thermal Current in 3D:

In a three-dimensional system, the electrons move with velocities distributed equally in all directions (isotropic distribution). The velocity v has components  $v_x$ ,  $v_y$ , and  $v_z$  along the x, y, and z-axes, respectively.

The total velocity squared is:

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

Since the distribution is isotropic (all directions are equivalent):

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle$$

Thus, the total velocity squared is split equally among the three components:

$$v^2=3\langle v_x^2
angle$$

This gives:

$$\langle v_x^2 
angle = rac{1}{3} v^2$$

### 2. Averaging Over Directions:

In Eq. (1.49), the thermal current density involves the x-component of the velocity:

$$j_q = n v^2 au rac{d\epsilon}{dT} \left( -rac{dT}{dx} 
ight)$$

When extending to 3D, we replace v with the x-component  $v_x$ , and average over all directions:

$$\langle v_x^2 
angle = {1 \over 3} v^2$$

This introduces the  $\frac{1}{3}$  factor in Eq. (1.50).

### 3. Resulting Equation:

In three dimensions, the thermal current density becomes:

$$j_q = rac{1}{3} v^2 au c_v (-
abla T)$$

This is Eq. (1.50), where  $c_v$  is the electronic specific heat.

### 2. In the Context of Thermal Conductivity:

• In the derivation of thermal conductivity:

$$c_v = n \frac{d\epsilon}{dT}$$

represents the electronic specific heat per unit volume. It quantifies how much energy an electron contributes to the thermal conduction process as temperature changes.

The Drude Theory of Metals <b>Thermal Conductivity OF A METAL</b>			
<b>1-D</b> Contribution to Thermal Current Density:			
$j_q = \frac{n}{2}nv[\varepsilon(T[x - vt]) - \varepsilon(T[x + vt])]$	(1.48)		
Thermal Current Density Approximation:			
$j_q = nv^2 \tau \frac{d\varepsilon}{dT} \left( -\frac{dT}{dx} \right)$	(1.49)		
<b>3-D Thermal Current Density:</b>			
$j_q = \frac{1}{3}v^2\tau c_v(-\nabla T)$	(1.50)		

Thermal conductivity ( $\kappa$ ) in terms of specific heat ( $c_v$ ):

$$\kappa = \frac{1}{3}v^{2}\tau c_{v} = \frac{1}{3}\ell v c_{v}$$
(1.51)

## **Thermal Conductivity OF A METAL**

 $\Box$  We can have the next fromula without  $\tau$ :

$$\frac{\kappa}{\sigma} = \frac{\frac{1}{3}c_v m v^2}{ne^2} \tag{1.52}$$

□ It was natural for Drude to apply the classical ideal gas laws in evaluating the electronic specific heat and mean square velocity. He therefore in effect took  $c_v$  to be  $\frac{3}{2}nk_B$  and  $\frac{1}{2}mv^2$  to be  $\frac{3}{2}k_BT$ , where  $k_B$  is Boltzmann's constant. This leads to the result:

$$\frac{\kappa}{\sigma} = \frac{3}{2} \left(\frac{k_B}{e}\right)^2 T \tag{1.53}$$

□ The right side of (1.53) is proportional to T and depends only on the universal constants  $k_B$  and e, in complete agreement with the law of Wiedemann and Franz

## **Thermal Conductivity OF A METAL**

Equation (1.53) gives a Lorenz number:

$$\frac{\kappa}{\sigma T} = \frac{3}{2} \left(\frac{k_B}{e}\right)^2 = 1.11 \times 10^{-8} \text{ watt-} \Omega/\text{K}^2 \tag{1.54}$$

This is about half the typical value given in Table 1.6.

In his original calculation of the electrical conductivity, Drude erroneously found half the correct result.

□ As a result of which he found a value  $K/\sigma T$ =2.22xI0<sup>-8</sup> watt- $\Omega/K^2$  in extraordinary agreement with experiment.

in Thermal Conductivity. point of agreement. () Wiedemann - Frant law the law says:  $\frac{k}{0} = \frac{\pi^2 H_B^2}{3e^2}$ . T 3 Electron Contribution to heat transport 3 linear dependence on T. \* disagreenent: heat capagity Cu () Overestimation of it predicts that all electrons contribute equally for Cu - over estimation

3 T dependence at Law T: Drude Model Predicts that thermal Conductivity Remains 2 - at all T Cexp. ~ ~ ~ ~ ~ at LawT) 3 ignare phanon contribution a it fails at high T 5 error in Lorentz Number: it agrees with the Law, but Det accurate Number.

## Thermal Conductivity OF A METAL

### **Summary of Agreement and Disagreement**

Aspect	Agreement	Disagreement
Wiedemann- Franz Law	Correctly predicts the linear relationship between $\kappa/\sigma$ and $T.$	Lorenz number is underestimated by a factor of 2 in the Drude model.
Role of Electrons	Identifies conduction electrons as primary heat carriers in metals.	Overestimates their contribution by assuming all electrons participate equally (ignoring Fermi-Dirac statistics).
Temperature Dependence	Predicts $\kappa \propto T$ at high temperatures, consistent with experiments.	Fails to predict $\kappa \propto T^2$ behavior at low temperatures.
Heat Transport Mechanisms	Provides insights into the relationship between mean free path, velocity, and specific heat.	Does not account for phonon contributions or electron-phonon scattering at high temperatures.
Application to Simple Metals	Works reasonably well for alkali and free-electron-like metals.	Fails for transition metals, alloys, and anisotropic materials.

