

104 PHYS

Ch. 27

Direct-Current Circuits

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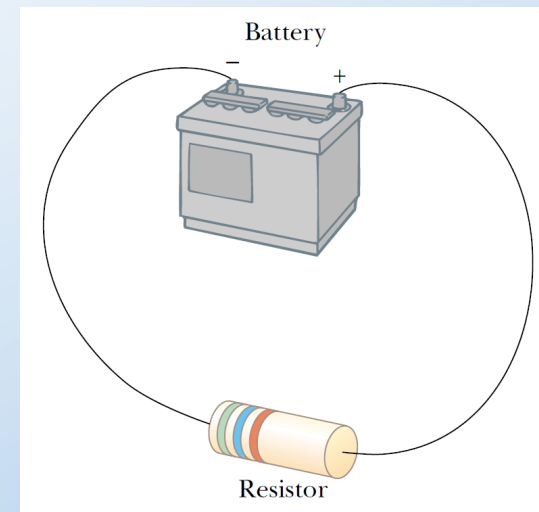
sec. 27.02 Resistors in Series and Parallel

sec. 27.03 Kirchhoff's Rules

- Most of the circuits analyzed are assumed to be in **steady state**, which means that currents in the circuit are constant in magnitude and direction.
- The current in the circuit that is constant in magnitude and direction is called a **direct current** (DC), while the current that changes direction periodically is called an **alternating current** (AC).
- A battery is called either a source of electromotive force or, more commonly, a source of emf.
- **The emf $\mathcal{E}$ of a battery is the maximum possible voltage that the battery can provide between its terminals.**
- Because a real battery is made of matter, there is resistance to the flow of charge within the battery. This resistance is called **internal resistance** r .
- For an idealized battery with zero internal resistance, the potential difference across the battery (called its **terminal voltage**) equals its emf.
- For a real battery, the terminal voltage is not equal to the emf for a battery in a circuit in which there is a current.



An assortment of batteries that can be used to provide energy for various devices. Batteries provide a voltage with a fixed polarity, resulting in a direct current in a circuit, that is, a current for which the drift velocity of the charges is always in the same direction.



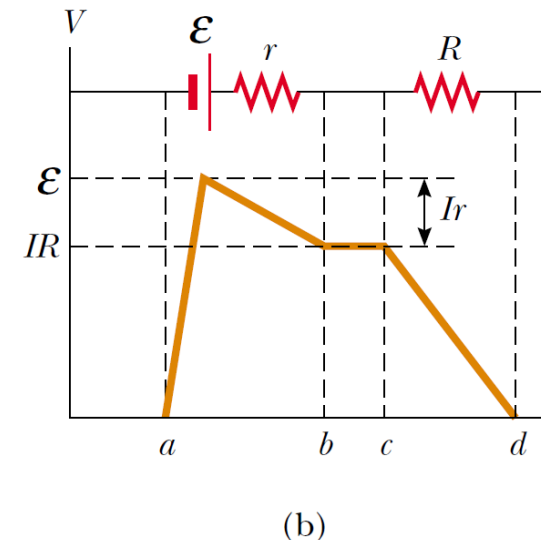
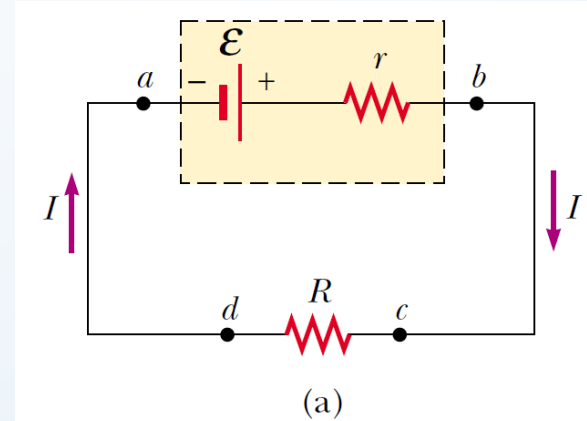
A circuit consisting of a resistor connected to the terminals of a battery.

- Now imagine moving through the battery from a to b and measuring the electric potential at various locations.
 - As we pass from the negative terminal to the positive terminal, the potential increases by an amount $\mathcal{E}$.
 - As we move through the resistance r , the potential decreases by an amount Ir , where I is the current in the circuit.
- Thus, the terminal voltage of the battery $\Delta V = V_b - V_a$ is:

$$\Delta V = \mathcal{E} - Ir$$

- $\mathcal{E}$ is equivalent to the open-circuit voltage—that is, the terminal voltage when the current is zero.
- The emf is the voltage labeled on a battery. The actual potential difference between the terminals of the battery depends on the current in the battery.
- The terminal voltage ΔV must equal the potential difference across the external resistance R , often called the **load resistance**.
- The potential difference across the load resistance is $\Delta V = IR$. Therefore $\mathcal{E}$ can be expressed as:

$$\mathcal{E} = IR + Ir$$



(a) Circuit diagram of a source of emf $\mathcal{E}$ (in this case, a battery), of internal resistance r , connected to an external resistor of resistance R . (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

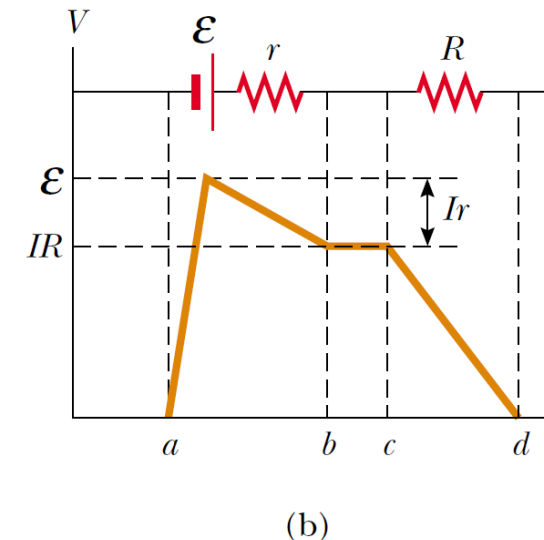
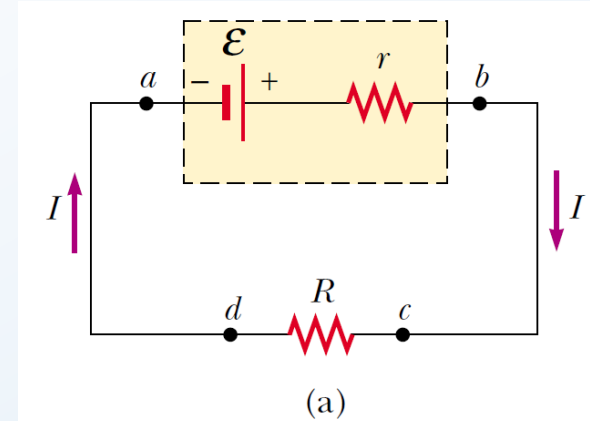
- Solving for the current gives:

$$I = \frac{\mathcal{E}}{R + r}$$

- The current in this simple circuit depends on both the load resistance R external to the battery and the internal resistance r . If R is much greater than r , as it is in many real-world circuits, we can neglect r .
- If we multiply $\mathcal{E}$ by the current I , we obtain:

$$I\mathcal{E} = I^2R + I^2r$$

- Because power $\mathcal{P} = I\Delta V$, the total power output $I\mathcal{E}$ of the battery is delivered to the external load resistance in the amount I^2R and to the internal resistance in the amount I^2r .



(a) Circuit diagram of a source of emf $\mathcal{E}$ (in this case, a battery), of internal resistance r , connected to an external resistor of resistance R . (b) Graphical representation showing how the electric potential changes as the circuit in part (a) is traversed clockwise.

Example 27.01

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω . Its terminals are connected to a load resistance of 3.00 Ω . (a) Find the current in the circuit and the terminal voltage of the battery. (b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

(a) The current is given as:

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0}{3.00 + 0.05} = 3.93 \text{ A}$$

We find the terminal voltage:

$$\Delta V = \mathcal{E} - Ir = 12.0 - (3.93 \times 0.05) = 11.8 \text{ V}$$

The power delivered to the load resistor is:

$$\mathcal{P}_R = I^2 R = (3.93)^2 \times 3.00 = 46.3 \text{ W}$$

(b) The power delivered to the internal resistance is:

$$\mathcal{P}_r = I^2 r = (3.93)^2 \times 0.05 = 0.772 \text{ W}$$

***To check this result, we can calculate the voltage across the load resistance R:**

$$\Delta V = IR = 3.93 \times 3.00 = 11.8 \text{ V}$$

Example 27.01

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω . Its terminals are connected to a load resistance of 3.00 Ω . (a) Find the current in the circuit and the terminal voltage of the battery. (b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Hence, the power delivered by the battery is the sum of these quantities, or 47.1 W. You should check this result, using the expression $\mathcal{P} = I\mathcal{E}$.

What If? As a battery ages, its internal resistance increases. Suppose the internal resistance of this battery rises to 2.00 Ω toward the end of its useful life. How does this alter the ability of the battery to deliver energy?

Let us connect the same 3.00 Ω load resistor to the battery. The current in the battery now is

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.0}{3.00 + 2.00} = 2.40 \text{ A}$$

And the terminal voltage is:

$$\Delta V = \mathcal{E} - Ir = 12.0 - (2.40 \times 2.00) = 7.2 \text{ V}$$

Example 27.01

A battery has an emf of 12.0 V and an internal resistance of 0.05 Ω . Its terminals are connected to a load resistance of 3.00 Ω . (a) Find the current in the circuit and the terminal voltage of the battery. (b) Calculate the power delivered to the load resistor, the power delivered to the internal resistance of the battery, and the power delivered by the battery.

Notice that the terminal voltage is only 60 % of the emf. The powers delivered to the load resistor and internal resistance are:

$$\mathcal{P}_R = I^2 R = (2.40)^2 \times 3.00 = 17.3 \text{ W}$$

$$\mathcal{P}_r = I^2 r = (2.40)^2 \times 2.00 = 11.5 \text{ W}$$

Notice that 40 % of the power from the battery is delivered to the internal resistance. In part (b), this percentage is 1.6 %. Consequently, even though the emf remains fixed, the increasing internal resistance significantly reduces the ability of the battery to deliver energy.

Question 27.01

A battery has an emf of 15.0 V. The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R . (a) What is the value of R ? (b) What is the internal resistance of the battery?

$$(a) \quad \mathcal{P}_R = \frac{(\Delta V)^2}{R}$$

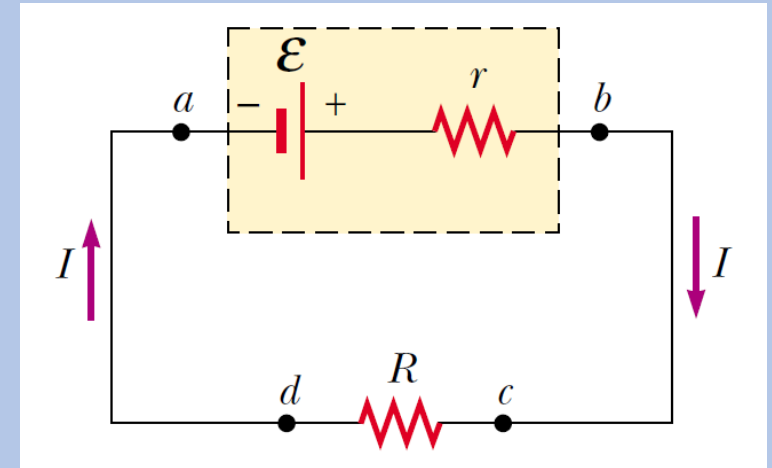
$$R = \frac{(\Delta V)^2}{\mathcal{P}_R} = \frac{(11.6)^2}{20.0} = 6.73 \, \Omega$$

$$(b) \quad \Delta V = IR$$

$$I = \frac{\Delta V}{R} = \frac{11.6}{6.73} = 1.72 \, \text{A}$$

$$\Delta V = \mathcal{E} - Ir$$

$$r = \frac{\mathcal{E} - \Delta V}{I} = \frac{15.0 - 11.6}{1.72} = 1.97 \, \Omega$$



Question 27.02

(a) What is the current in a $5.60\ \Omega$ resistor connected to a battery that has a $0.200\ \Omega$ internal resistance if the terminal voltage of the battery is $10.0\ \text{V}$? (b) What is the emf of the battery?

(a) $\Delta V = IR$

$$I = \frac{\Delta V}{R} = \frac{10.0}{5.60} = 1.79\ \text{A}$$

(b) $\Delta V = \mathcal{E} - Ir$

$$\mathcal{E} = \Delta V + Ir = 10 + (1.79 \times 0.200) = 10.4\ \text{V}$$

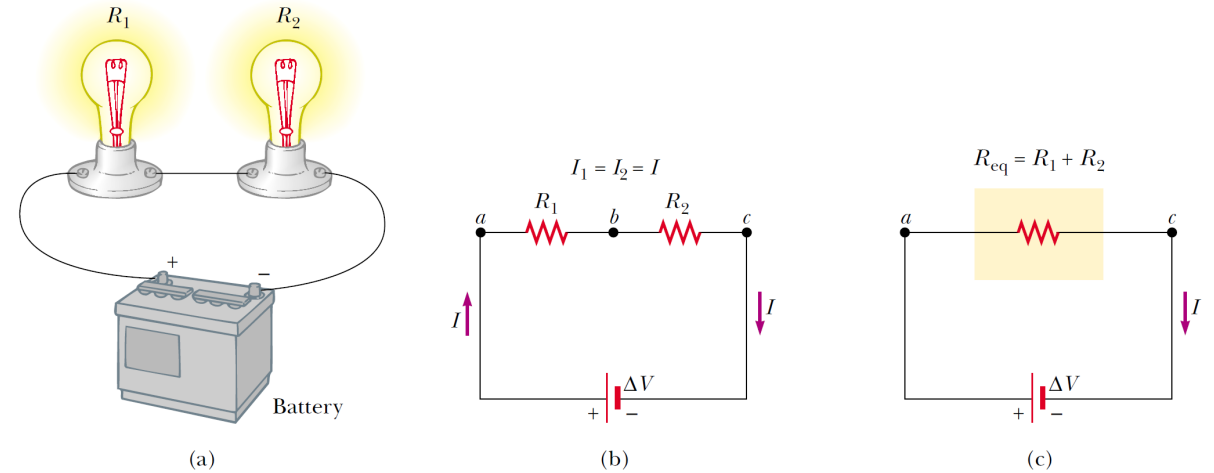
Resistors in Series

- When two or more resistors are connected together, they are said to be in series.
- In a series connection, if an amount of charge Q exits resistor R_1 , charge Q must also enter the second resistor R_2 .
- For a series combination of two resistors, the currents are the same in both resistors because the amount of charge that passes through R_1 must also pass through R_2 in the same time interval.
- The potential difference applied across the series combination of resistors will divide between the resistors.
- Because the voltage drop from a to b equals IR_1 and the voltage drop from b to c equals IR_2 , the voltage drop from a to c is:

$$\Delta V = IR_1 + IR_2 = I(R_1 + R_2)$$

- The potential difference across the battery is also applied to the **equivalent resistance** R_{eq} :

$$\Delta V = IR_{eq}$$



(a) A series connection of two lightbulbs with resistances R_1 and R_2 . (b) Circuit diagram for the two-resistor circuit. The current in R_1 is the same as that in R_2 . (c) The resistors replaced with a single resistor having an equivalent resistance $R_{eq} = R_1 + R_2$.

Resistors in Series

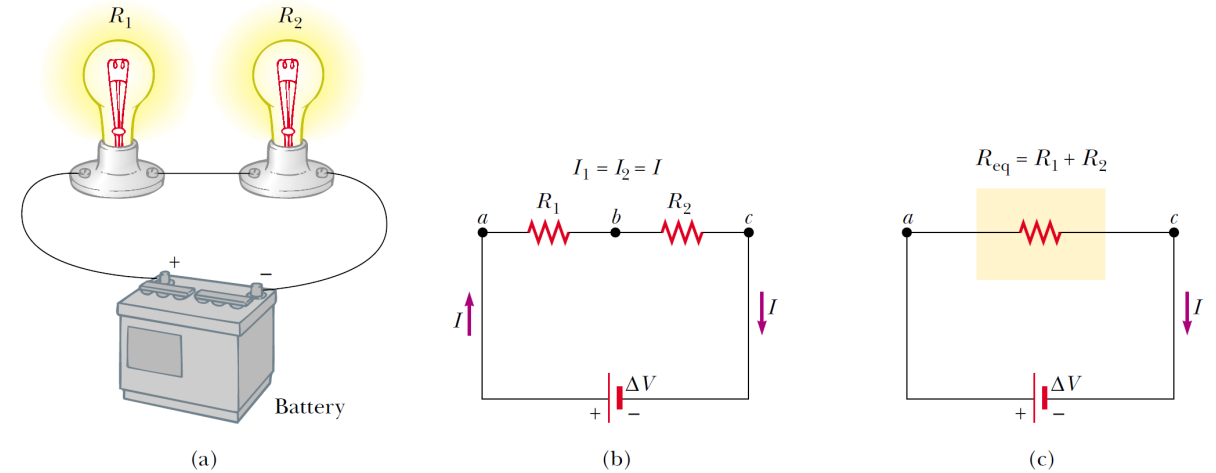
- We see that we can replace the two resistors in series with a single equivalent resistance whose value is the sum of the individual resistances:

$$\Delta V = IR_{eq} = I(R_1 + R_2) \rightarrow R_{eq} = R_1 + R_2$$

- The equivalent resistance of three or more resistors connected in series is:

$$R_{eq} = R_1 + R_2 + R_3 + \dots$$

- This relationship indicates that **the equivalent resistance of a series connection of resistors is the numerical sum of the individual resistances and is always greater than any individual resistance.**



(a) A series connection of two lightbulbs with resistances R_1 and R_2 . (b) Circuit diagram for the two-resistor circuit. The current in R_1 is the same as that in R_2 . (c) The resistors replaced with a single resistor having an equivalent resistance $R_{eq} = R_1 + R_2$.

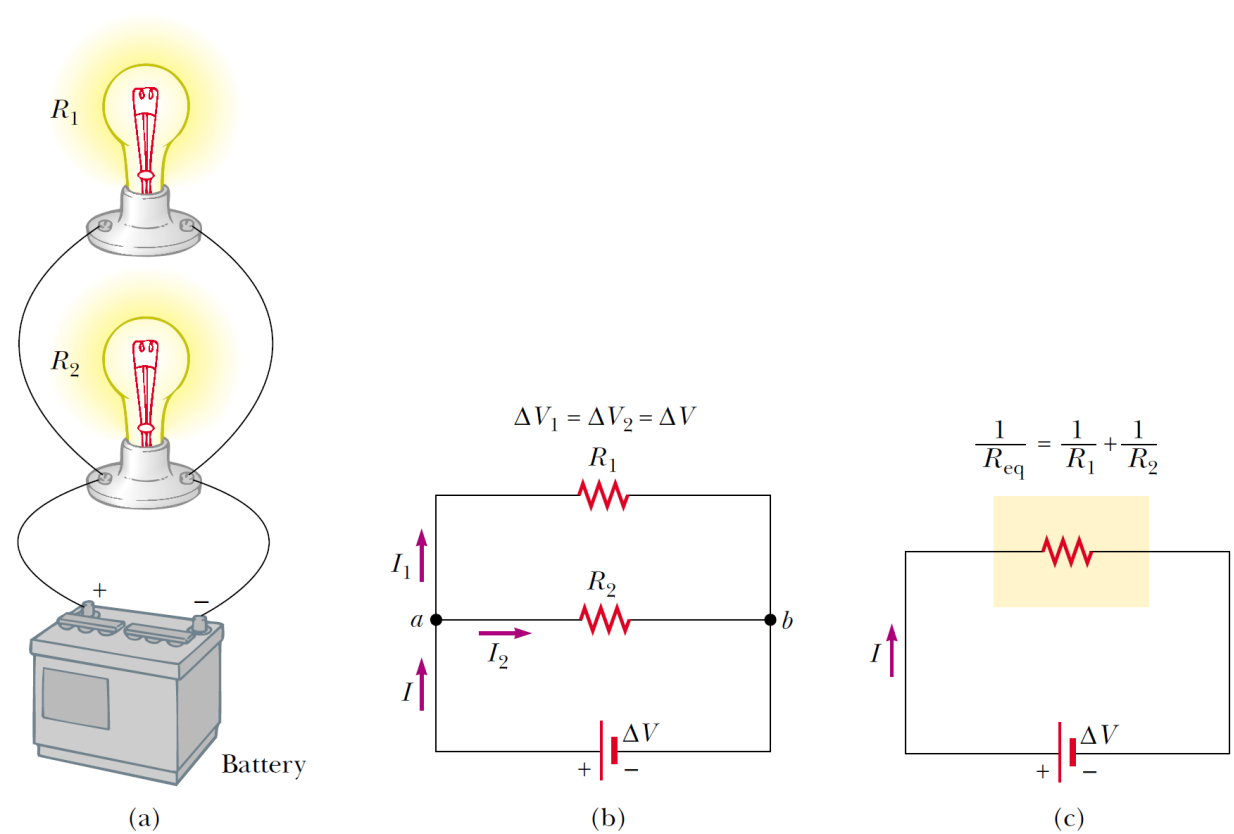
Resistors in Parallel

- Consider two resistors connected in parallel:
 - When charges reach point *a*, called a junction, they split into two parts, with some going through R_1 and the rest going through R_2 .
 - A **junction** is any point in a circuit where a current can split.
 - This split results in less current in each individual resistor than the current leaving the battery.
- Because electric charge is conserved, the current I that enters point *a* must equal the total current leaving that point:

$$I = I_1 + I_2$$

- When resistors are connected in parallel, the potential differences across the resistors is the same.
- Because the potential differences across the resistors are the same, the expression $\Delta V = IR$ gives:

$$I = I_1 + I_2 = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} = \Delta V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{\Delta V}{R_{eq}}$$



(a) A parallel connection of two lightbulbs with resistances R_1 and R_2 . (b) Circuit diagram for the two-resistor circuit. The potential difference across R_1 is the same as that across R_2 . (c) The resistors replaced with a single resistor having an equivalent resistance given by $R_{eq} = R_1 R_2 / (R_1 + R_2)$.

Resistors in Parallel

- From this result, we see that the equivalent resistance R_{eq} of two resistors in parallel is given by:

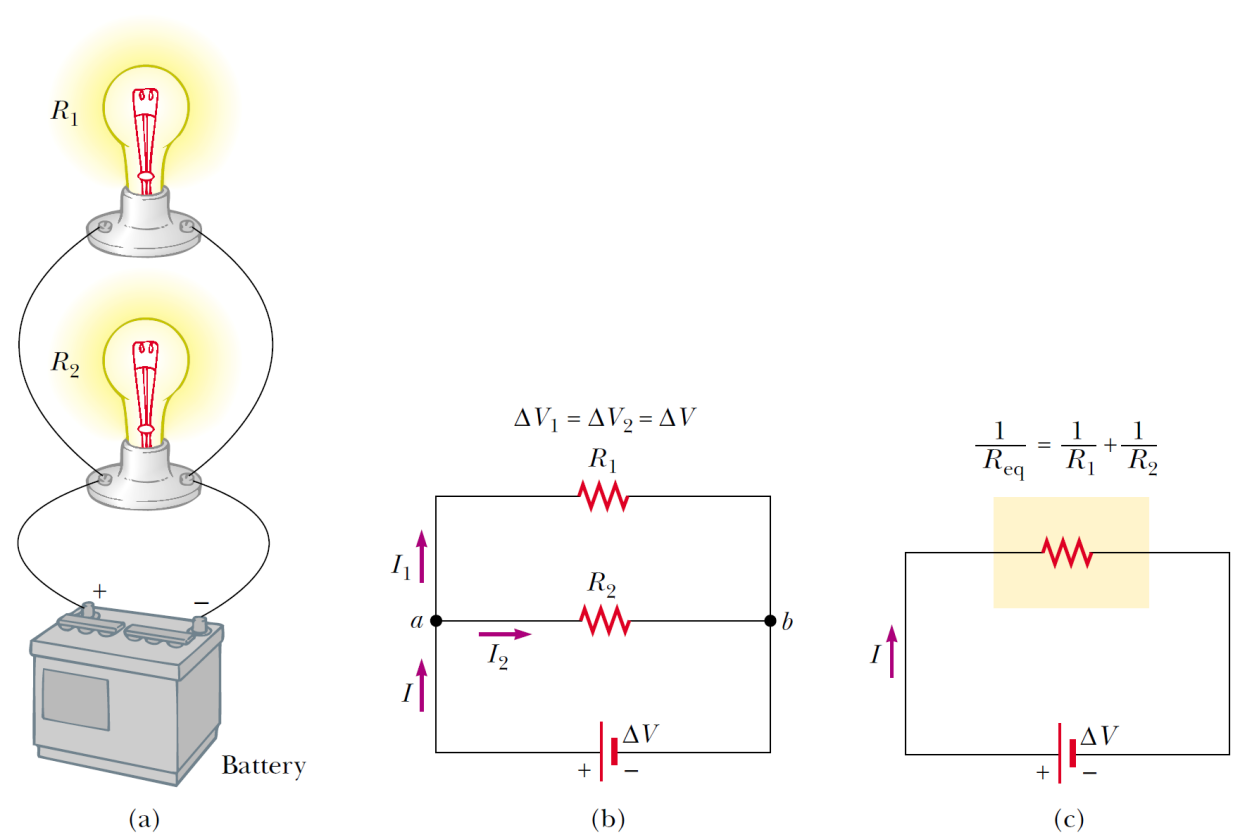
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

- An extension of this analysis to three or more resistors in parallel gives:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

- We can see from this expression that **the inverse of the equivalent resistance of two or more resistors connected in parallel is equal to the sum of the inverses of the individual resistances. Furthermore, the equivalent resistance is always less than the smallest resistance in the group.**



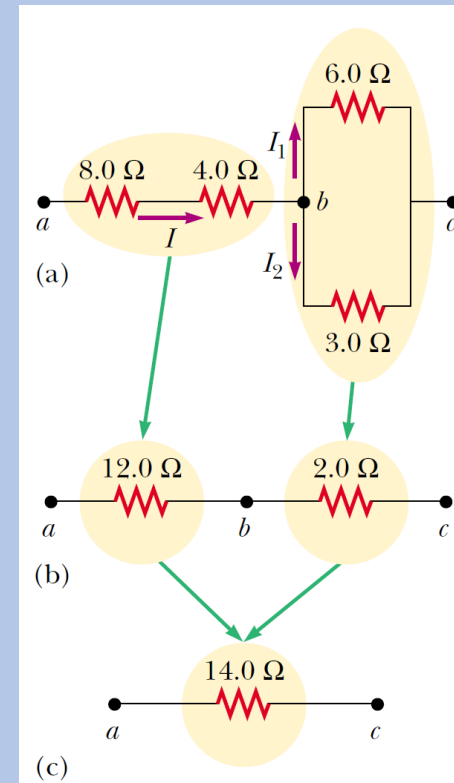
Example 27.04

Four resistors are connected as shown in Fig. (a). (a) Find the equivalent resistance between points a and c . (b) What is the current in each resistor if a potential difference of 42 V is maintained between a and c ?

- (a) The combination of resistors can be reduced in steps, as shown in the following figure. The $8.0\ \Omega$ and $4.0\ \Omega$ resistors are in series; thus, the equivalent resistance between a and b is $12.0\ \Omega$. The $6.0\ \Omega$ and $3.0\ \Omega$ resistors are in parallel, so, we find that the equivalent resistance from b to c is $2.0\ \Omega$. Hence, the equivalent resistance from a to c is $14.0\ \Omega$.
- (b) The currents in the $8.0\ \Omega$ and $4.0\ \Omega$ resistors are the same because they are in series. In addition, this is the same as the current that would exist in the $14.0\ \Omega$ equivalent resistor subject to the 42 V potential difference. Therefore, using $R = \Delta V/I$ and the result from part (a), we obtain

$$I = \frac{\Delta V_{ac}}{R_{eq}} = \frac{42}{14.0} = 3.0\ \text{A}$$

This is the current in the $8.0\ \Omega$ and $4.0\ \Omega$ resistors. When this 3.0 A current enters the junction at b , however, it splits, with part passing through the $6.0\ \Omega$ resistor (I_1) and part through the $3.0\ \Omega$ resistor (I_2). Because the potential difference is ΔV_{bc} across each of these parallel resistors, we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0\ \text{A}$, we find that $I_1 = 1.0\ \text{A}$ and $I_2 = 2.0\ \text{A}$.



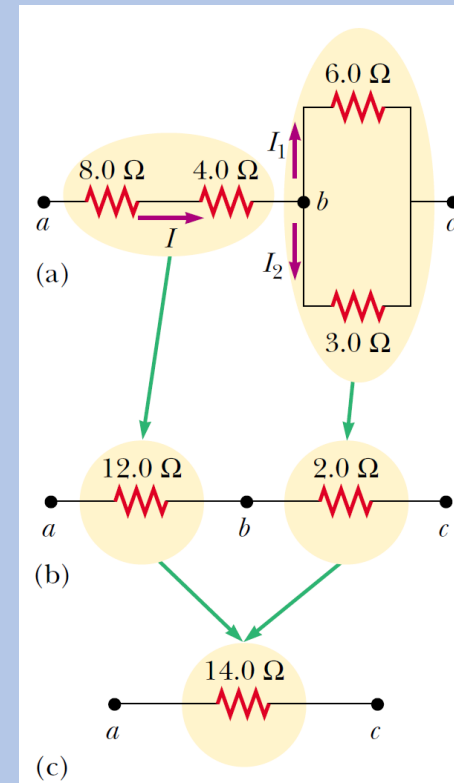
The original network of resistors is reduced to a single equivalent resistance.

Example 27.04

Four resistors are connected as shown in Fig. (a). (a) Find the equivalent resistance between points a and c . (b) What is the current in each resistor if a potential difference of 42 V is maintained between a and c ?

We could have guessed this at the start by noting that the current in the $3.0\ \Omega$ resistor has to be twice that in the $6.0\ \Omega$ resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0\text{ V}$ and $\Delta V_{ab} = (12.0\ \Omega)I = 36\text{ V}$; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42\text{ V}$, as it must.



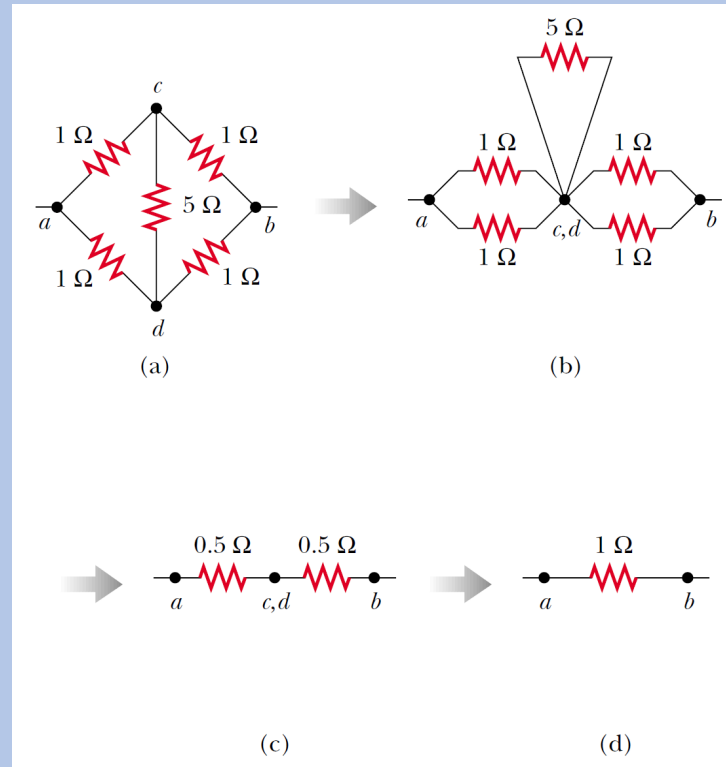
The original network of resistors is reduced to a single equivalent resistance.

Question 27.03

Consider five resistors connected as shown in Fig. (a). Find the equivalent resistance between points a and b .

If we inspect this system of resistors, we realize that we cannot reduce it by using our rules for series and parallel connections. We can, however, assume a current entering junction a and then apply symmetry arguments. Because of the symmetry in the circuit (all $1\ \Omega$ resistors in the outside loop), the currents in branches ac and ad must be equal; hence, the electric potentials at points c and d must be equal.

This means that $\Delta V_{cd} = 0$ and there is no current between points c and d . As a result, points c and d may be connected together without affecting the circuit, as in Fig. (b). Thus, the $5\ \Omega$ resistor may be removed from the circuit and the remaining circuit then reduced as in Fig. (c) and (d). From this reduction we see that the equivalent resistance of the combination is $1\ \Omega$. Note that the result is $1\ \Omega$ regardless of the value of the resistor connected between c and d .



Because of the symmetry in this circuit, the $5\ \Omega$ resistor does not contribute to the resistance between points a and b and therefore can be disregarded when we calculate the equivalent resistance.

Example 27.05

Three resistors are connected in parallel as shown in Fig. (a). A potential difference of 18.0 V is maintained between points *a* and *b*. (a) Find the current in each resistor. (b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors. (c) Calculate the equivalent resistance of the circuit.

- (a) The resistors are in parallel, and so the potential difference across each must be 18.0 V. Applying the relationship $\Delta V = IR$ to each resistor gives:

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0}{3.00} = 6.00 \text{ A}, \quad I_2 = \frac{\Delta V}{R_2} = \frac{18.0}{6.00} = 3.00 \text{ A}, \quad I_3 = \frac{\Delta V}{R_3} = \frac{18.0}{9.00} = 2.00 \text{ A}$$

- (b) We apply the relationship $\mathcal{P} = I^2 R$ to each resistor and obtain:

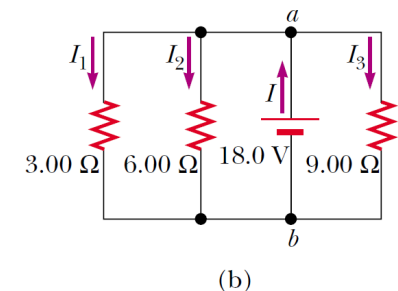
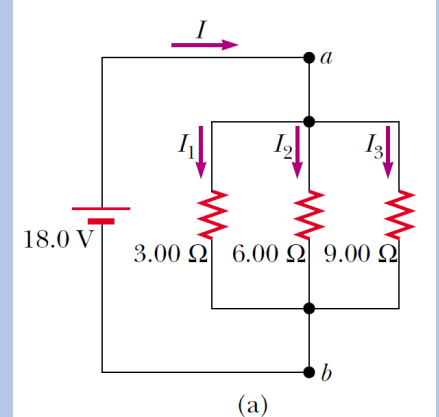
$$\mathcal{P}_1 = I_1^2 R_1 = (6.00)^2 \times 3.00 = 108 \text{ W}$$

$$\mathcal{P}_2 = I_2^2 R_2 = (3.00)^2 \times 6.00 = 54.0 \text{ W}$$

$$\mathcal{P}_3 = I_3^2 R_3 = (2.00)^2 \times 9.00 = 36.0 \text{ W}$$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 198 W.

(a) Three resistors connected in parallel. The voltage across each resistor is 18.0 V. (b) Another circuit with three resistors and a battery. Is this equivalent to the circuit in part (a) of the figure?



Example 27.05

Three resistors are connected in parallel as shown in Fig. (a). A potential difference of 18.0 V is maintained between points a and b . (a) Find the current in each resistor. (b) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors. (c) Calculate the equivalent resistance of the circuit.

(c) We can use $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$ to find R_{eq} :

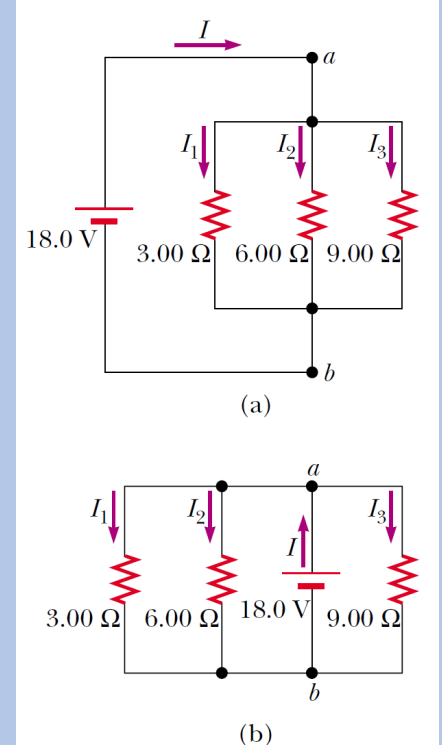
$$\frac{1}{R_{eq}} = \frac{1}{3.00} + \frac{1}{6.00} + \frac{1}{9.00}$$

$$R_{eq} = \frac{18.0}{11.0} = 1.64 \, \Omega$$

What If? What if the circuit is as shown in Fig. (b) instead of as in Fig. (a)? How does this affect the calculation?

There is no effect on the calculation. The physical placement of the battery is not important. In Fig. (b), the battery still applies a potential difference of 18.0 V between points a and b , so the two circuits in both figures are electrically identical.

(a) Three resistors connected in parallel. The voltage across each resistor is 18.0 V. (b) Another circuit with three resistors and a battery. Is this equivalent to the circuit in part (a) of the figure?



Question 27.04

Additional question

(a) Find the equivalent resistance between points a and b in the following figure. (b) A potential difference of 34.0 V is applied between points a and b . Calculate the current in each resistor.

$$(a) \quad R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{7.00 \times 10.0}{7.00 + 10.0} = 4.12 \, \Omega$$

$$R_{eq} = R_1 + R_{23} + R_4 = 4.00 + 4.12 + 9.00 = 17.1 \, \Omega$$

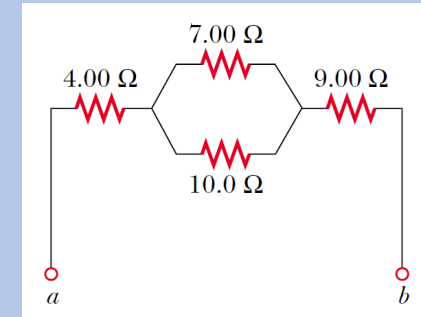
$$(b) \quad I_1 = I_4 = I_{23} = I = \frac{\Delta V}{R_{eq}} = \frac{34.0}{17.1} = 1.99 \, \text{A}$$

$$\Delta V_{23} = I_{23} R_{23} = 1.99 \times 4.12 = 8.18 \, \text{V}$$

$$\Delta V_{23} = \Delta V_2 = \Delta V_3$$

$$I_2 = \frac{\Delta V_{23}}{R_2} = \frac{8.18}{7.00} = 1.17 \, \text{A}$$

$$I_3 = \frac{\Delta V_{23}}{R_3} = \frac{8.18}{10.0} = 0.818 \, \text{A}$$



Question 27.05

Four copper wires of equal length are connected in series. Their cross-sectional areas are 1.00 cm^2 , 2.00 cm^2 , 3.00 cm^2 , and 5.00 cm^2 . A potential difference of 120 V is applied across the combination. Determine the voltage across the 2.00 cm^2 wire.

$$\Delta V = IR_{eq} = I \left(\frac{\rho l}{A_1} + \frac{\rho l}{A_2} + \frac{\rho l}{A_3} + \frac{\rho l}{A_4} \right) = I \rho l \left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4} \right)$$

$$I \rho l = \frac{\Delta V}{\left(\frac{1}{A_1} + \frac{1}{A_2} + \frac{1}{A_3} + \frac{1}{A_4} \right)} = \frac{120}{\left(\frac{1}{1 \times 10^{-4}} + \frac{1}{2 \times 10^{-4}} + \frac{1}{3 \times 10^{-4}} + \frac{1}{5 \times 10^{-4}} \right)} = 0.0059 \text{ V} \cdot \text{m}^2$$

$$\Delta V_2 = \frac{I \rho l}{A_2} = \frac{0.0059}{2 \times 10^{-4}} = 29.5 \text{ V}$$

Question 27.06

Additional question

Consider the circuit shown in the following figure. Find (a) the potential difference between points a and b and (b) the current in the $20.0\ \Omega$ resistor.

If we turn the given diagram on its side, we find that it is the same as Fig. (a). The $20.0\ \Omega$ and $5.00\ \Omega$ resistors are in series, so the first reduction is shown in Fig. (b). In addition, since the $10.0\ \Omega$, $5.00\ \Omega$, and $25.0\ \Omega$ resistors are then in parallel, we can solve for their equivalent resistance as:

$$R_{2345} = \left(\frac{1}{10.0} + \frac{1}{5.0} + \frac{1}{25.0} \right)^{-1} = 2.94\ \Omega$$

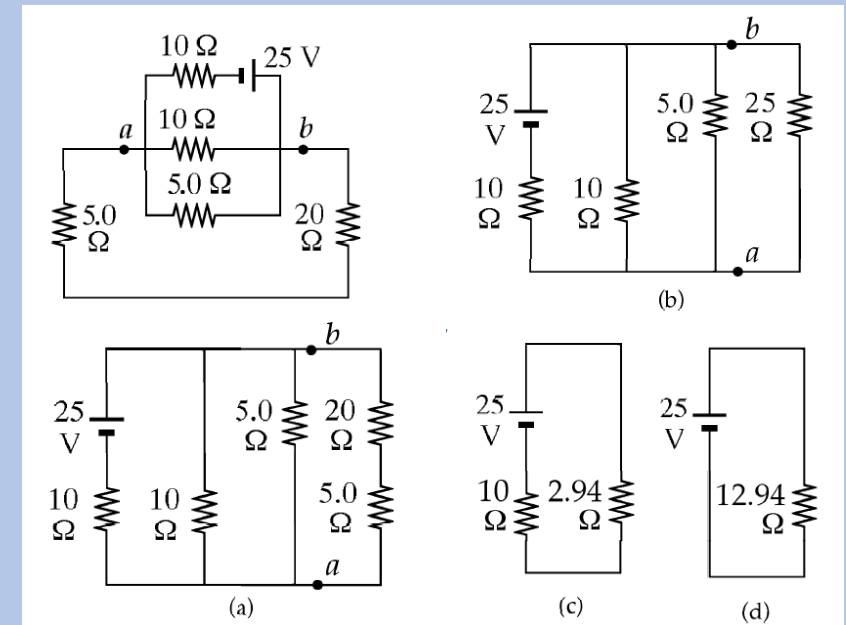
This is shown in Fig. (c), which in turn reduces to the circuit shown in Fig. (d).

Next, we work backwards through the diagrams applying $I = \Delta V/R$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The $12.94\ \Omega$ resistor is connected across $25.0\ \text{V}$, so the current through the battery in every diagram is:

$$I = \frac{\Delta V}{R_{eq}} = \frac{25.0}{12.94} = 1.93\ \text{A}$$

In Fig. (c), this $1.93\ \text{A}$ goes through the $2.94\ \Omega$ equivalent resistor to give a potential difference of:

$$\Delta V_{45} = \Delta V_2 = \Delta V_3 = IR_{2345} = 1.93 \times 2.94 = 5.68\ \text{V}$$



Question 27.06

Additional question

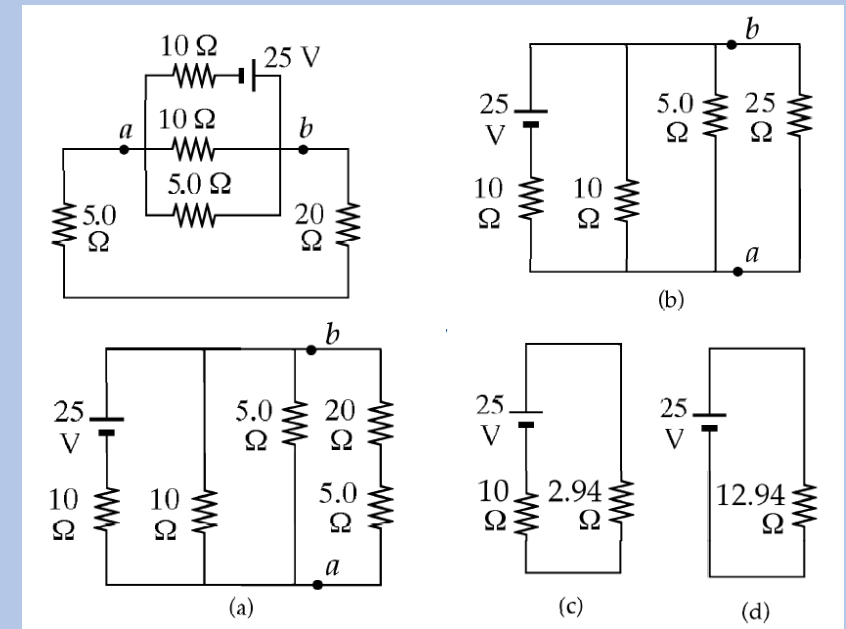
Consider the circuit shown in the following figure. Find (a) the potential difference between points a and b and (b) the current in the $20.0\ \Omega$ resistor.

From Fig. (b), we see that this potential difference is the same across ΔV_{ab} , the $10.0\ \Omega$ resistor, and the $5.00\ \Omega$ resistor. Therefore:

(a) $\Delta V_{ab} = 5.68\ \text{V}$

(b) Since the current through the $20.0\ \Omega$ resistor is also the current through the $25.0\ \Omega$ line ab ,

$$I_4 = I_5 = I_{45} = \frac{\Delta V_{45}}{R_{45}} = \frac{5.68}{25.0} = 0.227\ \text{A} = 227\ \text{mA}$$



Problem 27.13

Calculate the power delivered to each resistor in the circuit shown in the following figure.

$$R_{23} = \frac{3.00 \times 1.00}{3.00 + 1.00} = 0.75 \, \Omega$$

$$R_{eq} = 2.00 + 0.75 + 4.00 = 6.75 \, \Omega$$

$$I_1 = I_{23} = I_4 = I = \frac{\Delta V}{R_{eq}} = \frac{18.0}{6.75} = 2.67 \, \text{A}$$

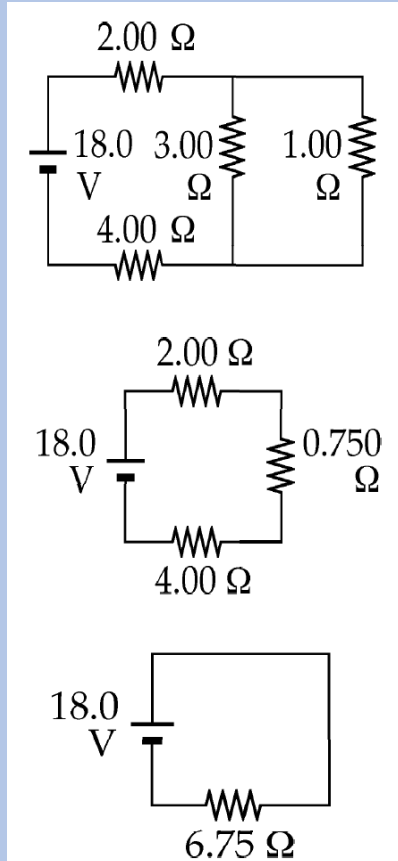
$$\Delta V_2 = \Delta V_3 = \Delta V_{23} = I_{23} R_{23} = 2.67 \times 0.75 = 2.00 \, \text{V}$$

$$\mathcal{P}_1 = I_1^2 R_1 = (2.67)^2 \times 2.00 = 14.2 \, \text{W}$$

$$\mathcal{P}_2 = \frac{(\Delta V_2)^2}{R_2} = \frac{(2)^2}{3} = 1.33 \, \text{W}$$

$$\mathcal{P}_3 = \frac{(\Delta V_3)^2}{R_3} = \frac{(2)^2}{1} = 4.00 \, \text{W}$$

$$\mathcal{P}_4 = I_4^2 R_4 = (2.67)^2 \times 4.00 = 27.4 \, \text{W}$$



- The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff's rules**:

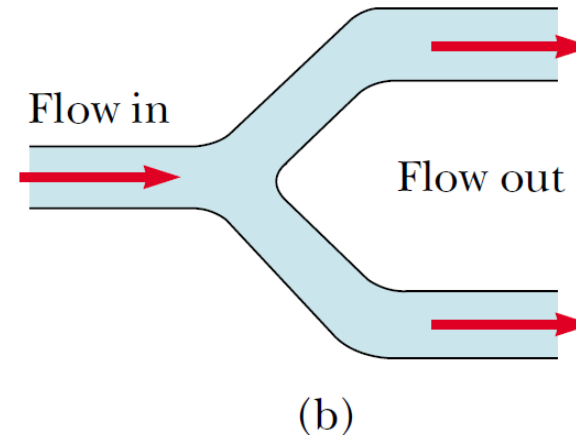
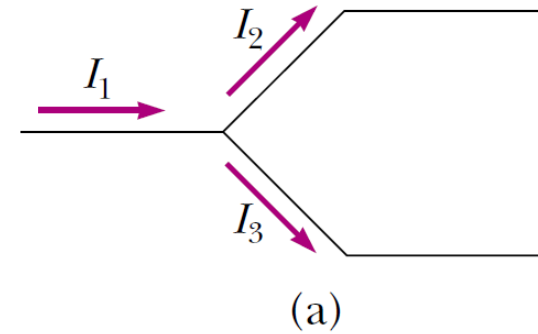
1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{in} = \sum I_{out}$$

2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{closed\ loop} \Delta V = 0$$

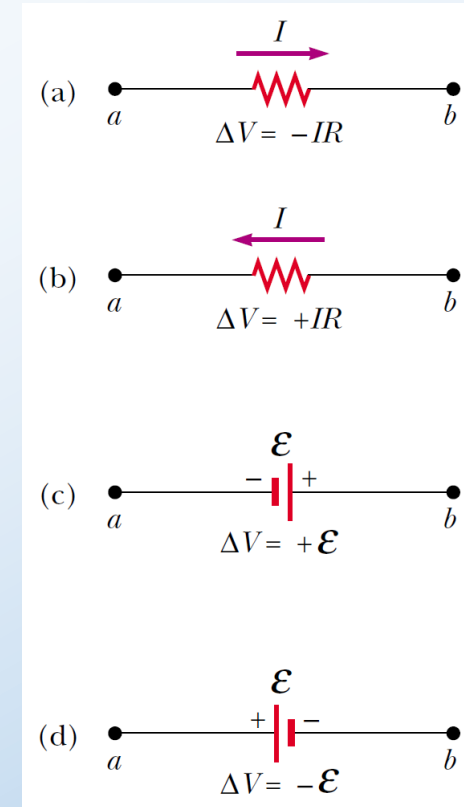
- Kirchhoff's first rule is a statement of conservation of electric charge.
- Kirchhoff's second rule follows from the law of conservation of energy.



(a) Kirchhoff's junction rule. Conservation of charge requires that all charges entering a junction must leave that junction. Therefore, $I_1 = I_2 + I_3$.

(b) A mechanical analog of the junction rule: the amount of water flowing out of the branches on the right must equal the amount flowing into the single branch on the left.

- When applying Kirchhoff's second rule in practice, we imagine traveling around the loop and consider changes in electric potential, rather than the changes in potential energy described in the preceding paragraph. You should note the following sign conventions when using the second rule:
 - Because charges move from the high-potential end of a resistor toward the low-potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$ (Fig. (a)).
 - If a resistor is traversed in the direction opposite the current, the potential difference ΔV across the resistor is $+IR$ (Fig. (b)).
 - If a source of emf (assumed to have zero internal resistance) is traversed in the direction of the emf (from $-$ to $+$), the potential difference ΔV is $+\mathcal{E}$ (Fig. (c)). The emf of the battery increases the electric potential as we move through it in this direction.
 - If a source of emf (assumed to have zero internal resistance) is traversed in the direction opposite the emf (from $+$ to $-$), the potential difference ΔV is $-\mathcal{E}$ (Fig. (d)). In this case the emf of the battery reduces the electric potential as we move through it.
- **In general, in order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.**
- **Any capacitor acts as an open branch in a circuit;** that is, the current in the branch containing the capacitor is zero under steady-state conditions.



Rules for determining the potential differences across a resistor and a battery. (The battery is assumed to have no internal resistance.) Each circuit element is traversed from left to right.

PROBLEM SOLVING HINTS

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a direction to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; its magnitude will be correct and the direction is opposite to that which you assigned.

Example 27.06

A single-loop circuit contains two resistors and two batteries, as shown in the following figure. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit. (b) What power is delivered to each resistor? What power is delivered by the 12 V battery?

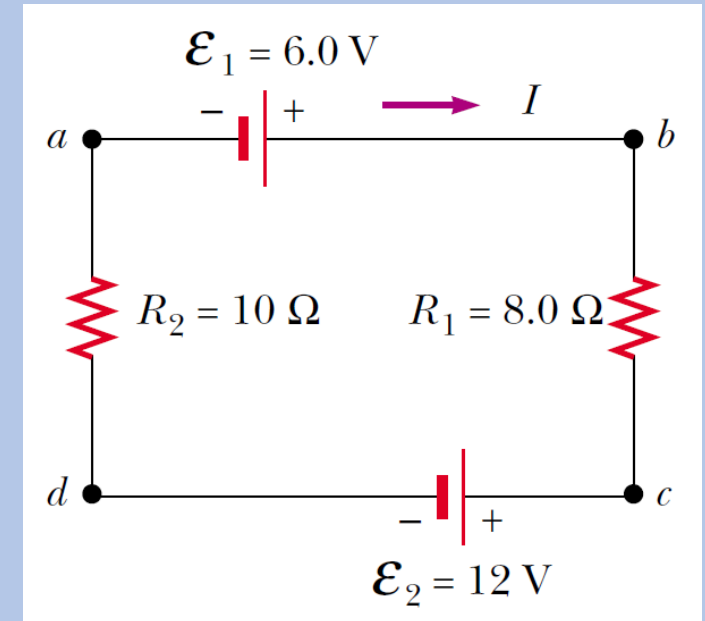
- (a) We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in the following figure. Traversing the circuit in the clockwise direction, starting at a , we see that $a \rightarrow b$ represents a potential difference of $+\mathcal{E}_1$, $b \rightarrow c$ represents a potential difference of $-IR_1$, $c \rightarrow d$ represents a potential difference of $+\mathcal{E}_2$, and $d \rightarrow a$ represents a potential difference of $-IR_2$. Applying Kirchhoff's loop rule gives:

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in the following figure, we obtain:

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0 - 12}{8.0 + 10} = -0.33 \text{ A}$$



A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

Example 27.06

A single-loop circuit contains two resistors and two batteries, as shown in the following figure. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit. (b) What power is delivered to each resistor? What power is delivered by the 12 V battery?

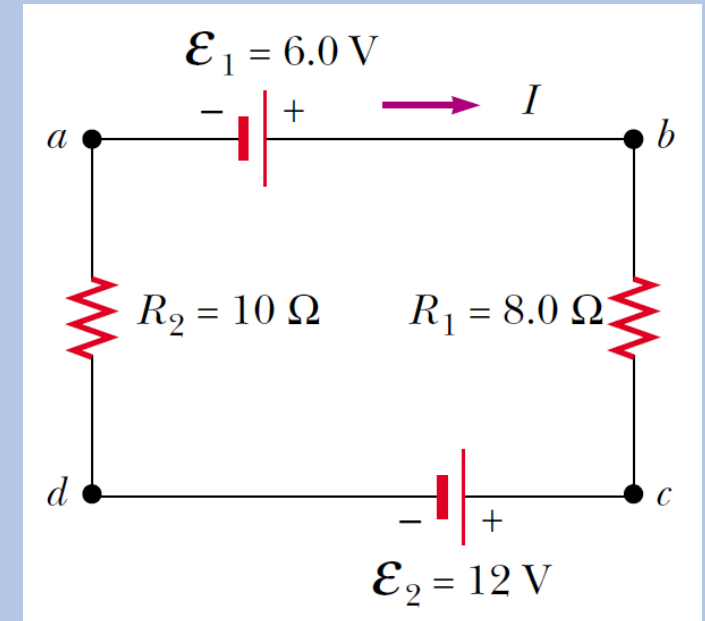
The negative sign for I indicates that the direction of the current is opposite the assumed direction. Notice that the emfs in the numerator subtract because the batteries have opposite polarities in the following figure. In the denominator, the resistances add because the two resistors are in series.

$$(b) \quad \mathcal{P}_1 = I^2 R_1 = (0.33)^2 \times 8.0 = 0.87 \text{ W}$$

$$\mathcal{P}_2 = I^2 R_2 = (0.33)^2 \times 10 = 1.1 \text{ W}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0 \text{ W}$.

The 12 V battery delivers power $I\mathcal{E}_2 = 4.0 \text{ W}$. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6 V battery, which is being charged by the 12 V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6 V battery.



A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

Example 27.06

A single-loop circuit contains two resistors and two batteries, as shown in the following figure. (Neglect the internal resistances of the batteries.) (a) Find the current in the circuit. (b) What power is delivered to each resistor? What power is delivered by the 12 V battery?

What If? What if the polarity of the 12 V battery were reversed? How would this affect the circuit?

Because the polarities of the two batteries are now in the same direction, the signs of $\mathcal{E}_1$ and $\mathcal{E}_2$ are the same.

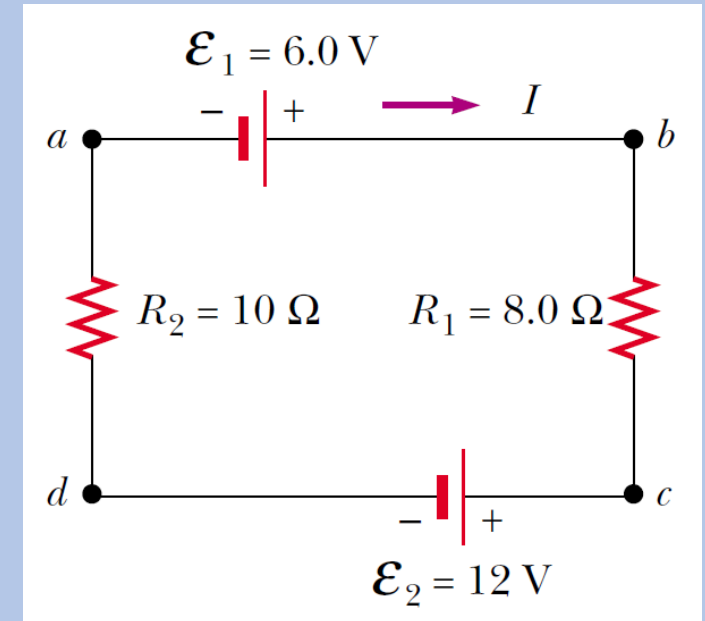
$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2} = \frac{6.0 + 12}{8.0 + 10} = 1.0 \text{ A}$$

The new powers delivered to the resistors are:

$$\mathcal{P}_1 = I^2 R_1 = (1.0)^2 \times 8.0 = 8.0 \text{ W}$$

$$\mathcal{P}_2 = I^2 R_2 = (1.0)^2 \times 10 = 10 \text{ W}$$

This totals 18 W, nine times as much as in the original circuit, in which the batteries were opposing each other.



A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

Example 27.07

Additional example

Find the currents I_1 , I_2 , and I_3 in the circuit shown in the following figure.

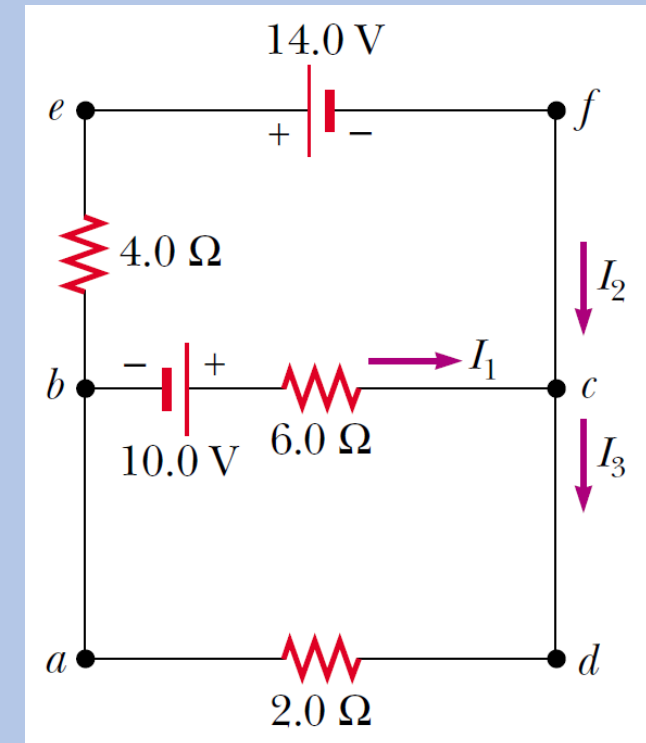
Conceptualize by noting that we cannot simplify the circuit by the rules of adding resistances in series and in parallel. (If the 10.0 V battery were taken away, we could reduce the remaining circuit with series and parallel combinations.) Thus, we categorize this problem as one in which we must use Kirchhoff's rules. To analyze the circuit, we arbitrarily choose the directions of the currents as labeled in the following figure. Applying Kirchhoff's junction rule to junction c gives:

$$I_1 + I_2 = I_3 \quad (1)$$

We now have one equation with three unknowns— I_1 , I_2 , and I_3 . There are three loops in the circuit— $abcda$, $befcb$, and $ae fda$. We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops $abcda$ and $befcb$ and traversing these loops clockwise, we obtain the expressions:

$$10 - 6I_1 - 2I_3 = 0 \quad (abcda) \quad (2)$$

$$-14 + 6I_1 - 10 - 4I_2 = 0 \quad (befcb) \quad (3)$$



A circuit containing different branches.

Example 27.07

Additional example

Find the currents I_1 , I_2 , and I_3 in the circuit shown in the following figure.

Note that in loop $befcb$ we obtain a positive value when traversing the $6.0\ \Omega$ resistor because our direction of travel is opposite the assumed direction of I_1 . Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives:

$$10 - 6I_1 - 2(I_1 + I_2) = 0$$

$$10 = 8I_1 + 2I_2 \quad (4)$$

Dividing each term in Equation (3) by 2 and rearranging gives:

$$-12 = -3I_1 + 2I_2 \quad (5)$$

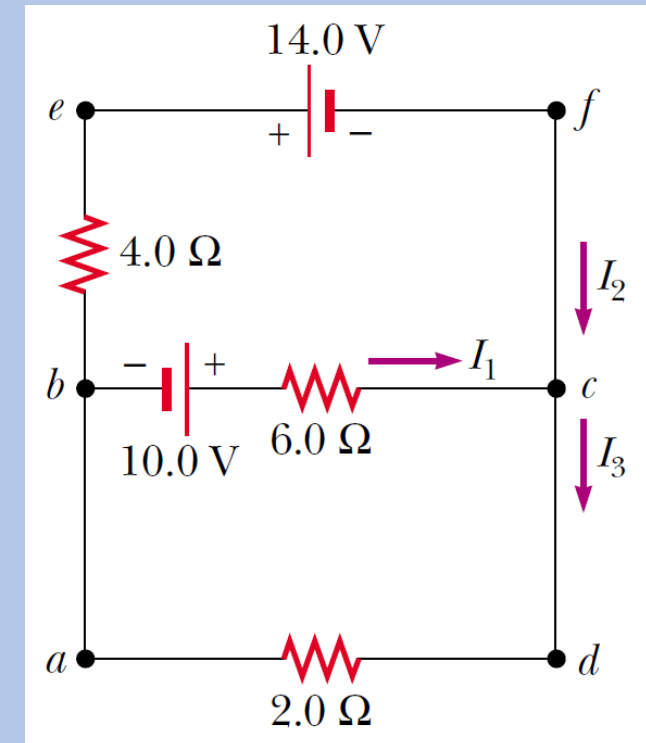
Subtracting Equation (5) from Equation (4) eliminates I_2 , giving:

$$22 = 11I_1$$

$$I_1 = 2\text{ A}$$

Using this value of I_1 in Equation (5) gives a value for I_2 :

$$2I_2 = 3I_1 - 120$$



A circuit containing different branches.

Example 27.07

Additional example

Find the currents I_1 , I_2 , and I_3 in the circuit shown in the following figure.

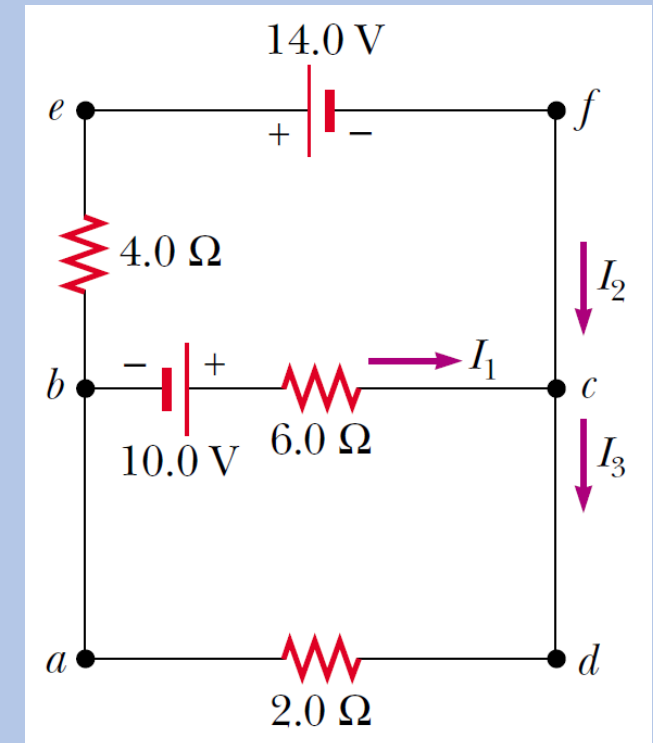
$$2I_2 = (3 \times 2) - 120 = -6 \text{ A}$$

$$I_2 = -3 \text{ A}$$

Finally,

$$I_3 = I_1 + I_2 = -1 \text{ A}$$

To finalize the problem, note that I_2 and I_3 are both negative. This indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct. What would have happened had we left the current directions as labeled in the following figure but traversed the loops in the opposite direction?



A circuit containing different branches.

Question 27.07

Using Kirchhoff's rules, (a) find the current in each resistor in the following figure. (b) Find the potential difference between points c and f . Which point is at the higher potential?

(a) Applying Kirchhoff's junction rule to junction c gives:

$$I_1 + I_2 = I_3 \quad (1)$$

The loops $abcfa$ and $fcdef$, traversed clockwise, gives:

$$70 - 60 + (3 \times 10^3)I_2 - (2 \times 10^3)I_1 = 0 \quad (abcfa)$$

$$I_1 = (5 \times 10^{-3}) + 1.5I_2 \quad (abcfa) \quad (2)$$

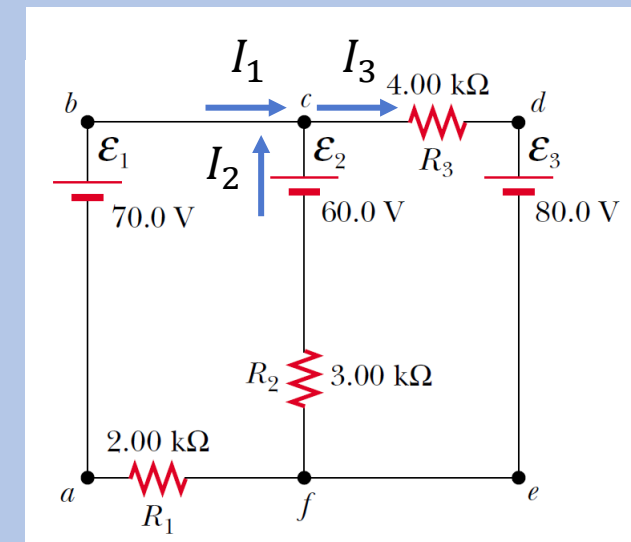
$$-(3 \times 10^3)I_2 + 60 - (4 \times 10^3)I_3 - 80 = 0 \quad (fcdef)$$

$$I_3 = -(5 \times 10^{-3}) - 0.75I_2 \quad (fcdef) \quad (3)$$

Substituting Equation (2) and (3) into Equation (1), gives:

$$(5 \times 10^{-3}) + 1.5I_2 + I_2 = -(5 \times 10^{-3}) - 0.75I_2$$

$$I_2 = -3.08 \times 10^{-3} \text{ A} = -3.08 \text{ mA}$$



Question 27.07

Using Kirchhoff's rules, (a) find the current in each resistor in the following figure. (b) Find the potential difference between points c and f . Which point is at the higher potential?

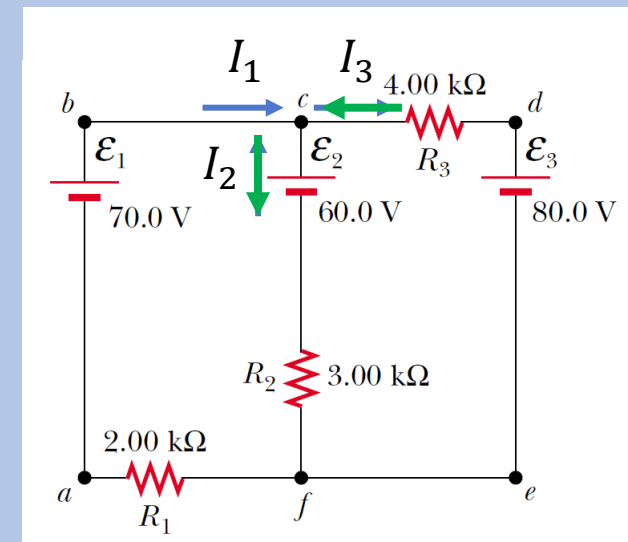
Using I_2 in Equations (2) and (1) gives:

$$I_1 = 0.38 \times 10^{-3} \text{ A} = 0.38 \text{ mA}$$

$$I_3 = -2.7 \times 10^{-3} \text{ A} = -2.7 \text{ mA}$$

$$(b) \Delta V_{cf} = -60 - (3.08 \times 10^{-3} \times 3 \times 10^3) = -69.24 \text{ V}$$

Point c is at higher potential.



Problem 27.35

(a) Under steady-state conditions, find the unknown currents I_1 , I_2 , and I_3 in the multiloop circuit shown in the following figure. (b) What is the charge on the capacitor?

- (a) First note that because the capacitor represents an open circuit, there is no current between g and b along path $ghab$ under steady-state conditions. Therefore, when the charges associated with I_1 reach point g , they all go toward point b through the 8.00 V battery; hence, $I_{gb} = I_1$. Labeling the currents as shown in the following figure at junction c , we obtain:

$$I_1 + I_2 = I_3 \quad (1)$$

The loops $defcd$ and $cfghbc$, traversed clockwise, gives:

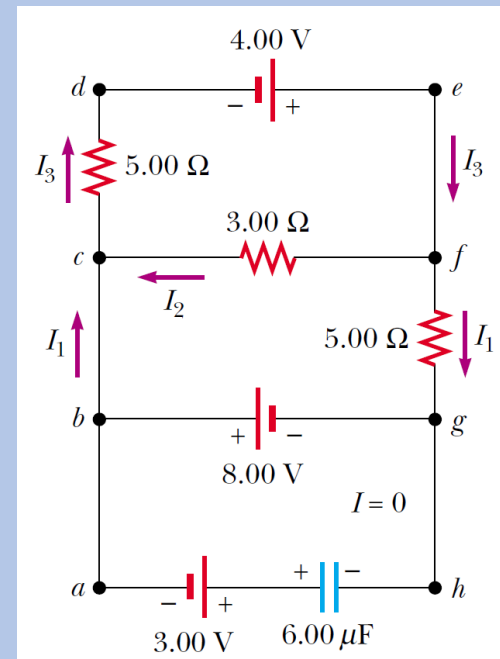
$$4 - 3I_2 - 5I_3 = 0 \quad (defcd)$$

$$I_3 = 0.8 - 0.6I_2 \quad (defcd) \quad (2)$$

$$3I_2 - 5I_1 + 8 = 0 \quad (cfghbc)$$

$$I_1 = 0.6I_2 + 1.6 \quad (cfghbc) \quad (3)$$

Substituting Equation (2) and (3) into Equation (1), gives:



A multiloop circuit. Kirchhoff's loop rule can be applied to any closed loop, including the one containing the capacitor.

Problem 27.35

(a) Under steady-state conditions, find the unknown currents I_1 , I_2 , and I_3 in the multiloop circuit shown in the following figure. (b) What is the charge on the capacitor?

$$0.6I_2 + 1.6 + I_2 = 0.8 - 0.6I_2$$

$$I_2 = -0.364 \text{ A}$$

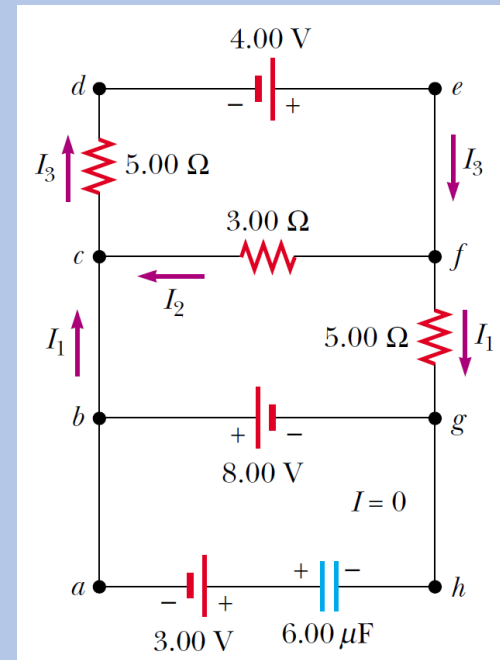
Because our value for I_2 is negative, we conclude that the direction of I_2 is from c to f in the 3.00Ω resistor. Despite this interpretation of the direction, however, we must continue to use this negative value for I_2 in subsequent calculations because our equations were established with our original choice of direction.

Using $I_2 = -0.364 \text{ A}$ in Equations (3) and (1) gives:

$$I_1 = 1.38 \text{ A}, \quad I_3 = 1.02 \text{ A}$$

- (b) We can apply Kirchhoff's loop rule to loop $bghab$ (or any other loop that contains the capacitor) to find the potential difference ΔV_{cap} across the capacitor. We use this potential difference in the loop equation without reference to a sign convention because the charge on the capacitor depends only on the magnitude of the potential difference.

Moving clockwise around this loop, we obtain:



A multiloop circuit. Kirchhoff's loop rule can be applied to any closed loop, including the one containing the capacitor.

Problem 27.35

(a) Under steady-state conditions, find the unknown currents I_1 , I_2 , and I_3 in the multiloop circuit shown in the following figure. (b) What is the charge on the capacitor?

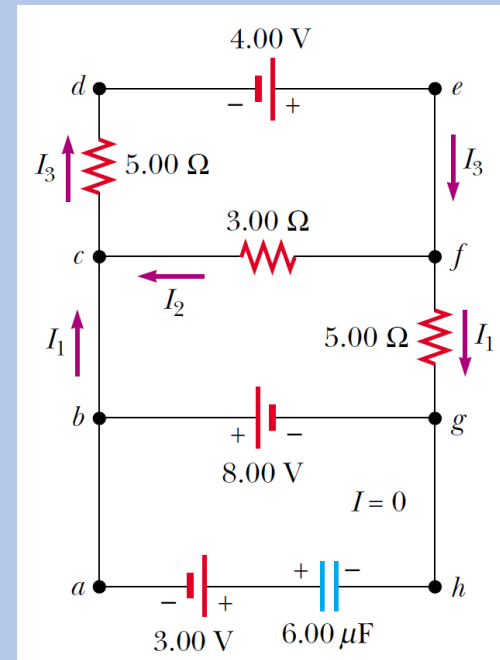
$$-8 + \Delta V_{cap} - 3 = 0$$

$$\Delta V_{cap} = 11 \text{ V}$$

Because $Q = C\Delta V_{cap}$, the charge on the capacitor is:

$$Q = 6 \times 11 = 66 \mu\text{C}$$

Why is the left side of the capacitor positively charged?



A multiloop circuit. Kirchhoff's loop rule can be applied to any closed loop, including the one containing the capacitor.

Question 27.08

The ammeter shown in the following figure reads 2.00 A. Find I_1 , I_2 , and $\mathcal{E}$.

$$15 - 7I_1 - (2 \times 5) = 0$$

$$5 = 7I_1$$

$$I_1 = 0.714 \text{ A}$$

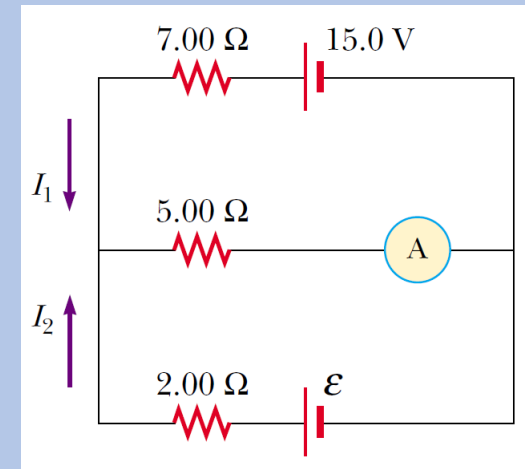
$$I_3 = I_1 + I_2 = 2 \text{ A}$$

$$0.714 + I_2 = 2 \text{ A}$$

$$I_2 = 1.29 \text{ A}$$

$$\mathcal{E} - (1.29 \times 2) - (2 \times 5) = 0$$

$$\mathcal{E} = 12.6 \text{ V}$$



Problem 27.17

Additional problem

Determine the current in each branch of the circuit shown in the following figure.

Applying Kirchhoff's junction rule to junction c gives:

$$I_1 + I_2 = I_3 \quad (1)$$

The loops $abcfa$ and $fcdef$, traversed clockwise, gives:

$$8I_1 - 5I_2 - I_2 - 4 = 0 \quad (abcfa)$$

$$I_1 = 0.75I_2 + 0.5 \quad (abcfa) \quad (2)$$

$$4 + I_2 + 5I_2 + 3I_3 + I_3 - 12 = 0 \quad (fcdef)$$

$$I_3 = 2 - 1.5I_2 \quad (fcdef) \quad (3)$$

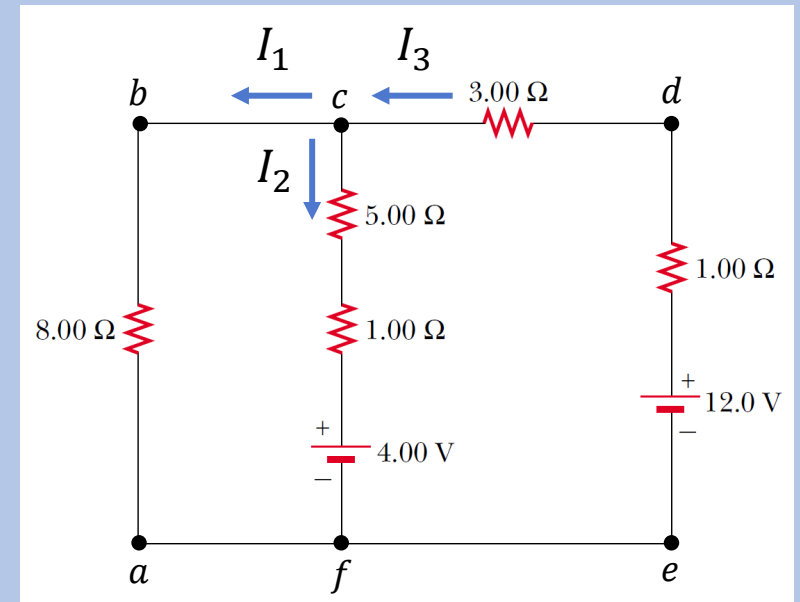
Substituting Equation (2) and (3) into Equation (1), gives:

$$0.75I_2 + 0.5 + I_2 = 2 - 1.5I_2$$

$$I_2 = 0.462 \text{ A} = 462 \text{ mA}$$

Using I_2 in Equations (2), (3), and (5) gives:

$$I_1 = 0.847 \text{ A} = 847 \text{ mA} \quad , \quad I_3 = 1.309 \text{ A}$$



Question 27.09

Additional question

In the circuit of the following figure, determine the current in each resistor and the voltage across the 200 Ω resistor.

Applying Kirchhoff's junction rule to junction d gives:

$$I_1 + I_2 + I_4 = I_3 \quad (1)$$

The loops $abcha$, $hcdgh$ and $gdefg$, traversed clockwise, gives:

$$-200I_1 - 40 + 80I_2 = 0 \quad (abcha)$$

$$I_1 = 0.4I_2 - 0.2 \quad (abcha) \quad (2)$$

$$-80I_2 + 40 + 360 - 20I_3 = 0 \quad (hcdgh)$$

$$I_3 = -4I_2 + 20 \quad (hcdgh) \quad (3)$$

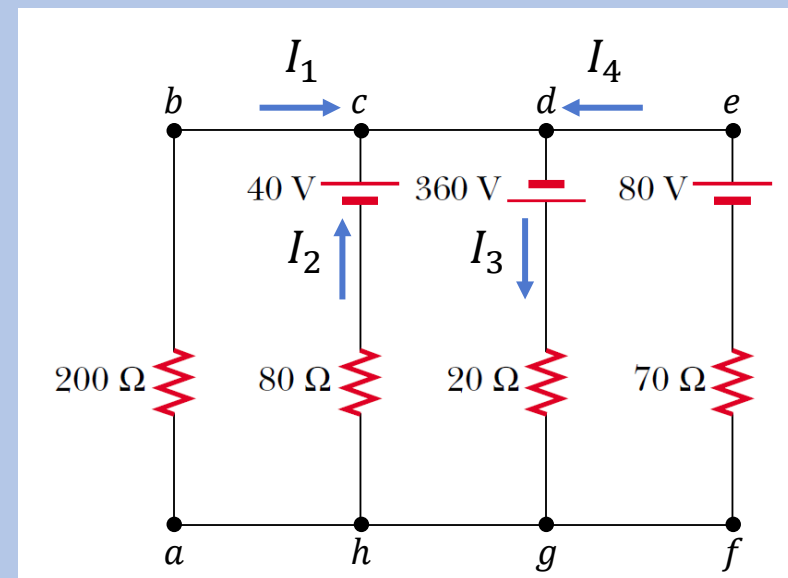
$$20I_3 - 360 - 80 + 70I_4 = 0 \quad (gdefg)$$

$$20I_3 - 440 + 70I_4 = 0 \quad (gdefg) \quad (4)$$

Substituting Equation (3) into Equation (4), gives:

$$-80I_2 + 70I_4 - 40 = 0$$

$$I_4 = 1.143I_2 + 0.571 \quad (5)$$



Question 27.09

Additional question

In the circuit of the following figure, determine the current in each resistor and the voltage across the 200 Ω resistor.

Substituting Equation (2), (3), and (5) into Equation (1), gives:

$$0.4I_2 - 0.2 + I_2 + 1.143I_2 + 0.571 = -4I_2 + 20$$

$$I_2 = 3 \text{ A} \quad (@ 80 \Omega)$$

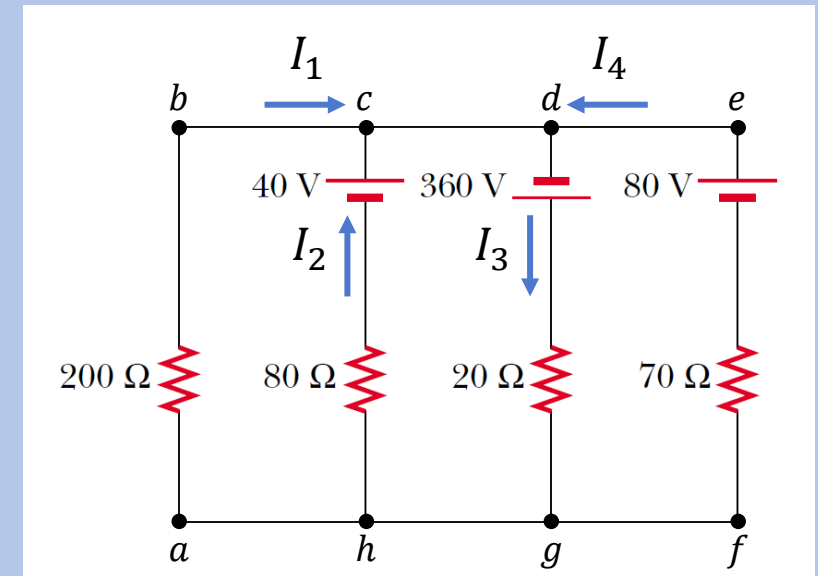
Using I_2 in Equations (2), (3), and (5) gives:

$$I_1 = 1 \text{ A} \quad (@ 200 \Omega)$$

$$I_3 = 8 \text{ A} \quad (@ 20 \Omega)$$

$$I_4 = 4 \text{ A} \quad (@ 70 \Omega)$$

$$\Delta V_{@200 \Omega} = 200I_1 = 200 \text{ V}$$



Question 27.10

Calculate the power delivered to each resistor shown in the following figure.

Applying Kirchhoff's junction rule to junction c to the second diagram gives:

$$I_1 = I_2 + I_3 \quad (1)$$

The loops $abcfa$ and $fcdef$, traversed clockwise, gives:

$$50 - 2I_1 - 2I_2 = 0 \quad (abcfa)$$

$$I_1 = 25 - I_2 \quad (abcfa) \quad (2)$$

$$2I_2 + 20 - 2I_3 = 0 \quad (fcdef)$$

$$I_3 = I_2 + 10 \quad (fcdef) \quad (3)$$

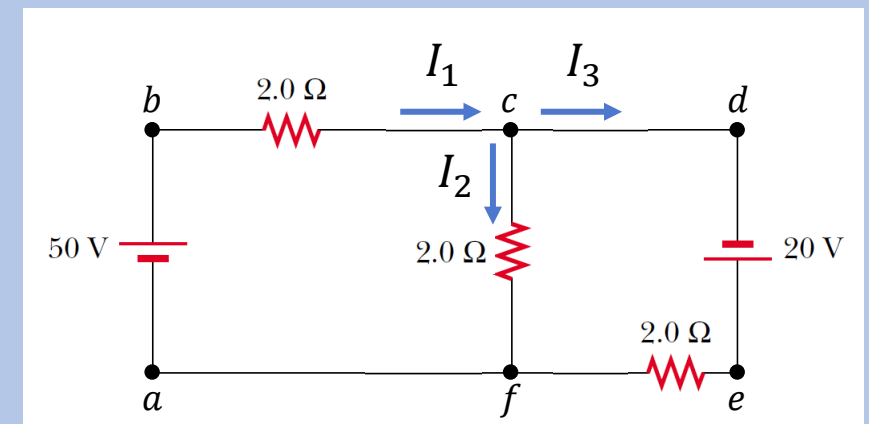
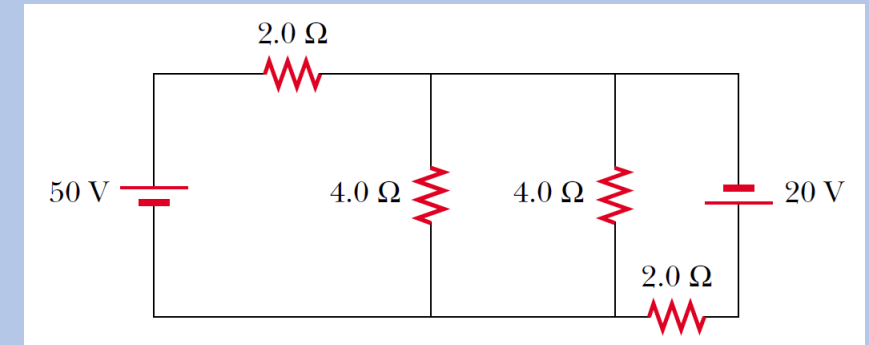
Substituting Equation (2) and (3) into Equation (1), gives:

$$25 - I_2 = I_2 + I_2 + 10$$

$$I_2 = 5 \text{ A}$$

Using I_2 in Equations (2), (3), and (5) gives:

$$I_1 = 20 \text{ A} , \quad I_3 = 15 \text{ A}$$



Question 27.10

Calculate the power delivered to each resistor shown in the following figure.

The powers delivered to the resistors are:

$$\mathcal{P}(@2\ \Omega, I_1) = 2I_1^2 = 2 \times (20)^2 = 800\ \text{W}$$

$$\mathcal{P}(@4\ \Omega, 4\ \Omega, I_2) = 4 \times \left(\frac{I_2}{2}\right)^2 = 4 \times \left(\frac{5}{2}\right)^2 = 25\ \text{W}$$

$$\mathcal{P}(@2\ \Omega, I_3) = 2I_3^2 = 2 \times (15)^2 = 450\ \text{W}$$

